COMPACT GROUPS OF REAL POWER NEED NOT BE METRIZABLE

DONALD HACKLER

We show that the metrizability of compact (separable Abelian) groups of power $c = 2^\omega$ can be neither proved nor disproved in ZFC. The *local weight* of a topological group is the least power of a basis at the identity (in a locally compact group, the least power of an open family whose intersection is the identity). It is known that a topological group is metrizable just in case it has local weight ω [K].

One can show that a (locally) compact group of power κ has local weight $< \kappa$ (for $\kappa = \omega_1$ this is [J]). Hence the continuum hypothesis implies that each (locally) compact group of power c is metrizable.

On the other hand, it is consistent that there be compact groups of power c and uncountable local weight. In fact, it is consistent to assume Martin's Axiom plus $c > \omega_1$, and Martin's Axiom implies that $2^\kappa = c$ whenever $\omega \leq \kappa < c$ (see [ST] and [MS]). For each such κ , $G = Z_2^\kappa$ is a compact separable Abelian group of power $2^\kappa = c$. G is not metrizable, since $0 = (0, 0, 0, \dots)$ is not the intersection of $< \kappa$ basic open sets.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN 53706

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