GROUPS WITH A FINITE NUMBER OF SYLOW Λ -SUBGROUPS

EUGENE SCHENKMAN

ABSTRACT. Two theorems on groups with a finite number of Sylow subgroups are proved.

THEOREM 1. *If for some set of primes Λ the group G has a finite number of Sylow Λ -subgroups, then G is a finite extension of a Λ' -extension of $O_\Lambda(G)$.*

THEOREM 2. *Let the set of all primes be a disjoint union of sets of primes Λ such that for each Λ , the group G has a finite number of Sylow Λ -subgroups. Then the set T of periodic elements of G is a normal subgroup and T/R is an FC group with R denoting $\prod_\Lambda O_\Lambda(G)$.*

Various authors (cf. [1, pp. 362–369])—Baer, Neumann, Haimo, Dickman, Eremin—have studied FC groups and conditions of finiteness for certain classes of elements or subgroups. Černikov studied thin FL groups (cf. [2, p. 139]). This note considers groups with finiteness conditions on Sylow subgroups and aims to prove the two theorems mentioned above.

PROOF OF THEOREM 1. For $i = 1, \dots, n$, let P_i be the Sylow Λ -subgroups of G and let $N_i = \mathcal{N}(P_i)$ be their normalizers. If $D = \bigcap_{i=1}^n N_i$, then the index $|G : D|$ is finite since $|G : N_i| = n$ is finite. Since N_i has exactly one Sylow Λ -subgroup, the same is true of D . In fact the Sylow Λ -subgroup of D is $O_\Lambda(G)$ and Theorem 1 follows.

PROOF OF THEOREM 2. As in the proof and with the notation of Theorem 1, $O_\Lambda(G)$ is of finite index in P_i ; for in N_i , $P_i/O_\Lambda(G)$ is isomorphic to $P_i D/D$ and $|G : D|$ is finite. Thus $G/O_\Lambda(G)$ has a finite number of finite Sylow Λ -subgroups for each Λ . It follows from the theorem of Dickman that all the Sylow Λ -subgroups of $G/O_\Lambda(G)$ are contained in a finite normal subgroup and hence that the Sylow Λ -subgroups of G generate a finite extension of $O_\Lambda(G)$. Thus T is a normal subgroup and T/R is an FC group as is asserted in Theorem 2.

After submitting this paper the author learned that Professor Derek Robinson was aware of these results for the cases where Λ consists of one prime.

BIBLIOGRAPHY

1. G. Baumslag, *Reviews on infinite groups*. Parts 1, 2, Amer. Math. Soc., Providence, R. I., 1974. MR 50 #2311.
2. D. J. S. Robinson, *Finiteness conditions and generalized solvable groups*. Part 1, *Ergebnisse der Mathematik und ihrer Grenzgebiete*, Band 62, Springer-Verlag, Berlin and New York, 1972. MR 48 #11314.

DEPARTMENT OF MATHEMATICS, PURDUE UNIVERSITY, WEST LAFAYETTE, INDIANA 47907

Received by the editors September 25, 1975.

AMS (MOS) subject classifications (1970). Primary 20E20; Secondary 20F15.

Key words and phrases. Sylow subgroup, FC group.

© American Mathematical Society 1976