ON THE SPACE OF PIECEWISE LINEAR HOMEOMORPHISMS OF A MANIFOLD

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ABSTRACT. Let M^n be a compact PL manifold, $n \neq 4$; if $n = 5$, suppose ∂M is empty. Let $H(M)$ be the space of homeomorphisms on M and $H^*(M)$ the elements of $H(M)$ which are isotopic to PL homeomorphisms. It is shown that the space of PL homeomorphisms, $PLH(M)$, has the finite dimensional compact absorption property in $H^*(M)$ and hence that $(H^*(M), PLH(M))$ is an (l_2, l_2^f) -manifold pair if and only if $H(M)$ is an l_2 -manifold. In particular, if M^2 is a 2-manifold, $(H(M^2), PLH(M^2))$ is an (l_2, l_2^f) -manifold pair.

1. Introduction. Let M be a compact piecewise linear (PL) manifold, possibly with boundary. We shall study the pair $(H(M), PLH(M))$, where $H(M)$ denotes the space of all homeomorphisms of M onto itself and $PLH(M)$ the subspace of all piecewise linear homeomorphisms. All function spaces will be assumed to have the compact-open topology.

For some years now there has been considerable interest in the question of whether $H(M)$ is an l_2 -manifold; i.e., a separable metric space which is locally homeomorphic to l_2 , the hilbert space of square-summable sequences. For an arbitrary compact manifold M , it is known that $H(M)$ is uniformly locally contractible (Chernavskii [6] and Edwards and Kirby [8]) and that $H(M) \times l_2$ is homeomorphic to $H(M)$ (Geoghegan [11]). By a theorem of Toruńczyk [20], $H(M)$ is an l_2 -manifold if and only if $H(M)$ is an ANR. When M is a 2-manifold, Luke and Mason [18] have shown that $H(M)$ is an ANR (hence an l_2 -manifold). But it is still unknown whether or not $H(M^n)$ is an ANR when $n > 2$.

In the PL case more is known. $PLH(M)$ is the countable union of finite-dimensional compacta (Geoghegan [12]), and is uniformly locally contractible (Edwards, see [13] and Gauld [10]). Haver [13] has shown that any such space is an ANR. Toruńczyk [20] has shown that any such ANR becomes an l_2^f -manifold when multiplied by l_2^f (where l_2^f denotes the subspace of l_2 consisting of those sequences having only finitely many nonzero entries). Hence $PLH(M) \times l_2^f$ is an l_2^f -manifold. Finally, Keesling and Wilson [16] have shown that $PLH(M) \times l_2^f$ is homeomorphic to $PLH(M)$. Hence $PLH(M)$ is an l_2^f -manifold.

In general it is not true that $PLH(M)$ is dense in $H(M)$. For example, Kirby and Siebenmann (see [17]) have shown that $H(S^2 \times S^3)$ has a component containing no PL homeomorphism. They have also shown that if the

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third cohomology group of M^n with $\mathbb{Z}_2$ -coefficients is trivial, and if $n \geq 6$, or $n = 5$ and ∂M is empty, $PLH(M^n)$ is dense in $H(M^n)$. In low dimensions, $PLH(M^n)$ is always dense in $H(M^n)$: this is obvious when $n = 1$; see Rado [19] when $n = 2$; see Bing [4] when $n = 3$.

For our purposes we can avoid reference to the difficult work of Kirby and Siebenmann. Using earlier work of Connell [7] we can prove

THEOREM 1. *Let M^n be a compact PL manifold, $n \neq 4$; if $n = 5$ suppose ∂M is empty. Then the closure of $PLH(M)$ in $H(M)$ is the union of components of $H(M)$.*

Let $H^*(M)$ be the subset of $H(M)$ consisting of those homeomorphisms which are isotopic to PL homeomorphisms. The above remarks show that $H^*(M)$ is often equal to $H(M)$. Theorem 1 says that [with certain dimension restrictions] $PLH(M^n)$ is always dense in $H^*(M^n)$.

A pair of spaces (X, X') is an (l_2, l_2^f) -manifold pair if X is an l_2 -manifold, and if there exist an open cover $\mathcal{U}$ of X and open embeddings $\{f_U: U \rightarrow l_2 \mid U \in \mathcal{U}\}$ such that for each U , $f_U(U \cap X') = f_U(U) \cap l_2^f$. In other words X' sits in X , locally, as l_2^f sits in l_2 .

THEOREM 2. *Let M^n be a compact PL manifold, $n \neq 4$; if $n = 5$, suppose ∂M is empty. Then $(H^*(M^n), PLH(M^n))$ is an (l_2, l_2^f) -manifold pair if and only if $H(M)$ is an l_2 -manifold.*

Theorem 2 is proved by showing that $PLH(M^n)$ has the “finite-dimensional compact absorption property” in $H^*(M^n)$: see §3 for the definition. This theorem, when combined with our opening remarks, yields

COROLLARY 1. *If M^2 is a compact PL 2-manifold, $(H(M^2), PLH(M^2))$ is an (l_2, l_2^f) -manifold pair.*

Theorem 1 combined with the fact that each of $PLH(M)$ and $H(M)$ is uniformly locally contractible immediately gives:

COROLLARY 2. *Let M^n be as in Theorem 2. Then the inclusion $PLH(M^n) \rightarrow H^*(M^n)$ is a weak homotopy equivalence (a homotopy equivalence if $n = 2$).*

COROLLARY 3. *The inclusion of the identity component of $PLH(M^n)$ into the identity component of $H(M^n)$ is a weak homotopy equivalence (a homotopy equivalence if $n = 2$).*

We will let $H_0(I^n)$ denote the set of elements of $H(I^n)$ which equal the identity when restricted to the boundary of I^n and $PLH_0(I^n) = PLH(I^n) \cap H_0(I^n)$. If X is any space, let 1_X denote the identity homeomorphism on X . If $f \in H(M)$, let $N_\epsilon(f) = \{h \in H(M) \mid d(h, f) < \epsilon\}$ where d is a metric on M .

2. Proof of Theorem 1. As explained in §1, the low-dimensional cases are well known.

LEMMA 1. *If $n > 4$, $PLH_0(I^n)$ is dense in $H_0(I^n)$.*

PROOF. Let $H_0'(I^n)$ be the group of homeomorphisms of I^n which fix a neighborhood of ∂I^n . More precisely:

$$H_0'(I^n) = \{h \in H_0(I^n) \mid \text{for some } 0 < a < 1,$$

$$h(x) = x \text{ whenever } d(x, 0) \geq a\}.$$

By [9, Theorem 3], $H_0'(I^n)$ is a simple subgroup of $H_0(I^n)$. $H_0'(I^n)$ is clearly dense in $H_0(I^n)$. Consider

$$H_0''(I^n) = \{h \in H_0'(I^n) \mid \text{If } T \text{ is a PL structure on } I^n \text{ and if } \epsilon > 0,$$

$$\text{there exists } f \in H_0'(I^n) \text{ such that}$$

$$f \text{ is PL relative to } T \text{ and } d(f, h) < \epsilon\}.$$

CLAIM (following Connell). $H_0''(I^n)$ is normal in $H_0'(I^n)$.

PROOF OF CLAIM. Let $h \in H_0''(I^n)$ and $g \in H_0'(I^n)$. We must show $g^{-1}hg \in H_0''(I^n)$. Let T be a PL structure on I^n and let $\epsilon > 0$ be given. There exists $\delta > 0$ such that for all $x, y \in I^n$, $d(x, y) < \delta$ implies $d(g^{-1}(x), g^{-1}(y)) < \epsilon$. Let T_1 be the PL structure on I^n which is the image of T under g . Since $h \in H_0''(I^n)$ there exists $f \in H_0'(I^n)$ which is PL with respect to T_1 and satisfies $d(h, f) < \delta$. Thus $d(g^{-1}hg, g^{-1}fg) < \epsilon$. Now $g^{-1}fg$ is PL with respect to T . It follows that $g^{-1}hg \in H_0''(I^n)$; the Claim is proved.

Since $H_0'(I^n)$ is dense and simple, it only remains to show that $H_0''(I^n)$ contains some $h_0 \neq 1_{I^n}$. When $n > 6$, Connell shows on p. 331 of [7] that any nontrivial symmetric radial expansion of I^n which fixes a neighborhood of ∂I^n can serve as h_0 . The restriction $n > 6$ arises from the radial engulfing lemma used in [7]. But in [3], Bing proves a stronger engulfing lemma which makes Connell's proof valid for $n > 4$; see [3, p. 3] and [7, p. 337].

PROOF OF THEOREM 1. Let H be a component of $H(M)$. We show that $H \cap PLH(M)$ is either dense in H or is empty.

We first show the PL homeomorphisms are dense in a neighborhood of 1_M : i.e., that there is a $\delta > 0$ such that whenever $g \in N_\delta(1_M)$ and $\epsilon > 0$ are given, there exists $h \in PLH(M) \cap N_\epsilon(g)$.

Choose an open cover $\{B_1, \dots, B_p\}$ of M so that for each i , $\bar{B}_i$ is a PL n -ball and so that $D_i = \bar{B}_i \cap \partial M$ is either empty or a PL $(n-1)$ -ball. In [8, p. 19] it is shown that there is a $\delta > 0$ so that each $g \in N_\delta(1_M)$ can be written as the composition $g = g_p g_{p-1} \cdots g_1$ of p homeomorphisms such that each g_i is supported by B_i . Since each g_i is uniformly continuous, there exist positive numbers $\eta_1, \dots, \eta_p$ such that if for each i the homeomorphism h_i satisfies $d(h_i, g_i) < \eta_i$, then $d(h_p h_{p-1} \cdots h_1, g_p g_{p-1} \cdots g_1) < \epsilon$.

If $\bar{B}_i \cap \partial M = \emptyset$, $g_i|_{\bar{B}_i}$ lies in $H_0(\bar{B}_i)$ and, by the previous lemma, there exists a PL homeomorphism $\tilde{h}_i \in H_0(\bar{B}_i)$ such that $d(\tilde{h}_i, g_i|_{\bar{B}_i}) < \eta_i$. Extend $\tilde{h}_i$ by the identity to form $h_i \in PLH(M)$ and note that $d(h_i, g_i) < \eta_i$. If $\bar{B}_i \cap \partial M = D_i \neq \emptyset$ (and is therefore a PL $(n-1)$ -ball), g_i maps D_i homeomorphically onto itself and fixes ∂D_i . By the previous lemma there is a PL homeomorphism $f_i \in H_0(D_i)$ approximating $g_i|_{D_i}$. Extending f_i by the identity we get a PL homeomorphism $\tilde{f}_i: \partial \bar{B}_i \rightarrow \partial \bar{B}_i$ which approximates $g_i|_{\partial \bar{B}_i}$. Thus $\tilde{f}_i g_i^{-1}|_{\partial \bar{B}_i}$ approximates $1_{\partial \bar{B}_i}$. Coning at a point in the interior of B_i , we obtain a homeomorphism F_i of B_i which agrees with $\tilde{f}_i g_i^{-1}$ on $\partial \bar{B}_i$ and which approximates 1_{B_i} . Let $g_i' = F_i g_i: \bar{B}_i \rightarrow \bar{B}_i$. Note that g_i' approximates g_i and is PL on $\partial \bar{B}_i$. Omitting epsilons, we may say that $d(g_i', g_i|_{B_i}) < \eta_i/2$. Note

that g_i' fixes $\partial \bar{B}_i \setminus \text{int } D_i$. By coning, construct a PL homeomorphism $g_i'' : \bar{B}_i \rightarrow \bar{B}_i$ which extends $g_i' | \partial B_i$. Then $g_i'(g_i'')^{-1} \in H_\partial(B_i)$, so there exists $\tilde{g}_i \in PLH_\partial(B_i)$ such that $d(\tilde{g}_i, g_i'(g_i'')^{-1}) < \eta_i/2$. Extend $\tilde{g}_i g_i''$ by the identity to $h_i \in PLH(M)$ and note that $d(h_i, g_i) < \eta_i$. The required h is $h_p \cdots h_1$.

Now let H be a component of $H(M)$ such that $H \cap PLH(M) \neq \emptyset$. Let $A = \text{cl}_H(H \cap PLH(M))$. We will show that A is open in H , from which it will follow that $A = H$. Let $f \in A$. Let δ be as in the first part of the proof, and assume without loss of generality that $N_{2\delta}(f) \subset H$. The local connectedness of $H(M)$ is used here. Let $g_0 \in N_\delta(f)$. We must show that $g_0 \in A$. Let $\eta > 0$ be given: we may assume $\eta \leq \delta$. Since $d(g_0 f^{-1}, 1_M) < \delta$, there exists $h \in PLH(M)$ such that $d(h, g_0 f^{-1}) < \eta/2$ and hence $d(hf, g_0) < \eta/2$. Since h is uniformly continuous, there exists $\gamma > 0$ such that $d(h(x), h(y)) < \eta/2$ whenever $d(x, y) < \gamma$. Choose $f' \in PLH(M)$ such that $d(f', f) < \gamma$. Then $hf' \in PLH(M)$ and $d(hf', g_0) \leq d(hf', hf) + d(hf, g_0) < \eta$. Therefore $hf' \in N_\eta(g_0) \cap H$. Since hf' is PL, $g_0 \in A$.

3. Proof of Theorem 2. A subset X' of a metric space X is said to have the *finite-dimensional compact absorption property* (f.d. cap) if $X' = \bigcup_{n=1}^\infty A_n$ where (i) each A_n is finite dimensional and compact, and (ii) given a finite-dimensional compactum A , a closed subset of B of A , an embedding $f: A \rightarrow X$ such that $f(B) \subset X'$, and a number $\epsilon > 0$, there exists an embedding $h: A \rightarrow X$ such that $d(f, h) < \epsilon$, $h(A) \subset X'$ and $h(b) = f(b)$ whenever $b \in B$.

This property characterizes (l_2, l_2^f) -manifold pairs: precisely

PROPOSITION 1 (WEST [21]). *Let (X, X') be a pair of metric spaces such that X' has the f.d. cap in X . Then (X, X') is an (l_2, l_2^f) -manifold pair if and only if X is an l_2 -manifold.*

Where there is no confusion, we will not distinguish between a complex P , and its underlying point set, $|P|$. Let $\overline{PLH}(M)$ be the closure of $PLH(M)$ in $H(M)$.

LEMMA 2. *Let P be a finite complex and B a closed subset of P . Suppose $f: P \rightarrow \overline{PLH}(M)$ is a map such that $f(B) \subset PLH(M)$ and $f(P \setminus B) \cap f(B) = \emptyset$. Then given $\epsilon > 0$, there is a map $g: P \rightarrow PLH(M)$ such that $d(f, g) < \epsilon$, $g|_B = f|_B$ and $g(P \setminus B) \cap g(B) = \emptyset$.*

PROOF. Let $n = \dim P$. Whenever W is a closed subset of $P \setminus B$ let

$$\eta_W = 1/4 \min(\epsilon, d(f(W), f(B))).$$

Using the fact that $PLH(M)$ is uniformly locally contractible, form a triangulation T of $P \setminus B$ satisfying:

(a) for each simplex σ of T there is a sequence $0 < \delta_{\sigma,0} < \delta_{\sigma,1} < \cdots < \delta_{\sigma,n} \leq \eta_\sigma$ such that any subset of $PLH(M)$ of diameter $< 3\delta_{\sigma,i}$ is contractible in a subset of $PLH(M)$ of diameter $< \delta_{\sigma,i+1}$, $0 \leq i \leq n-1$; and

(b) $\text{diam } f(\sigma) < \delta_{\sigma,0}$. (First choose a triangulation $\tilde{T}$, next choose the $\delta_{\sigma,i}$'s with reference to $\tilde{T}$ to satisfy (a); then subdivide $\tilde{T}$ to get T satisfying (b).)

Define $g: T^0 \rightarrow PLH(M)$ as follows. If v is a vertex of T^0 , let $g(v)$ be any

point of $PLH(M)$ such that $d(g(v), f(v)) < \delta_{\sigma,0}$ for each of the (finite number of) simplexes σ of T which contains v as a vertex.

We next define $g: T^1 \rightarrow PLH(M)$. Let τ be a 1-simplex of T^1 and let $\sigma_0 < T$ be a simplex of which τ is a face. Then $\text{diam} f(\tau) \leq \text{diam} f(\sigma_0)$. Hence $\text{diam} g(\partial\tau) < 3\delta_{\sigma_0,0}$. We can therefore define

$$g|_{\tau}: \tau \rightarrow PLH(M)$$

extending $g|_{\partial\tau}$ in such a way that $\text{diam} g(\tau) < \delta_{\sigma,1}$ for each of the (finite number of) simplexes σ of T which contains τ as a face.

Assume, inductively, that we have defined $g|_{T^j}: T^j \rightarrow PLH(M)$, such that if τ is a j -simplex of T , then $\text{diam} g(\tau) < \delta_{\sigma,j}$ for each of the (finite number of) simplexes σ of T which contains τ as a face.

Let τ be a $(j+1)$ -simplex of T . Note that $\text{diam} g(\partial\tau) < 3\delta_{\sigma,j}$ for each simplex σ of T which contains τ as a face. Therefore $g|_{\partial\tau}$ can be extended to τ in such a way that $\text{diam} g(\tau) < \delta_{\sigma,j+1}$ for each simplex σ of T of which τ is a face.

Thus, by induction we define a continuous map $g: T \rightarrow PLH(M)$ so that if τ is a simplex of T , $\text{diam} g(\tau) < \eta_\tau$. Finally define g on B to agree with f .

We now show that $d(f, g) < \epsilon$. If $p \in B$, $d(f(p), g(p)) = 0$. If $p \in P \setminus B$ let τ be a simplex of T containing p : $d(f(p), g(p)) \leq 3\eta_\tau < \epsilon$.

To show that $g(P \setminus B) \cap g(B) = \emptyset$, we observe that if $p \in P \setminus B$ and p lies in the simplex τ of T ,

$$\begin{aligned} d(g(p), g(B)) &\geq d(f(p), f(B)) - d(f(p), g(p)) \\ &\geq d(f(p), f(B)) - 3\eta_\tau > 0. \end{aligned}$$

Finally we show that g is continuous on B . Let $g \in B$ and $\eta > 0$ be given. Choose $\delta > 0$ so that if $d(p, q) < \delta$, $d(f(p), f(q)) < \eta/2$. Assume $p \in P \setminus B$. Then

$$\begin{aligned} d(g(p), g(q)) &\leq d(g(p), f(p)) + d(f(p), f(q)) \\ &\leq d(f(p), f(B)) + d(f(p), f(q)) \leq 2d(f(p), f(q)) < \eta. \end{aligned}$$

Let s denote the countable infinite product of open unit intervals and $s_f = \{\{x_i\} \in s \mid \text{for at most finitely many } i, x_i \neq 0\}$ with metric $d(x, y) = \sum_i \frac{1}{2} |x_i - y_i|$.

LEMMA 3. *Let P be a finite complex, B a closed subset of P , V an open subset of s_f . Let $g: P \rightarrow V$ be a continuous map. Then given $\epsilon > 0$, there is a map $h: P \rightarrow V$ such that $h|_B = g|_B$, $d(g, h) < \epsilon$, $h(B) \cap h(P \setminus B) = \emptyset$ and $h|_{P \setminus B}$ is injective.*

PROOF. This is standard infinite-dimensional topology: we sketch the proof, leaving epsilonics to the reader. Let $p_m: s_f \rightarrow s_f$ be the projection onto the first m coordinates. If m is large, g is homotopic in V to $p_m g$ by a small homotopy. If m is large enough, $p_m g$ is homotopic in V to an embedding $h': P \rightarrow V$ by a small homotopy. $h' g^{-1}|_{g(B)}$ is a homeomorphism of $g(B)$ onto $h'(B)$ which is homotopic to the identity by a small homotopy. By Theorem 2.25 of [5], there

is a homeomorphism k of s_f close to the identity, extending $h'g^{-1}|g(B)$. The required h is $k^{-1}h': P \rightarrow V$.

PROOF OF THEOREM 2. By Theorem 1, $H^*(M)$ is an l_2 -manifold if and only if $H(M)$ is. By Proposition 1 it is enough to show that $PLH(M)$ has the f.d. cap in $H^*(M)$. By [12, Theorem 1.9], $PLH(M)$ is the countable union of finite-dimensional compacta. We must verify (ii) the f.d. cap.

Suppose we are given a finite-dimensional compactum A , a compact subset B , an embedding $f: A \rightarrow H^*(M)$ such that $f(B) \subset PLH(M)$, and $\epsilon > 0$. By Theorem 1, $\overline{PLH}(M) = H^*(M)$. Since $H^*(M)$ is locally contractible, f extends to a map $f': P \rightarrow H^*(M)$ where P is a finite complex (see [15, p.150]). As explained in §1, $PLH(M)$ is an l_2^f -manifold. By Theorem 7 of [14], $PLH(M)$ is homeomorphic to an open subset of l_2^f , and hence to an open subset of s_f since l_2^f and s_f are homeomorphic [1], [2]. Thus, Lemmas 2 and 3 give a map $h: P \rightarrow PLH(M)$ with $d(f', h) < \epsilon$, $h|B' = f'|B'$ where $B' = f'^{-1}(f(B))$, $h(B') \cap h(P \setminus B') = \emptyset$ and $h|P \setminus B'$ is injective. $h|A$ is the desired embedding.

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