A REMARK ON THE GROTHENDIECK RESIDUE MAP¹

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ABSTRACT. The purpose of this note is to give a direct proof that a global integral over a compact complex manifold X can be evaluated on the zero set of a meromorphic vector field on X with isolated zeros via a Grothendieck residue morphism. A special case of this evaluation is the meromorphic vector field theorem of Baum and Bott [1]. The present proof suggests some complements of the M.V.F. Theorem which are contained in Theorem 2.

1. Introduction. Statement of results. Given a finite, nontrivial, but possibly unreduced subvariety Z of a connected compact complex manifold X of dimension n , and given $\omega \in H^n(X, \Omega^n)$, the global integral $\int_X \omega$ can always be evaluated as a sum of residues (in the sense of Grothendieck) on Z . The proofs of this fact in the literature are valid in algebraic geometry and are necessarily complicated, cf. [7], [13]. On the other hand, what we shall show is that if one makes the assumption that Z is the variety of zeros of a meromorphic vector field V on X , then a reformulation of the fundamental commutative diagram of [13], which makes the contribution from Z explicit, can be proven simply (Theorem 1). Moreover this approach immediately suggests some complements to the Meromorphic Vector Field Theorem [1], [2], which are given in Theorem 2.

To state our results precisely, let T denote the holomorphic tangent bundle of X , L a holomorphic line bundle on X , and Θ (resp. $\mathcal{L}$) the sheaf of germs of holomorphic sections of T (resp. L). Contraction by $V \in H^0(X, T \otimes L)$ on holomorphic p -forms is an operator $i(V): \Omega^p \rightarrow \Omega^{p-1} \otimes \mathcal{L}$ on the sheaf level, which defines a sheaf of ideals $I_Z = i(V)(\Omega^1 \otimes \mathcal{L}^{-1})$ in $\Omega^0 = \mathcal{O}$. The subvariety Z defined by I_Z is called the *variety of zeros* of V . The structure sheaf of Z is by definition the sheaf of rings $\mathcal{O}_Z = \mathcal{O}/I_Z$. For any sheaf $\mathcal{F}$ of $\mathcal{O}$ -modules on X , let $\mathcal{F}_Z = \mathcal{F} \otimes_{\mathcal{O}} \mathcal{O}_Z$. The Grothendieck Residue Map we will associate to V is a map $\text{Res}: H^0(X, \mathcal{L}_Z^n) \rightarrow \mathbb{C}$ about which we shall prove the following result. It will always be assumed that Z is finite but nontrivial.

THEOREM 1. *There exists a map $m: H^0(X, \mathcal{L}_Z^n) \rightarrow H^n(X, \Omega^n)$ (depending only on V) such that*

Received by the editors May 28, 1976 and, in revised form, April 20, 1977.

AMS (MOS) subject classifications (1970). Primary 32A25, 14F25.

¹Supported in part by a grant from the National Research Council.

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$$\begin{array}{ccc}
 H^0(X, \mathcal{L}_Z^n) & \xrightarrow{\text{Res}} & C \\
 (-1)^n m \downarrow & \nearrow \text{tr} & \\
 H^n(X, \Omega^n) & &
 \end{array} \quad (1)$$

commutes, where tr is the map $(1/2\pi i)^n \int_X$ (cf. [13]).

Given an ad-invariant linear map $p: \mathfrak{gl}(n, C)^{\otimes k} \rightarrow C$, there is a natural element $p(V_0)$ of $H^0(X, \mathcal{L}_Z^k)$ such that in the case $k = n$, $\text{tr}(m(p(V_0)))$ is a characteristic number of the virtual bundle $T - L^{-1}$. The assertion $(-1)^n \text{tr}(m(p(V_0))) = \text{Res } p(V_0)$ is of course the M.V.F. Theorem. In §5, we will show that in addition one can use (1) to prove

THEOREM 2. Suppose $\deg p = k < n$ and that $\sigma \in H^0(X, L)$. Let $\sigma^{n-k} p(V_0)$ denote the image of $\sigma^{n-k} \otimes p(V_0)$ under the natural pairing $H^0(X, L^{n-k}) \otimes H^0(X, \mathcal{L}_Z^k) \rightarrow H^0(X, \mathcal{L}_Z^n)$. Then $\text{Res } \sigma^{n-k} p(V_0) = 0$.

To prove Theorem 1, we employ a double complex with differentials $i(V)$ and $\bar{\partial}$ to compute $H^0(X, \mathcal{L}_Z^n)$. The mapping m is an edge morphism in this double complex. (1) is a consequence of the projector trick of Bott [3] and an interesting local integral representation formula for Res (Lemma 4), which simply amounts to combining a local I.R.F. for the partial derivatives of a holomorphic function at a point with the computational algorithm for the local residue involving the Nullstellensatz.

2. A double complex. For a fixed integer m , let $A^{p,q}(L^m)$ denote C^∞ , L^m -valued forms on X of type (p, q) . The operators $i(V): A^{p,q}(L^m) \rightarrow A^{p-1,q}(L^{m+1})$ and $\bar{\partial}: A^{p,q}(L^m) \rightarrow A^{p,q+1}(L^m)$ satisfy $i(V)^2 = \bar{\partial}^2 = \bar{\partial}i(V) + i(V)\bar{\partial} = 0$, so $D = i(V) + \bar{\partial}$ is a total differential for the complex $C_m^k = \sum_p C_m^{p,k-p}$ formed from the double complex $\{C_m^{p,q} = A^{p,q}(L^{m-p}), i(V), \bar{\partial}\}$.

LEMMA 1. Assuming that Z is finite, then $H^0(C_m) \cong H^0(X, \mathcal{L}_Z^m)$.

Let $\mathcal{E}^{p,q}(\mathcal{L}^k)$ denote the sheaf of germs of C^∞ , L^k -valued (p, q) forms on X . To prove Lemma 1, we need

LEMMA 2. For any k we have a fine resolution of $\mathcal{L}_Z^k$:

$$\begin{aligned}
 0 \rightarrow \mathcal{L}_Z^k \xrightarrow{h} \mathcal{E}^{0,0}(\mathcal{L}^k)/i(V)\mathcal{E}^{1,0}(\mathcal{L}^{k-1}) \xrightarrow{\bar{\partial}} \mathcal{E}^{0,1}(\mathcal{L}^k)/i(V)\mathcal{E}^{1,1}(\mathcal{L}^{k-1}) \\
 \xrightarrow{\bar{\partial}} \dots \xrightarrow{\bar{\partial}} \mathcal{E}^{0,n}(\mathcal{L}^k)/i(V)\mathcal{E}^{1,n}(\mathcal{L}^{k-1}) \rightarrow 0
 \end{aligned} \quad (2)$$

where h is the natural inclusion.

PROOF. One first notes that for any k the Dolbeault resolution

$$0 \rightarrow \mathcal{L}^k \rightarrow \mathcal{E}^{0,0}(\mathcal{L}^k) \xrightarrow{\bar{\partial}} \mathcal{E}^{0,1}(\mathcal{L}^k) \rightarrow \dots \rightarrow \mathcal{E}^{0,n}(\mathcal{L}^k) \rightarrow 0$$

is an exact sequence of $\mathcal{O}$ -modules from which (2) is obtained via tensorisation by $\mathcal{O}_Z$. Lemma 2 follows from the fact that $\mathcal{E}^{p,q}(\mathcal{L}^k)$ is a flat $\mathcal{O}$ -module,

by the Malgrange Preparation Theorem [9].

This lemma does not assume Z is finite. Finiteness of Z is used to conclude exactness of

$$0 \rightarrow \Omega^n \otimes \mathcal{L}^{k-n} \xrightarrow{i(V)} \Omega^{n-1} \otimes \mathcal{L}^{k-n+1} \rightarrow \dots \rightarrow \Omega^1 \otimes \mathcal{L}^{k-1} \xrightarrow{i(V)} \mathcal{L}^k. \quad (3)$$

To prove Lemma 1, consider the exact sequence obtained from (2):

$$0 \rightarrow H^0(X, \mathcal{L}_Z^k) \xrightarrow{h} C_k^{0,0}/i(V)C_k^{1,0} \xrightarrow{\bar{\partial}} C_k^{0,1}/i(V)C_k^{1,1}. \quad (4)$$

For $s \in H^0(X, \mathcal{L}_Z^k)$, choose $s_0 \in C_k^{0,0}$ such that $h(s) = s_0$ modulo $i(V)C_k^{1,1}$. By (4), $\bar{\partial}s_0 = i(V)s_1$ for some $s_1 \in C_k^{1,1}$. Since $i(V)\bar{\partial}s_1 = -\bar{\partial}i(V)s_1 = \bar{\partial}^2s_0 = 0$, there exists by (3) an $s_2 \in C_k^{1,2}$ so that $\bar{\partial}s_1 = i(V)s_2$. Continuing in this manner, one gets a total cocycle $S = s_0 - s_1 + \dots + (-1)^n s_n$ of C_k of degree 0, and the map $s \rightarrow S$ induces a morphism $\phi_k: H^0(X, \mathcal{L}_Z^k) \rightarrow H^0(C_k)$. To show that ϕ_k is an isomorphism, we produce its inverse. Let S be a total cocycle in C_k^0 denoted as above. Then $\bar{\partial}s_0$ is in $i(V)C_k^{1,1}$. Hence by (4), there exists a unique $s \in H^0(X, \mathcal{L}_Z^k)$ such that $h(s) = s_0 \bmod i(V)C_k^{1,0}$. Then $S \rightarrow s$ induces the inverse of ϕ_k , and the proof of Lemma 1 is complete.

One now defines $m: H^0(X, \mathcal{L}_Z^n) \rightarrow H^n(X, \Omega^n)$ ($n = \dim X$) by composing ϕ_k^{-1} with the natural edge morphism $H^0(C_n) \rightarrow H^0(X, \Omega^n)$ induced by mapping S in C_n^0 to the component s_n of S in $A^{n,n}(L^0)$.

3. The morphism Res. Let U be an open ball about the origin O of C^n , which is the only common zero of $a_1, \dots, a_n \in H^0(U, \mathcal{O})$. The local residue at O of $\omega \in H^0(U, \Omega^n)$ with respect to $a_1, \dots, a_n$ is defined in [11] as

$$\text{Res}\left(a_1 \cdot \omega \cdot a_n\right) = \left(\frac{1}{2\pi i}\right)^n \int_{\partial D \times \dots \times \partial D} \frac{\omega}{a_1 \cdot \dots \cdot a_n} \quad (5)$$

where D is a disc about O in C chosen so that $\partial D \times \dots \times \partial D$ misses the hypersurface $\{z: (a_1 \cdot \dots \cdot a_n)(z) = 0\}$ in U . The local residue defined by (5) coincides with the Grothendieck residue, and hence can be computed by the well-known algorithm [1], [2], [7], [11], [13]. If $a_1, \dots, a_n \in H^0(U, \mathcal{L})$, then

$$\text{Res}\left(a_1 \cdot \omega \cdot a_n\right)$$

is unambiguously defined for all $\omega \in H^0(U, \Omega^n \otimes \mathcal{L}^n)$.

Suppose $V \in H^0(X, T \otimes L)$ has isolated zeros Z , and let $(z_1, \dots, z_n)$ denote local coordinates for X near $\zeta \in Z$ with $z_i(\zeta) = 0$ for each i . One may locally express $V = \sum a_i \partial / \partial z_i$, where $a_1, \dots, a_n$ are local sections of L whose only common zero is ζ . Given $s \in \mathcal{L}_\zeta^n$, the local residue of s at ζ is defined to be

$$\text{Res}(s)_\zeta = \text{Res}\left(\frac{s \, dz_1 \cdot \dots \cdot dz_n}{a_1 \cdot \dots \cdot a_n}\right). \quad (6)$$

Under change of coordinates, Res transforms in a manner so as to imply that (6) depends only on V and s . In fact, $\text{Res}(s)_\zeta = 0$ if $s \in (I_Z \mathcal{L}^n)_\zeta$ (by [7]), and consequently Res is defined as a morphism $\text{Res}_\zeta: \mathcal{L}_{Z,\zeta}^n \rightarrow C$. Define $\text{Res}: H^0(X, \mathcal{L}_Z^n) \rightarrow C$ by $\text{Res} = \sum_Z \text{Res}_\zeta$.

4. The proof of Theorem 1. In order to localize $\omega = m(S) \in H^n(X, \Omega^n)$, we will use Bott's projector trick. Recall that projector for V is an L^{-1} valued $(1, 0)$ form π on $X - Z$ such that $\pi(V) = 1$. Bott's key observation concerning projectors can be rephrased in this context as the observation that the differential form

$$\tau = \pi(s_0(\bar{\partial}\pi)^{n-1} - s_1(\bar{\partial}\pi)^{n-2} + \cdots + (-1)^{n-1}s_{n-1}) \quad (7)$$

on $X - Z$ satisfies $s_n = (-1)^{n-1} d\tau$ provided $S = s_0 + s_1 + \cdots + s_n$ is a total cocycle of C_n^0 . If $\{W_j\}$ is a finite covering of Z by disjoint coordinate balls such that $W_i \cap Z = \{\zeta_i\}$, then by Stokes' Theorem

$$\int_X s_n = (-1)^n \sum \int_{\partial W_j} \tau. \quad (8)$$

Now the right-hand side of (8) can be vastly simplified by the following observation.

LEMMA 3. *For any $S \in H^0(C)$, there is a representing cocycle $S = s_0 + \cdots + s_n$ such that s_0 is holomorphic near Z , and, if $i > 0$, then $s_i = 0$ near Z .*

PROOF. s_0 can obviously be so chosen. The existence of the g_i follows from repeated application of (3).

The proof of Theorem 1 now results from the following local integral representation formula for Res.

LEMMA 4. *For any projector π for V and for any $\zeta \in Z$,*

$$\text{Res}(s)_\zeta = \left(\frac{1}{2\pi i} \right)^n \int_{\partial W} s\pi(\bar{\partial}\pi)^{n-1} \quad (9)$$

where $s \in \mathcal{L}_\zeta^n$ and W is a sufficiently small ball centered at ζ .

PROOF. Since Z is finite, $\mathcal{L}_{Z,\zeta}^n \subset H^0(X, \mathcal{L}_Z^n)$. Therefore, $m(s)$ is defined and $\int_X m(s) = \int_{\partial W} s\pi(\bar{\partial}\pi)^{n-1}$. Hence (9) is independent of π . Let π be the projector for V defined in a neighborhood W of ζ as follows. Let $V = \sum a_i \partial/\partial z_i$ on W as above. By Hilbert's Nullstellensatz, there exist positive integers $\alpha_1, \dots, \alpha_n$ and $b_{ij} \in H^0(W, L^{-1})$ such that $z_i^{\alpha_i} = \sum b_{ij} a_j$. Set $\pi = u^{-1} \sum \bar{z}_i^{\alpha_i} b_{ij} dz_j$, where $u = \sum (z_i \bar{z}_i)^{\alpha_i}$. Note

$$\begin{aligned} \pi(\bar{\partial}\pi)^{n-1} &= (n-1)! (-1)^{n(n-1)/2} u^{-n} \det \|b_{ij}\| \\ &\quad \times \sum (-1)^{i-1} \bar{z}_i^{\alpha_i} d\bar{z}_1^{\alpha_1} \wedge \cdots \wedge \widehat{d\bar{z}_i^{\alpha_i}} \wedge \cdots \wedge d\bar{z}_n^{\alpha_n} \wedge dz_1 \wedge \cdots \wedge dz_n. \end{aligned}$$

Now, by applying a standard I.R.F. for the partial derivatives of a holomorphic function g defined in a neighbourhood of $\bar{W}$ [10, p. 56], one gets

$$\left(\frac{1}{2\pi i} \right)^n \int_{\partial W} g\pi(\bar{\partial}\pi)^{n-1} = 1/(\alpha-1)! D^\alpha (\det \|b_{ij}\| g)(\zeta) \quad (10)$$

where α and $\alpha-1$ denote, respectively, the multi-indices $(\alpha_1, \dots, \alpha_n)$ and $(\alpha_1-1, \dots, \alpha_n-1)$. Lemma 4 follows from the fact that the r.h.s. of (10) is

$$\text{Res}_S \left(\frac{g \, dz_1 \cdots dz_n}{a_1, \dots, a_n} \right),$$

by the algorithm.

To finish the proof of (1), let $s \in H^0(X, \mathbb{C}_Z^n)$ and let $S = s_0 + s_1 + \cdots + s_n$ be a total cocycle of C_n^0 representing $\phi(s)$ satisfying all the conclusions of Lemma 3. Then $(-1)^n m(S) = (-1)^n s_n$, so

$$\left(-\frac{1}{2\pi i} \right)^n \int_X s_n = \left(\frac{1}{2\pi i} \right)^n \sum \int_{\partial W_j} s_0 \pi (\bar{\partial} \pi)^{n-1} = \text{Res}(s)$$

by (9) and Lemma 4.

5. Some residue formulas. If V is a global section of the tangent sheaf Θ of X (i.e. a holomorphic vector field on X), then the Lie bracket $Y \rightarrow [V, Y]$ induces a C -linear map $\tilde{V}: \Theta \rightarrow \Theta$ lifting the derivation $V: \Theta \rightarrow \Theta$, i.e. $\tilde{V}(fY) = V(f)Y + f\tilde{V}(Y)$. Any sheaf $\mathcal{F}$ of Θ -modules admitting such a lifting $\tilde{V}$ is called V -equivariant and $\tilde{V}$ is called an equivariant lift of V . Note that $\tilde{V}$ defines $V_0 \in H^0(X, \text{Hom}_\Theta(\Theta, \Theta)_Z)$. Analogously, an equivariant lift $\tilde{V}: \mathcal{F} \rightarrow \mathcal{F} \otimes L$ of $V \in H^0(X, T \otimes L)$ defines an element $V_0 \in H^0(X, (\text{Hom}_\Theta(\mathcal{F}, \mathcal{F}) \otimes \mathbb{C})_Z)$. Now Θ is generally not equivariant for an arbitrary $V \in H^0(X, T \otimes L)$, however $\tilde{V}$ admits a well-defined localization $V_0 \in H^0(X, (\text{Hom}_\Theta(\Theta, \Theta) \otimes \mathbb{C})_Z)$. In fact, choose a covering $\{U_\alpha\}$ of X such that both $L|_{U_\alpha}$ and $T|_{U_\alpha}$ are locally trivial for each α , and on U_α write $V = w_\alpha \otimes t_\alpha$ where $w_\alpha \in H^0(U_\alpha, \Theta)$, $t_\alpha \in H^0(U_\alpha, \mathbb{C})$, and t_α is nowhere vanishing. For $Y \in H^0(U_\alpha, \Theta)$, set $\tilde{V}_\alpha(Y) = [w_\alpha, Y] \otimes t_\alpha$. Note that on $U_\alpha \cap U_\beta$, $\tilde{V}_\beta(Y) - \tilde{V}_\alpha(Y) = t_\beta t_\alpha^{-1} i(Y) d(t_\beta t_\alpha^{-1}) \otimes V$, consequently the $\tilde{V}_\alpha$ patch on Z giving V_0 as asserted.

Let $p(V_0)$ denote the element of $H^0(X, \mathbb{C}_Z^k)$ obtained by applying the ad-invariant symmetric linear map $p: \text{gl}(n, C)^{\otimes k} \rightarrow C$ to V_0 . In order to construct the class $\phi_k(p(V_0))$ in $H^0(C_k)$, first choose a local holomorphic connection D_α for $T|_{U_\alpha}$, and let $D = \sum \rho_\alpha D_\alpha$ be the connection of type $(1, 0)$ on T where ρ_α is a partition of unity fitted to $\{U_\alpha\}$. Following [1], consider the Θ -linear map $\psi_\#: \Omega^1 \rightarrow \text{Hom}(\Theta, \Theta) \otimes \Omega^1$ so that if $\omega \in \Omega_x^1$, and $Y \in \Theta_x$ then $i(Y)\psi_\#\omega$ is the element of $\text{Hom}(\Theta_x, \Theta_x)$ such that $i(Y)\psi_\#\omega(w) = \omega(w)Y$. Let $\Gamma \in A^{1,0}(\text{Hom}(T, T))$ be given by $\Gamma = \sum \rho_\alpha \{\psi_\#(t_\alpha^{-1} dt_\alpha) - D_\alpha\}$, and let $K^* = \bar{\partial} \Gamma \in A^{1,1}(\text{Hom}(T, T))$. Finally, define $\tau = \sum \rho_\alpha \{\tilde{V}_\alpha - i(V)D_\alpha\} \in A^{0,0}(\text{Hom}(T, T) \otimes L)$. The following lemma is proved by a local calculation which will be omitted.

LEMMA 5. $i(V)K^* + \bar{\partial}\tau = 0$ in $A^{0,1}(\text{Hom}(T, T) \otimes L)$.

One may therefore perform the ad-invariant map construction to get a class $p(K^* + \tau) = p((K^* + \tau)^{\otimes k})$ in C_k^0 where $k = \deg p$. Because of Lemma 5, it follows that $p(K^* + \tau)$ is a total cocycle. Note

$$p(K^* + \tau) = p(K^{*\otimes k}) + kp(K^{*\otimes k-1} \otimes \tau) + \cdots + p(\tau^{\otimes k}).$$

Note that by definition, $p(\tau^{\otimes k})$ is an extension of $p(V_0)$ to X . If $k = \deg p =$

n , we therefore get by (1) that

$$\left(\frac{1}{2\pi i}\right)^n \int_X p(K^{\otimes n}) = \text{Res } p(V_0)$$

which is part of the M.V.F. Theorem. The rest of the theorem is an identification of $(1/2\pi i)^n \int_X p(K^{\otimes n})$ with a characteristic number of $T - L^{-1}$.

In order to prove Theorem 2, note that in the case $\deg p = k < n$, if $\sigma \in H^0(X, L)$, then $\sigma^{n-k}p(V_0)$ lies in $H^0(X, \mathcal{L}_Z^n)$ and $m\phi_n(\sigma^{n-k}p(V_0)) = (-1)^n m(\sigma^{n-k} \times p(K^{\otimes k}))$ since $\phi_n(\sigma^{n-k}p(V_0))$ is represented by the total cocycle $\sigma^{n-k}p(K^* + \tau)$ of C_n^0 (due to the fact that σ^{n-k} commutes with both $i(V)$ and $\bar{\partial}$). But clearly $m(\sigma^{n-k}p(K^* + \tau)) = 0$ since $\deg p < n$. Consequently $\text{Res}(\sigma^{n-k}p(V_0)) = 0$, by (1), as asserted.

REMARK. The notion of V -equivariance is studied in [1] and in [4] from a different viewpoint. In [4] it is shown that if X is projective, then given a holomorphic vector bundle E on X , there exists a $V \in H^0(X, T \otimes L)$ with isolated zeros for which E is V equivariant. Thus E 's characteristic numbers can be computed as above.

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