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THE REGULAR CONTINUOUS IMAGE OF A MINIMAL REGULAR SPACE IS NOT NECESSARILY MINIMAL REGULAR

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ABSTRACT. Herrlich has shown that the regular continuous image of a regular-closed space is regular-closed. In this paper, an example is given to show that Herrlich's result cannot be extended to a corresponding result for minimal regular spaces. Also, a modification of this example shows that a continuous function from a minimal regular space onto a regular space is not necessarily a closed function.

In the subsequent discussion, all regular spaces and compact spaces are assumed to be Hausdorff. Two of the fundamental properties of compact spaces are that the Hausdorff continuous image of a compact space is compact and its consequence that any continuous function from a compact space to a Hausdorff space is closed. Herrlich has shown [4] that the regular continuous image of a regular-closed space is regular-closed. In the following, we give an example of a minimal regular space whose regular continuous image is not minimal regular. We also give an example which shows that a continuous function from a minimal regular (and hence regular-closed) space to a regular space is not necessarily closed.

Definition [3]. A regular filter-base on a topological space is an open filter-base $\mathcal{F}$ such that for each $U \in \mathcal{F}$ there exists $V \in \mathcal{F}$ such that $\bar{V} \subseteq U$.

[2] and [3] contain standard working characterizations of regular-closed spaces and minimal regular spaces in terms of regular filter-bases.

Example 1. This is an example of a minimal regular space whose regular continuous image is not minimal regular. Let (Z, τ) be the noncompact minimal regular space constructed in [2] and let (T, σ) be the subspace of (Z, τ) given by $T = \{p\} \cup \bigcup \{Z_k | k \in N\}$. In [4] it is shown that (T, σ) is regular-closed, but not minimal regular since the filter-base $\mathcal{G} = \{G \subseteq T | G \in \sigma, p \in G, T - G \text{ is compact}\}$ is a regular filter-base on T with a unique adherent

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point p which does not converge to p on T. Now define the following function f from Z onto T:

$$f(z) = \begin{cases} z, & \text{if } z \in T, \\ p, & \text{if } z = q, \\ (-n+1, x, y), & \text{if } z = (n, x, y) \text{ and } n \leq 0. \end{cases}$$

Geometrically, the function f "reflects" the nonpositive half of the space (Z, τ) about the Z_1-sheet onto the space T, and f is a continuous function from Z onto T. This yields the desired example since (Z, τ) is minimal regular and (T, σ) is not.

Example 2. This is an example of a continuous function from a minimal regular space onto a regular space which is not a closed function. Let (Z, τ) and (T, σ) be as given in Example 1. Since (T, σ) is not minimal regular, there exists a strictly weaker regular topology σ' on T. Let i be the identity mapping of (T, σ) onto (T, σ'). Clearly i is continuous, but not closed. Now let $g = i \circ f$ where f is given in Example 1. Then g is a continuous function from the minimal regular space (Z, τ) onto the regular space (T, σ'). Since i is not closed, there exists a closed set F in (T, σ) which is not closed in (T, σ'). Then $g(f^{-1}(F)) = (i \circ f)(f^{-1}(F)) = F$. But $f^{-1}(F)$ is closed in (Z, τ) and F is not closed in (T, σ'). Hence, g is not a closed function and we have our example.

We conclude this paper with a question to which an affirmative answer would solve problem 14 in [3] proposed by Banaschewski in [1]. *If (X, τ) is a minimal regular space having the property that every continuous function from (X, τ) onto a regular space is closed, then is (X, τ) compact?*

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