PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 60, October 1976

ERRATUM TO "ON THE AUTOMORPHISM GROUP OF A LIE GROUP"

DAVID WIGNER

D. Z. Djokovic and G. P. Hochschild have pointed out that the argument in the third paragraph of the proof of Theorem 1 is insufficient. The fallacy lies in assuming that the fixer of S in $K(S)$ is an algebraic subgroup. This is of course correct for complex groups, since a connected complex semisimple Lie group is algebraic, but false for real Lie groups. By Proposition 1, the theorem is correct for real solvable Lie groups and the given proof is correct for complex groups, but the theorem fails for real semisimple groups, as the following example, worked out jointly with Professor Hochschild, shows.

Let $G = SL(2, \mathbf{R})$; then $G_{\mathbf{C}} = SL(2, \mathbf{C})$ is the complexification of G and the complex conjugation τ in $SL(2, \mathbf{C})$ fixes exactly $SL(2, \mathbf{R})$. Let

$$H = PSL(2, \mathbf{R}) = SL(2, \mathbf{R})/\pm I;$$

then $H_{\mathbf{C}} = PSL(2, \mathbf{C}) = SL(2, \mathbf{C})/\pm I$. The complex conjugation induced by τ in $PSL(2, \mathbf{C})$ fixes the image M in $PSL(2, \mathbf{C})$ of the matrix $\left(\begin{smallmatrix} 0 & i \\ i & 0 \end{smallmatrix}\right)$ in $SL(2, \mathbf{C})$. Therefore M is in the real algebraic hull of $H = PSL(2, \mathbf{R})$. Conjugation by M on $PSL(2, \mathbf{R})$ acts on $\pi_1(PSL(2, \mathbf{R})) \cong \mathbf{Z}$ by multiplication by -1.

Now let $\tilde{H}$ be the universal cover of $PSL(2, \mathbf{R})$ and D its center. Let $K = \tilde{H} \times \tilde{H}/\{\langle d, d\rangle | d \in D\}$. Then the inner automorphism group of K is $H \times H$ and its algebraic hull contains elements whose conjugation action on the fundamental group of $H \times H$ is multiplication by -1 on the fundamental group of the first factor and the identity on the fundamental group of the second factor. Such elements do not come from automorphisms of K. Therefore the automorphism group of K has no open algebraic subgroup.

The inner automorphisms of a connected real semisimple Lie group have finite index in the group of all automorphisms. As a result the following reformulation (suggested by Professor Hochschild) of the real case of Theorem 1 is valid.

THEOREM. *Let G be a connected group, with Lie algebra L. Identify* $\mathrm{Aut}(G)$ *with its image in* $\mathrm{Aut}(L)$. *Then the Lie algebra of* $\mathrm{Aut}(G)$ *coincides with the Lie algebra of an algebraic subgroup of* $\mathrm{Aut}(L)$.

REFERENCES

1. D. Wigner, *On the automorphism group of a Lie group*, Proc. Amer. Math. Soc. **45** (1974), 140–143. MR **50** #10152.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MICHIGAN, ANN ARBOR, MICHIGAN 48104