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AN EQUATION $f(x) = kx$ POSSESSING NO SOLUTIONS

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ABSTRACT. An example is given of a nonlinear operation $f: C[0, 1] \to C[0, 1]$ having for any $k \in R$ no solutions of the equation $f(x) = kx$.

The main result of [1] states that if f is a continuous operator on a Banach space then $f(x) = kx$ has a solution when $|k|$ is sufficiently large. Here is a counterexample showing that the result is false.

Let $f: C[0, 1] \to C[0, 1]$ be defined by

$$(fx)(t) = (\max[t, |x(t) - x(0)|])^{1/2}.$$

Suppose $x \in C[0, 1]$ and $k \in R$ satisfy $f(x) = kx$. Obviously $x(0) = 0$. Now find $0 < t_n \to 0$ such that $|x(t_n)| \geq t_n$ or, if such a sequence does not exist, the sequence $0 < s_n \to 0$ such that $|x(s_n)| \leq s_n$. In the first case we get $|x(t_n)|^{1/2} = |k| \cdot |x(t_n)|$, otherwise $s_n^{1/2} = |k| \cdot |x(s_n)| \leq |k| s_n$ which both lead to a contradiction when $n \to \infty$.

REFERENCE

1. S. Venkateswaran, *The existence of a solution of $f(x) = kx$ for a continuous not necessarily linear operator*, Proc. Amer. Math. Soc. 36 (1972), 313–314. MR 46 #7997.

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