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ON THE TOPOLOGICAL COMPLETION

HOWARD CURZER AND ANTHONY W. HAGER[1]

ABSTRACT. Let X be a Tychonoff space. As is well known, the points of the Stone-Čech compactification βX "are" the zero-set ultrafilters of X, and the points of the Hewitt real-compactification υX are the zero-set ultrafilters which are closed under countable intersection. It is shown here that a zero-set ultrafilter is a point of the Dieudonné topological completion δX iff the family of complementary cozero sets is σ-discretely, or locally finitely, additive. From this follows a characterization of those dense embeddings $X \subset Y$ such that each continuous metric space-valued function on X extends over Y, and a somewhat novel proof of the Katětov-Shirota Theorem.

All spaces shall be Tychonoff.

It is most convenient to view the class of topologically complete spaces as the class $\mathfrak{R}(\mathfrak{M})$ of closed subspaces of products from the class $\mathfrak{M}$ of metrizable spaces, that is, as the epireflective hull of $\mathfrak{M}$. (Dieudonné showed that a Tychonoff space X has a compatible complete uniformity iff X admits an embedding with closed range into a product of metrizable spaces [**D**].) The topological completion δX of a Tychonoff space X is the epireflection of X into $\mathfrak{R}(\mathfrak{M})$, that is, δX is the essentially unique topologically complete space containing X densely such that each continuous map $f\colon X \to Z$ $(Z \in \mathfrak{R}(\mathfrak{M}))$, admits a continuous extension $\delta f\colon \delta X \to Z$. This universal mapping property is implied by the weaker one for maps into spaces in $\mathfrak{M}$, by the standard technique used to show for βX that the universal mapping property for maps to $[0, 1]$ implies the property for maps to compact spaces. See, e.g., [**W**]. We shall use this fact below.

δX may be constructed as the closure of a suitable homeomorph of X in a large product of metrizable spaces, similar to the common construction of βX (e.g., [**W**]). The following is a more useful construction for our purpose. It depends on knowledge of βX.

1. LEMMA. *$\delta X = \bigcap\{(\beta f)^{-1}(M) \mid f\colon X \to M$ continuous, $M \in \mathfrak{M}\}$. (Here $\beta f\colon \beta X \to \beta M$ is the extension over the Stone-Čech compactifications.)*

We sketch a proof of 1. Let $Y = \bigcap_M\{(\beta f)^{-1}(M)\}$. Clearly, a continuous map $f\colon X \to M$ has the extension $\beta f|Y\colon Y \to M$, so it suffices to show that $Y \in \mathfrak{R}(\mathfrak{M})$. Since $\mathfrak{R}(\mathfrak{M})$ is productive and closed-hereditary, it is closed under intersection (seen by realizing an intersection as a diagonal in a product), so it

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suffices that each $(\beta f)^{-1}(M) \in \mathfrak{R}(\mathfrak{M})$. Now $\mathfrak{R}(\mathfrak{M})$ is closed-hereditary, and $A \times K \in \mathfrak{R}(\mathfrak{M})$ if $A \in \mathfrak{R}(\mathfrak{M})$ and K is compact–since $K \in \mathfrak{R}(\mathfrak{M})$; this implies that $\mathfrak{R}(\mathfrak{M})$ is closed under perfect pre-images **[HS]**. (A continuous map $g: A \to B$ is called perfect if $\beta g(\beta A - A) \subset \beta B - B$.) But clearly, each $\beta f|(\beta f)^{-1}(M)$ is perfect.

(Something close to 1 appears in **[F]**, from which we have borrowed the proof. Also see **[H]**.)

It might be noted that one can view the class of realcompact spaces as the epireflective hull $\mathfrak{R}(\mathbf{R})$ of the real line R (i.e., closed subspaces of powers of R), and then construct the epireflection vX–the Hewitt realcompactification–just as above:

$$vX = \bigcap\{(\beta f)^{-1}(R) | f \in C(X)\},$$

$C(X)$ being the ring of real-valued continuous functions.

In any event, $X \subset \delta X \subset vX \subset \beta X$.

Recall (say from **[GJ]**) that the points of βX and the z-ultrafilters on X (i.e., ultrafilters in the family of zero-sets of functions in $C(X)$) are associated one-to-one by $p \leftrightarrow \mathfrak{F}_p = \{Z | Z$ is a zero-set and $p \in \overline{Z}\}$ (the closure in βX); that $p \in vX$ iff $\mathfrak{F}_p$ is closed under countable intersection; that for $p \in vX$ and Z a zero-set, $Z \in \mathfrak{F}_p$ iff whenever $Z = Zf$, $(f \in C(X))$, then $\beta f(p) = 0$.

If $\mathfrak{F}$ is a z-ultrafilter on X, let $\operatorname{co}\mathfrak{F} = \{X - Z | Z \in \mathfrak{F}\}$. To say that $\mathfrak{F}$ is closed under countable intersection is to say that $\operatorname{co}\mathfrak{F}$ is closed under countable union, or, as we shall say, σ-additive. We introduce the stronger addition property, characterizing the points of δX.

A family $\mathfrak{D}$ of subsets of X will be called discrete if there is a continuous pseudometric d on X and $\varepsilon > 0$ such that if $A, B \in \mathfrak{D}$ with $A \neq B$, then $d(A, B) \geqslant \varepsilon$. A family is σ-discrete if it is the union of countably many discrete subfamilies.

Finally, a family $\mathcal{G}$ of subsets of X is discretely (or σ-discretely, or locally finitely) additive if $\bigcup \mathfrak{D} \in \mathcal{G}$ whenever $\mathfrak{D} \subset \mathcal{G}$ and $\mathfrak{D}$ is discrete (or σ-discrete, or locally finite).

2. THEOREM. *Let $p \in \beta X$. The following are equivalent.*

(a) $p \in \delta X$;

(b) co $\mathfrak{F}_p$ *is σ-discretely additive*;

(c) co $\mathfrak{F}_p$ *is locally finitely additive.*

The proof will use the association between continuous pseudometrics d on X and continuous functions f from X to metric spaces M; given d, $\langle M, \rho\rangle$ is the "metric identification" of $\langle X, d\rangle$ and f is the natural projection; given $f: X \to \langle M, \rho\rangle$, set $d(x,y) = \rho(f(x), f(y))$. It follows that each d possesses a continuous pseudometric expression δd over δX; since $d(A, B) \geqslant \varepsilon$ implies $\delta d(A, B) \geqslant \varepsilon$, a discrete (or σ-discrete) family in X is discrete (or σ-discrete) in δX. And, if $g: X \to Y$ is continuous, and $\mathfrak{D}$ is discrete (or σ-discrete) in Y, then $g^{-1}(\mathfrak{D})$ is discrete (or σ-discrete) in X.

Note this also: if $\mathfrak{D}$ is a discrete family of subsets of X, then each point of X has a neighborhood meeting at most one member of $\mathfrak{D}$. (If $d(D, E) \geqslant \varepsilon$ for $D \neq E$ in $\mathfrak{D}$, then $\{x | d(x,p) < \varepsilon/2\}$ is such a neighborhood of p.) Thus, if $\mathfrak{D}$ consists of cozero sets, say $\mathfrak{D} = \{\operatorname{coz} f_D | D \in \mathfrak{D}\}$, then $f = \sum \{f_D | D \in \mathfrak{D}\} \in C(X)$, and $\operatorname{coz} f = \bigcup \mathfrak{D}$.

PROOF OF 2. (a) *implies* (b): Let $p \in \delta X$. Since $\delta X \subset vX$, co$\mathfrak{F}_p$ is σ-additive, so we must show that co$\mathfrak{F}_p$ is discretely additive. Let $\mathfrak{D} \subset$ co$\mathfrak{F}_p$, with $\mathfrak{D}$ discrete. For $D \in \mathfrak{D}$, choose $f_D \in C(X)$ with $D = \operatorname{coz} f_D$; thus $Zf_D \in \mathfrak{F}_p$ and $\beta f_D(p) = 0$. Set $f = \sum \{f_D | D \in \mathfrak{D}\}$; as noted above, $f \in C(X)$ and $\operatorname{coz} f = \bigcup \mathfrak{D}$. We show that $\beta f(p) = 0$, i.e., that $\operatorname{coz} f \in$ co$\mathfrak{F}_p$.

As noted above, $\{\operatorname{coz} \delta f_D | D \in \mathfrak{D}\}$ is discrete in δX, so $\sum \{\delta f_D | D \in \mathfrak{D}\}$ is well defined and continuous. By uniqueness of extension, $\sum_D \{\delta f_D\} = \delta f$. Thus, $\beta f(p) = \delta f(p) = \sum_D (\delta f_D)(p)$. This last is 0, since for each D, $\delta f_D(p) = \beta f_D(p) = 0$.

(We have used the fact that for $p \in vX$, and Z a zero-set, in order that $Z \in \mathfrak{F}_p$, it is enough to find f with $Zf = Z$ and $\beta f(p) = 0$.)

(a) *implies* (c): It is possible to give an argument similar to the above, but the following is interesting.

Let $p \in \delta X$, and let $\mathfrak{D} = \{\operatorname{coz} f_D | D \in \mathfrak{D}\}$ be a locally finite subfamily of co$\mathfrak{F}_p$. As is well known and easily verified, $d(x,y) \equiv \sum_D |f_D(x) - f_D(y)|$ defines a continuous pseudometric on X; and d has a continuous pseudometric extension δd over δX. Let $Z_n = X \cap \{x \in \delta X | \delta d(x,p) \leqslant 1/n\}$. Any set closed in the δd-topology is a zero-set of δX, and so Z_n is a zero-set of δX. Clearly, $p \in \overline{Z}_n$, so that $Z_n \in \mathfrak{F}_p$. Since $p \in vX$, $Z \equiv \bigcap_n Z_n \in \mathfrak{F}_p$. Evidently, if $q \in Z$, then $\delta d(q,p) = 0$. Finally, $Z \subset \bigcap_D Zf_D$ (so $\bigcap_D Zf_D \in \mathfrak{F}_p$): for if $q \notin Zf_D, |f_D(q)| > 0$; since $d(x,y) \geqslant |f_D(x) - f_D(y)|$ for each x,y, it follows that

$$\delta d(q,p) \geqslant |\delta f_D(q) - \delta f_D(p)| = |\delta f_D(q)| > 0.$$

(c) *or* (b) *implies* (a): Essentially the same proof works in either case. Suppose that $p \notin \delta X$. By 1, choose continuous $f\colon X \to M$, M metrizable, with $\beta f(p) \notin M$. So f fails to extend continuously to p with values in M. Thus, fixing a metric ρ on M, the oscillation of f at p is nonzero, say $\geqslant \varepsilon$. By A. H. Stone's Theorem **[St]** (or see **[W]**), there is an open cover $\mathfrak{U}$ of $f(M)$ refining the collection of $\varepsilon/4$-spheres, which is σ-discrete with respect to ρ (or locally finite).

Thus $f^{-1}(\mathfrak{U})$ is σ-discrete in X, with respect to $d(x,y) = \rho(f(x),f(y))$ (or locally finite). Now $\mathfrak{U}$ consists of cozero-sets (because any open set in a metrizable space is cozero), and so does $f^{-1}(\mathfrak{U})$ (because $f^{-1}(\operatorname{coz} g) = \operatorname{coz}(g \circ f)$). And f oscillates $\leqslant \varepsilon/2$ on each member of $f^{-1}(\mathfrak{U})$.

Evidently, $\bigcup f^{-1}(\mathfrak{U}) = X \notin$ co$\mathfrak{F}_p$. We claim that $f^{-1}(\mathfrak{U}) \subset$ co$\mathfrak{F}_p$, i.e., that $p \in \overline{X - f^{-1}(U)}$ for each $U \in \mathfrak{U}$: for if not, and there is $U \in \mathfrak{U}$, and a neighborhood G of p with $G \cap (X - f^{-1}(U)) = \varnothing$, then $G \cap X \subset f^{-1}(U)$, and $\operatorname{osc}_G f \leqslant \varepsilon/2$, a contradiction. Thus co$\mathfrak{F}_p$ is not σ-discretely (or locally finitely) additive.

REMARK. The second half of the proof actually shows this. If X is dense in Y, and if there is continuous $f\colon X \to M$, M metrizable, which fails to extend to $p \in Y - X$, then there is a family $\{Z_a | a \in A\}$ of zero-sets (namely, the family $\{X - f^{-1}(U) | U \in \mathfrak{U}\}$ above) with $\{X - Z_a | a \in A\}$ σ-discrete (or locally finite), with $\bigcap_a Z_a = \varnothing$ and $p \in \bigcap_a \overline{Z}_a$, the closures taken in Y.

3. COROLLARY. *X is topologically complete iff each z-ultrafilter $\mathfrak{F}$ for which* co$\mathfrak{F}$ *is σ-discretely (or locally finitely) additive is of the form $\{Z | p \in \overline{Z}\}$ for some unique $p \in X$.*

PROOF. $X \in \mathfrak{R}(\mathfrak{M})$ iff $\delta X = X$.

The following useful proposition from **[GJ]** results from the association between z-ultrafilters and points of the epireflections βX or υX.

4. PROPOSITION. *Let X be dense in Y. The following are equivalent.*

(a) *Each continuous function from X to* $[0, 1]$ (*respectively,* R) *extends continuously over Y.*

(b) $Y \subset \beta X$ (*respectively,* $Y \subset \upsilon X$).

(c) *If* $\{Z_a | a \in A\}$ *is a finite* (*respectively, countable*) *family of zero-sets of X with* $\bigcap_a Z_a = \varnothing$ *then* $\bigcap_a \overline{Z}_a = \varnothing$ (*the closures in Y*).

Analogously, we can derive easily from 2 the following.

5. COROLLARY. *Let X be dense in Y. The following are equivalent.*

(a) *Each continuous function from X to a metrizable space extends continuously over Y.*

(b) $Y \subset \delta X$.

(c) *If* $\{Z_a | a \in A\}$ *is a family of zero-sets, with* $\{X - Z_a | a \in A\}$ *σ-discrete* (*or locally finite*), *and if* $\bigcap_a Z_a = \varnothing$, *then* $\bigcap_a \overline{Z}_a = \varnothing$ (*the closures in Y*).

PROOF. (a) *implies* (b): Assuming (a), continuous functions from X to metrizable spaces extend over δY. By uniqueness of epireflections, $\delta Y = \delta X$ (essentially), so $Y \subset \delta X$ (essentially).

(b) *implies* (c): Assume (b), and let $\{Z_a\}$ be a family as in (c), with $\bigcap_a \overline{Z}_a \neq \varnothing$. Thus, for some $p \in \delta X$, $p \in \bigcap_a \overline{Z}_a$, and each $Z_a \in \mathfrak{F}_p$. By 2, $\mathfrak{F}_p$ is σ-discretely or locally finitely additive, so $\bigcap_a Z_a \in \mathfrak{F}_p$. Since $\mathfrak{F}_p$ is a filter, $\bigcap_a Z_a \neq \varnothing$.

(c) *implies* (a): See the remark after 2.

REMARKS. (1) A proof of 4 is easily constructed by analogy with the above proof of 5.

(2) As noted in the introduction, the extension properties in 4(a) and 5(a) imply the stronger extension properties for maps into the epireflective hulls $\mathfrak{R}([0, 1])$ = compact spaces, $\mathfrak{R}(R)$ = realcompact spaces, $\mathfrak{R}(\mathfrak{M})$ = topologically complete spaces.

We conclude with a relatively simple proof, based on 2, of the Katětov-Shirota Theorem–or more exactly, of a version of the Gillman-Jerison version of the theorem. (See **[K]**, **[S]**, **[GJ]**.)

Recall that the set S has measurable power if there is a "nontrivial measure on S", i.e., a countably additive measure μ, defined for all subsets of S, taking values 0 and 1, with $\mu(\{p\}) = 0$ for each $p \in S$, and $\mu(S) = 1$.

6. THEOREM. $\delta X = \upsilon X$ *iff each discrete subset of X has nonmeasurable power.*

PROOF. What we shall show is that σ-additivity of co $\mathfrak{F}$ implies (σ-) discrete additivity for each z-ultrafilter iff the stated condition holds.

The "if" part is immediate from the following.

7. LEMMA. *Let $\mathfrak{F}$ be a z-ultrafilter. If* co $\mathfrak{F}$ *is σ-additive, then* co $\mathfrak{F}$ *is "nonmeasurably" discretely additive.*

PROOF. Let co $\mathfrak{F}$ be σ-additive, let $\mathfrak{D} \subset$ co $\mathfrak{F}$ be discrete of nonmeasurable power. For $\mathfrak{C} \subset \mathfrak{D}$, define $\mu(\mathfrak{C}) = 1$ if $\bigcup \mathfrak{C} \notin$ co $\mathfrak{F}$; $\mu(\mathfrak{C}) = 0$ if $\bigcup \mathfrak{C} \in$ co $\mathfrak{F}$.

Note that for each $\mathscr{C} \subset \mathfrak{D}$, $\bigcup\mathscr{C}$ is a cozero set because $\mathscr{C}$ is discrete. For $D \in \mathfrak{D}$, $\mu(\{D\}) = 0$ because $\mathfrak{D} \subset \operatorname{co}\mathfrak{F}$. We shall check that μ is a measure; thus μ will be identically 0, $\mu(\mathfrak{D}) = 0$ and $\bigcup\mathfrak{D} \in \operatorname{co}\mathfrak{F}$.

Let $\mathscr{C}_1, \mathscr{C}_2, \ldots$ be a sequence of disjoint subsets of $\mathfrak{D}$. We shall show that $\mu(\bigcup_n\mathscr{C}_n) = \sum_n \mu(\mathscr{C}_n)$. Let $A_n = \bigcup\{D \in \mathscr{C}_n\}$, $A = \bigcup_n A_n$. If $\mu(\bigcup_n\mathscr{C}_n) = 0$, then $A \in \operatorname{co}\mathfrak{F}$.

For each n, $A_n \subset A$, hence $A_n \in \operatorname{co}\mathfrak{F}$ and $\mu(\mathscr{C}_n) = 0$. Thus $\mu(\bigcup_n\mathscr{C}_n) = 0 = \sum_n \mu(\mathscr{C}_n)$. Now suppose $\mu(\bigcup_n\mathscr{C}_n) = 1$, *i.e.*, $A \notin \operatorname{co}\mathfrak{F}$. By σ-additivity, there is n_0 such that $A_{n_0} \notin \operatorname{co}\mathfrak{F}$. Let $n \neq n_0$, then $\mathscr{C}_n \cap \mathscr{C}_{n_0} = \varnothing$, $A_n \cap A_{n_0} = \varnothing$, and $(X - A_n) \cup (X - A_{n_0}) = X \in \mathfrak{F}$. Since $\mathfrak{F}$ is an ultrafilter, it is prime, and since $X - A_{n_0} \notin \mathfrak{F}$, we have $X - A_n \in \mathfrak{F}$, $A_n \in \operatorname{co}\mathfrak{F}$, and $\mu(\mathscr{C}_n) = 0$. So $1 = \mu(\bigcup_n\mathscr{C}_n) = \mu(\mathscr{C}_{n_0}) = \sum_n \mu(\mathscr{C}_n)$.

Conversely, let X contain the discrete set D of measurable power, and let μ be a nontrivial measure on D. Let $\mathfrak{F}$ be the family of zero-sets Z of X with $\mu(Z \cap D) = 1$. Evidently, $\mathfrak{F}$ is a filter with $\operatorname{co}\mathfrak{F}$ σ-additive.

We shall show that $\mathfrak{F}$ is maximal and not discretely additive. For use in both parts, choose d and ε with $d(p,q) \geqslant \varepsilon$ for $p \neq q$ in D, and for $p \in D$, set $C_p = \{x | d(p,x) < \varepsilon/4\}$. Note that C_p is a cozero-set, and $\{C_p | p \in \mathfrak{D}\}$ is discrete.

$\mathfrak{F}$ is maximal: Let Z_0 be a zero-set with $Z_0 \cap Z \neq \varnothing$ for each $Z \in \mathfrak{F}$. For each $p \in D - Z_0$, choose a zero-set Z_p with $p \in Z_p$, $Z_p \cap Z_0 = \varnothing$, and $Z_p \subset C_p$. Let $C_p = \operatorname{coz} f_p$ and $Z_p = Zg_p$. Then $\{\operatorname{coz}(f_p g_p) | p \in D - Z_0\}$ is discrete, and with $f = \sum\{f_p g_p | p \in D - Z_0\}$, we have $Zf = \bigcup\{Z_p | p \in D - Z_0\}$. Now $Z_0 \cap Zf = \varnothing$, so that $Zf \in \mathfrak{F}$. Since $D = (Zf \cap D) \cup (Z_0 \cap D)$, it follows that $\mu(Z_0 \cap D) = 1$, and $Z_0 \in \mathfrak{F}$.

$\mathfrak{F}$ is not discretely additive: Let $\mathfrak{D} = \{C_p | p \in D\}$. Evidently, $\mathfrak{D} \subset \operatorname{co}\mathfrak{F}$, while $\mu((X - \bigcup\mathfrak{D}) \cap D) = \mu(\varnothing) = 0$, so that $\bigcup\mathfrak{D} \notin \operatorname{co}\mathfrak{F}_p$.

REMARKS. (1) It is easy to show that each discrete subset of X has nonmeasurable power iff each locally finite subset of X has nonmeasurable power. This yields another version of 6.

(2) The proof of 6 given above (including the proof of 2) resembles to some degree the proof of 15.21 of **[GJ]**. 7 generalizes 12.3 of **[GJ]**.

(3) All known proofs of theorems close to 6 use the Stone Theorem on σ-discrete refinement. Our use of it is confined to 2, and our proof of 6 proper consists of fairly simple set-theoretic computations.

(4) **[DW]** and 2.4 of **[T]** use locally finite partitions of unity to characterize topological completeness; the ideas are somewhat similar to those of this paper.

(5) The referee points out that the space we are labelling δX (after its inventor Dieudonné **[D]**) has been labelled θX in **[B]** and μX in **[M]**.

(6) The referee points out that Buchwalter **[B]** has also and earlier obtained an identification of the points of δX among those of vX: As is well known, the points p of vX (i.e., the z-ultrafilters $\mathfrak{F}$ with $\operatorname{co}\mathfrak{F}$ σ-additive) correspond one-to-one with the unitary ring homomorphism h: $C(X) \to R$. In §4 of **[B]**, Buchwalter shows that the following conditions on h are equivalent: (a) $h \in \delta X$. (b) $h|E$ is continuous for every equicontinuous $E \subset C(X)$, E having the topology of simple convergence on X. (c) If $\{E_n\}$ is a sequence of equicontinuous subsets of $C(X)$, then there is $x \in X$ such that $h(f) = f(x)$ for each $f \in \bigcup_n E_n$.

As here 2 implies 6, the above implies a version of the Katětov-Shirota Theorem ([**B**, p. 55]).

Buchwalter's Theorem and our 2 establish that a homomorphism h satisfies (b) above iff the associated z-ultrafilter $\mathfrak{F} = \{Z(f) \mid f \in \ker h\}$ has $\mathrm{co}\,\mathfrak{F}$ σ-discretely additive.

(7) As is well known (e.g., [**D**]), δX can be viewed as the topological space underlying the completion of X equipped with its finest compatible uniformity. We have chosen to avoid connections with uniformities here, but a later paper [**CH**] will treat this thoroughly.

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Czechoslovakian Academy of Sciences, Prague, Czechoslovakia

Department of Mathematics, Wesleyan University, Middletown, Connecticut 06457