THE CAUCHY PROBLEM FOR HYPERBOLIC OPERATORS WITH VARIABLE MULTIPLE CHARACTERISTICS

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ABSTRACT. Let $P(t, x, D_t, D_x)$ be a hyperbolic differential operator with the principal symbol $p_m(t, x, \tau, \xi)$. We assume that P_m is denoted by $\prod_{j=1}^s (\tau - \lambda_j)^{m_j} \prod_{j=s+1}^{m-N+s} (\tau - \lambda_j)$ and $(\lambda_i - \lambda_j)(t, x, \xi) \neq 0$ if $(i, j) \neq (k, m - N + k)$ ($k = 1, \dots, s$), where $N = \sum_{j=1}^s m_j$ and $\lambda_j(t, x, \xi) \in C^\infty([0, T] \times R^n \times (R^n \setminus 0))$. Under a generalized condition of E. E. Levi, we shall show that the Cauchy problem $Pu = f$ in $[0, T] \times R^n$, $D_t^j u|_{t=0} = g_j$ ($j = 1, \dots, m - 1$) is well posed. When $m_j = 1$ ($j = 1, \dots, s$), our result coincides those of Ohya and Petkov.

1. Statement of the result. We shall consider the following differential operator:

$$P(t, x, D_t, D_x) = \sum_{|\alpha| \leq m} a_\alpha(t, x) D_t^{\alpha_0} D_x^{\alpha'},$$

where $(t, x) \in [0, T] \times R^n$ ($0 < T < \infty$), $D_t^{\alpha_0} D_x^{\alpha'} = D_t^{\alpha_0} D_{x_1}^{\alpha'_1} \cdots D_{x_n}^{\alpha'_n}$ and $D_t = -i\partial/\partial t$, $D_{x_j} = -i\partial/\partial x_j$. We assume that $a_{m,0,\dots,0}(t, x) = 1$ and $a_\alpha(t, x)$ belongs to $\mathcal{B}([0, T] \times R^n)$, which consists of all functions having that these arbitrary derivations are bounded in $[0, T] \times R^n$. Let (τ, ξ) be the covariable of (t, x) . Then we define the following symbols:

$$p_k(t, x, \tau, \xi) = \sum_{|\alpha| = k} a_\alpha(t, x) \tau^{\alpha_0} \xi^{\alpha'} \quad (k = 0, \dots, m).$$

First we shall impose the following assumptions on p_m :

$$p_m(t, x, \tau, \xi) = \prod_{j=1}^s (\tau - \lambda_j)^{m_j} \prod_{j=s+1}^{m-N+s} (\tau - \lambda_j), \quad (\text{A.1})$$

where $m_1 \geq m_2 \geq \cdots \geq m_s$, $N = \sum_{j=1}^s m_j$ and all functions $\lambda_j(t, x, \xi)$ ($j = 1, \dots, m - N + s$) are real and positively homogeneous of degree 1 with respect to ξ and belong to $\mathcal{B}([0, T] \times R^n \times S^{n-1})$ if $\xi \in S^{n-1}$. Here S^{n-1} is the unit sphere of R^n .

(A.2) For any couple $(i, j) \neq (k, m - N + k)$ ($k = 1, \dots, s$) we suppose the following:

$$|(\lambda_i - \lambda_j)(t, x, \xi)| \geq C, \quad (t, x, \xi) \in [0, T] \times R^n \times S^{n-1},$$

where C is a positive constant.

Throughout this note, the symbols of pseudo-differential operators are elements of $\mathcal{B}([0, T] \times R^n \times S^n)$ or $\mathcal{B}([0, T] \times R^n \times S^{n-1})$ if $(\tau, \xi) \in S^n$ or

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$\xi \in S^{n-1}$. Moreover we use the following notation. For symbols $a(t, x, \xi)$, $b(t, x, \xi)$ and $A(t, x, \tau, \xi)$, $A|_{\tau=a} \equiv 0 \bmod b$ means that there exists a symbol $c(t, x, \xi)$ such that

$$A(t, x, a(t, x, \xi), \xi) = c(t, x, \xi)b(t, x, \xi).$$

For the lower order terms of P we assume the following:

(A.3) For any k ($k = 1, \dots, s$) we can denote $P(t, x, D_t, D_x)$ by the following form:

$$P = \sum_{l=0}^{m_k} Q_{k,l}(t, x, D_t, D_x)(\Lambda_k(t, x, D_t, D_x))^l. \quad (1.1)$$

Here Λ_k is the pseudo-differential operator defined by the symbol $\tau - \lambda_k(t, x, \xi)$ and $Q_{k,l}$ ($l = 0, \dots, m_k$) is a pseudo-differential operator of order $m - m_k$, whose principal symbol $q_{k,l}(t, x, \tau, \xi)$ has the following property:

$$q_{k,l}|_{\tau=\lambda_k} \equiv 0 \bmod \lambda_k - \lambda_{m-N+k}. \quad (1.2)$$

Clearly if $\lambda_k = \lambda_{m-N+k}$, then the above condition (A.3) is that of E. E. Levi. In the final part of this note we denote (A.3) by the condition with respect to p_k , when $m_k = 1$ or 2 .

For a nonnegative integer k and $s \in R$ the function space $C^k([0, T]; H_s(R^n))$ consists of functions such that $D_t^j u(t)$ ($j = 0, \dots, k$) exists as an element of $H_{s-j}(R^n)$ and is continuous on the topology of $H_{s-j}(R^n)$. We use the following norm:

$$|||u(t)|||_{s,k}^2 = \sum_{j=0}^k \|D_t^j u(t)\|_{s-j}^2,$$

where $\|\cdot\|_{s-j}$ is the usual norm of $H_{s-j}(R^n)$.

Now we can state our theorem.

THEOREM. Let $P(t, x, D_t, D_x)$ be a differential operator of order m . If P satisfies the assumptions (A.1), (A.2) and (A.3), then the Cauchy problem $Pu = f$ in $[0, T] \times R^n$, $D_t^j u|_{t=0} = g_j$ ($j = 0, \dots, m-1$) is well posed, i.e., for $f \in C^{k-m+m_1+1}([0, T]; H_{s-m+m_1+1}(R^n))$ and $g_j \in H_{s-j+m_1}(R^n)$ there exists a unique solution $u(t, x) \in C^k([0, T]; H_s(R^n))$ such that

$$|||u(t)|||_{s,k} \leq C \left\{ \sum_{j=0}^{m-1} \|g_j\|_{s-j+m_1} + |||f(0)|||_{s-m+m_1, k-m+m_1} + \int_0^t |||f(r)|||_{s-m+m_1+1, k-m+m_1+1} dr \right\},$$

where $k \geq m - m_1 - 1$, $|||f(0)|||_{s-m+m_1, -1} = 0$ and $t \in [0, T]$.

This Theorem is the same as those of [3] and [4] when $m_j = 1$ ($j = 1, \dots, s$). Under a different situation, in [1] they consider the Cauchy problem of a triple case.

2. Reform of the condition (A.3). In this section, we state an equivalent condition of (A.3).

Taking care of multiplicities of the roots λ_j ($j = 1, \dots, s$), we can denote $p_m(t, x, \tau, \xi)$ by

$$(\tau - \lambda_{m-N+s}) \cdots (\tau - \lambda_{s+1}) \Phi_\mu^{n_\mu} \cdots \Phi_1^{n_1},$$

where $\Phi_\nu(t, x, \tau, \xi)$ ($\nu = 1, \dots, \mu$) is a polynomial of degree s with respect to τ and equal to $\prod_{j=1}^s (\tau - \lambda_j)$. Here $m_{s_{\nu-1}+1} = \cdots = m_{s_\nu}$ ($\nu = 1, \dots, \mu$). Remark that $s_1 < s_2 < \cdots < s_\mu = s$, $m_1 = \sum_{\nu=1}^\mu n_\nu$ and denote N_ν by $s_\nu n_\nu$. We introduce a product pseudo-differential operator $\Phi_\nu(t, x, D_t, D_x) = (\Lambda_{s_\nu} \cdots \Lambda_1)(t, x, D_t, D_x)$. Then we denote $\Delta_j(t, x, D_t, D_x)$ of order j ($j = 0, \dots, m$) by $\Delta_0 = 1$, $\Delta_1 = \Lambda_1, \dots, \Lambda_j, \dots, \Delta_N = \Phi_\mu^{n_\mu} \cdots \Phi_1^{n_1}, \dots, \Delta_{N+k} = \Lambda_{s+k} \cdots \Lambda_{s+1} \Delta_N, \dots, \Delta_m = \Lambda_{m-N+s} \cdots \Lambda_{s+1} \Delta_N$, where $\Delta_j = \Lambda_\delta \cdots \Lambda_1 \Phi_\nu^\sigma \Phi_{\nu-1}^{n_{\nu-1}} \cdots \Phi_1^{n_1}$ if $j = (N_1 + \cdots + N_{\nu-1}) + \sigma s_\nu + \delta$ ($\sigma = 0, \dots, n_\nu - 1, \delta = 0, \dots, s_{\nu-1}$). Then we have the following:

PROPOSITION 2.1. *Let $P(t, x, D_t, D_x)$ be a differential operator which satisfies the conditions (A.1) and (A.2). Then the condition (A.3) is equivalent to the following statement. We can denote P by*

$$P(t, x, D_t, D_x) = \sum_{i=0}^{m_1} Q_i(t, x, D_t, D_x) \Delta_{I(i)}, \quad (2.1)$$

where if $i = n_\mu + \cdots + n_{\nu+1} + \sigma$ ($1 \leq \sigma \leq n_\nu$), then $I(i) = N_1 + \cdots + N_\nu - s_\nu \sigma$ and Q_i ($i = 0, \dots, m_1$) is a pseudo-differential operator of order $M_i = m - i - I(i)$ and differential operator of t . Moreover the principal symbol $q_i(t, x, \tau, \xi)$ of Q_i satisfies the following condition:

$$q_i|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}} \quad \text{if } k \leq s_\nu. \quad (2.2)$$

Since any pseudo-differential operator of order l ($\leq m$) which is a differential operator of t is represented by $\Delta_0, \dots, \Delta_m$, we have the following:

PROPOSITION 2.2. *Let P be a differential operator which satisfies the conditions (A.1), (A.2), (2.1) and (2.2). Then P is denoted by*

$$P = \sum_{i=0}^{m_1} \sum_{j=0}^{M_i} R_{i,j}(t, x, D_x) \Delta_{I(i)+j}, \quad (2.3)$$

where $R_{i,j}$ is a pseudo-differential operator of order $M_i - j$ whose principal symbol is $r_{i,j}(t, x, \xi)$, and $r_{i,j}$ have the following property:

$$\sum_{j=0}^{k-1} r_{i,j} \Lambda_j \cdots \Lambda_{l|\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}, \quad k \leq s_\nu, \quad (2.4)$$

if $i = n_\mu + \cdots + n_{\nu+1} + \sigma$ ($1 \leq \sigma \leq n_\nu$).

3. Reduction to a first order system and the proof of the Theorem. Since the proof of the Theorem is inferred on the analogy of a simple case, we assume that $s = 2$, $m_1 = 2$ and $m_2 = 1$. Thus by (2.3) our considered operator

$P(t, x, D_t, D_x)$ in (1.1) is denoted by

$$P(t, x, D_t, D_x) = \Delta_m + \sum_{i=1}^2 \sum_{j=0}^{m-i} R_j^{m-i} \Delta_j,$$

where R_j^{m-i} is of order $m - i - j$. The condition (2.4) says that

$$R_0^{m-1}(t, x, D_x) = 0, \quad (3.1)$$

$$r_1^{m-1}(t, x, \xi) \equiv r_0^{m-2}(t, x, \xi) \equiv 0 \pmod{\lambda_1 - \lambda_{m-2}}, \quad (3.2)$$

$$(r_1^{m-1} + r_2^{m-1}(\lambda_2 - \lambda_1))(t, x, \xi) \equiv 0 \pmod{\lambda_2 - \lambda_{m-1}}. \quad (3.3)$$

We denote a pseudo-differential operator with the symbol $|\xi|$ by the term Λ and define a column vector

$$U = (\Lambda^{m-3} \Delta_0 u, \Lambda^{m-3} \Delta_1 u, \Lambda^{m-4} \Delta_2 u, \Lambda^{m-4} \Delta_3 u, \dots, \Lambda \Delta_{m-2} u, \Delta_{m-1} u) \quad (3.4)$$

and $F = (0, \dots, 0, f)$. Then by (3.1) the equation $Pu = f$ becomes the following first order system:

$$MU = (D_t - A(t, x, D_x))U + B(t, x, D_x)U = F,$$

where B is of order 0 and A is a first order pseudo-differential operator with the symbol

$$a(t, x, \xi) = \begin{pmatrix} \lambda_1, 0, & & & & & \\ & \lambda_1, |\xi|, & & \circ & & \\ & & \lambda_2, 0, & & & \\ & & & \lambda_3, |\xi|, & & \\ & & & & \circ & \\ & & & & & \lambda_{m-2}, |\xi| \\ a_{m1}, a_{m2}, a_{m3}, 0, & , & 0 & , & \lambda_{m-1} \end{pmatrix},$$

where $a_{m1} = -r_0^{m-2}(t, x, \xi/|\xi|)|\xi|$, $a_{m2} = -r_1^{m-1}(t, x, \xi/|\xi|)|\xi|$ and $a_{m3} = -r_2^{m-1}(t, x, \xi/|\xi|)|\xi|$. Then a has the following property.

PROPOSITION 3.1. *There exists a nonsingular matrix $N(t, x, \xi)$ whose components are homogeneous of degree 0 and belong to $\mathcal{B}([0, T] \times R^n \times (R^n \setminus 0))$ such that*

$$(aN)(t, x, \xi) = (ND)(t, x, \xi),$$

where $D = (d_{ij})$ is a $m \times m$ diagonal matrix with $d_{11} = d_{22} = \lambda_1$ and $d_{ii} = \lambda_{i-1}$ ($i = 2, \dots, m$).

PROOF. Since clearly an eigenvalue of $a(t, x, \xi/|\xi|)$ is $\tilde{\lambda}_j = \lambda_j(t, x, \xi/|\xi|)$ ($j = 1, \dots, m-1$), we shall seek an eigenvector of $\tilde{\lambda}_j$. The equation

$$(\mu I - a(t, x, \xi/|\xi|))n = 0,$$

where $n = (n_1, \dots, n_m)$, is equivalent to the following:

$$\begin{aligned} (\mu - \tilde{\lambda}_1)n_1 &= 0, \quad (\mu - \tilde{\lambda}_1)n_2 - n_3 = 0, \quad (\mu - \tilde{\lambda}_2)n_3 = 0, \\ (\mu - \tilde{\lambda}_3)n_4 &= n_5, \dots, (\mu - \tilde{\lambda}_{m-2})n_{m-1} = n_m, \\ (\mu - \tilde{\lambda}_{m-1})n_m &+ \tilde{r}_0^{m-2}n_1 + \tilde{r}_1^{m-1}n_2 + \tilde{r}_2^{m-1}n_3 = 0, \end{aligned} \quad (3.5)$$

where $\tilde{r}_j^{m-i}(t, x, \xi) = r_j^{m-i}(t, x, \xi/|\xi|)$. Since $\tilde{r}_1^{m-1}n_2 + \tilde{r}_2^{m-1}n_3 = (\tilde{r}_1^{m-1} + \tilde{r}_2^{m-1}(\mu - \tilde{\lambda}_1))n_2$, for example in fact we can take the following matrix $N(t, x, \xi)$ such that each column of N is an eigenvector of an eigenvalue $\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_{m-1}$ respectively in turn;

$$N(t, x, \xi) = \begin{pmatrix} 1 & ,0 & ,0 & ,0 & \dots & ,0 \\ 0 & ,1 & ,(\lambda_2 - \lambda_1)^{-1} & ,0 & \dots & ,0 \\ 0 & ,0 & ,1 & ,0 & \dots & ,0 \\ & & & ,1 & & \\ & & * & \cdot & * & \\ & & & & \cdot & \\ n_{m-11}, n_{m-12}, n_{m-13} & , & \circ & \cdot & & \\ n_{m1} & , n_{m2} & , n_{m3} & , & & ,1 \end{pmatrix}$$

where

$$\begin{aligned} n_{m1} &= \tilde{r}_0^{m-2}/(\tilde{\lambda}_1 - \tilde{\lambda}_{m-1}), \quad n_{m-11} = \tilde{r}_0^{m-2}/(\tilde{\lambda}_1 - \tilde{\lambda}_{m-1})(\tilde{\lambda}_1 - \tilde{\lambda}_{m-2}), \\ n_{m2} &= \tilde{r}_1^{m-1}/(\tilde{\lambda}_1 - \tilde{\lambda}_{m-1}), \quad n_{m-12} = \tilde{r}_1^{m-1}/(\tilde{\lambda}_1 - \tilde{\lambda}_{m-1})(\tilde{\lambda}_1 - \tilde{\lambda}_{m-2}), \\ n_{m3} &= (\tilde{r}_1^{m-1} + \tilde{r}_2^{m-1}(\tilde{\lambda}_2 - \tilde{\lambda}_1))/(\tilde{\lambda}_2 - \tilde{\lambda}_{m-1}), \quad n_{m-13} = n_{m3}/(\tilde{\lambda}_2 - \tilde{\lambda}_{m-2}). \end{aligned}$$

These are well defined by (A.2), (3.2) and (3.3) and $*$ are easily inductively determined from (A.2) and (3.5). Clearly the matrix $N(t, x, \xi)$ is positively homogeneous of degree 0 and a nonsingular matrix in $[0, T] \times R^n \times (R^n \setminus 0)$. This completes the proof of Proposition 3.1.

By Proposition 3.1 the following fact is well known (see [2]), which guarantees the validity of our theorem.

PROPOSITION 3.2. For any nonnegative integer k and $s \in R$ if $F(t, x) \in C^k([0, T]; H_s(R^n))$ and $U_0(x) \in H_s(R^n)$, then there exists a unique solution $U(t, x) \in C^k([0, T]; H_s(R^n))$ of the Cauchy problem for M with initial data on $t = 0$ such that

$$|||U(t)|||_{s,k} \leq C \left\{ |||U(0)|||_{s,k} + \int_0^t |||MU(r)|||_{s,k} dr \right\},$$

where C does not depend on $U(t, x)$ and $t \in [0, T]$.

REMARK. For a general case the vector U of (3.4) becomes the following:

$$U = (\Lambda^{m-m_1-1}\Delta_0 u, \dots, \Lambda^q \Delta_j, \dots, \Lambda^{m-N-1}\Delta_N, \dots, \Lambda \Delta_{m-2}, \Delta_{m-1}),$$

where $a_j = m - (n_\nu + \dots + n_\mu) + \sigma - j - 1$ if $j = N_1 + \dots + N_{\nu-1} + s_\nu \sigma + \delta$ ($\sigma = 0, \dots, n_\nu - 1, \delta = 0, \dots, s_\nu - 1$).

4. Some sufficient condition of (A.3) when $m_k = 1$ or 2. In this section we shall denote (A.3) by the conditions with respect to p_k when $m_k = 1$ or 2. We denote the subprincipal symbol of P by

$$p_{m-1}^s(x, \xi) = p_{m-1} + (i/2) \sum_{j=0}^n \frac{\partial^2 p_m}{\partial x_j \partial \xi_j}$$

and the Poisson bracket of $f(x, \xi)$ and $g(x, \xi)$ by

$$\{f, g\}(x, \xi) = \sum_{j=0}^n \left(\frac{\partial f}{\partial \xi_j} \frac{\partial g}{\partial x_j} - \frac{\partial f}{\partial x_j} \frac{\partial g}{\partial \xi_j} \right)(x, \xi),$$

where for simplicity we identify (t, τ) with (x_0, ξ_0) in this section. We state the following sufficient condition of (A.3), which is a generalized condition of that in [4].

PROPOSITION 4.1. *We have the following:*

(i) *When $m_k = 1$, the condition (A.3) is equivalent to the following:*

$$p_{m-1}^s + (i/2)r_0\{\Lambda_{m-N+k}, \Lambda_k\}|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}, \quad (4.1)$$

where $r_0(t, x, \tau, \xi) = p_m/(\tau - \lambda_k)(\tau - \lambda_{m-N+k})$.

(ii) *When $m_k = 2$, if the following three conditions hold, then (A.3) is valid;*

$$p_{m-1}^s(t, x, \lambda_k(t, x, \xi), \xi) = 0, \quad (4.2)$$

$$\partial p_{m-1}^s / \partial \tau|_{\tau=\lambda_k} \equiv \{\Lambda_{m-N+k}, \Lambda_k\}|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}, \quad (4.3)$$

$$\begin{aligned} p_{m-2} - \sum_{j=0}^n p_{m-1,j}^{(j)}/2 + \sum_{l,j=0}^n p_{m,lj}^{(lj)}/8 \\ - \sum_{j=0}^n r_0(\{\Lambda_{m-N+k}^{(j)}, \Lambda_k\} \Lambda_{kj} - \{\Lambda_{m-N+k,j}, \Lambda_k\} \Lambda_k^{(j)})/4 \equiv 0 \\ \pmod{\lambda_k - \lambda_{m-N+k}}, \end{aligned} \quad (4.4)$$

where $f_\beta^{(\alpha)} = (iD_\xi^\alpha)D_x^\beta f(x, \xi)$ and $r_0(t, x, \tau, \xi) = p_m/(\tau - \lambda_k)^2(\tau - \lambda_{m-N+k})$.

PROOF. (i) Since $R_0\Lambda_k$, where R_0 is a pseudo-differential operator with the symbol $p_m/(\tau - \lambda_k)$, satisfies the condition (A.3), we may show that the principal symbol $r(x, \xi)$ of $P - R_0\Lambda_k$ satisfies the condition: $r|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}$. By an easy computation we see that

$$(r - p_{m-1}^s) - \frac{i}{2} r_0\{\Lambda_{m-N+k}, \Lambda_k\}|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}.$$

This implies (4.1).

(ii) In this case if we denote the principal symbol of $P - R_0\Lambda_k^2$ by $r(t, x, \tau, \xi)$, where R_0 is a pseudo-differential operator defined by the symbol $p_m/(\tau - \lambda_k)^2$, then r is written by the following form:

$$r = p_{m-1}^s + (ir_0\{\Lambda_{m-N+k}, \Lambda_k\} + A\Lambda_{m-N+k})\Lambda_k, \quad (4.5)$$

where A is some positively homogeneous function of degree $m-3$. Thus by (4.2) and (4.5) we can denote P by $R_0\Lambda_k^2 + R_1\Lambda_k + R_2$, where R_j is a pseudo-differential operator of order $m-2$ and the principal symbol is written by $r_j(t, x, \tau, \xi)$. By (4.5) if we assume (4.3), then we have $r_1|_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}$.

For simplicity we denote the homogeneous symbol of order i of a pseudo-differential operator $Q(t, x, D_t, D_x)$ by $\sigma_i(Q)$. Then we have

$$r_2|_{\tau=\lambda_k} = p_{m-2} - \sigma_{m-2}(R_0\Lambda_k^2) - \sum_{j=0}^n r_1^{(j)}\Lambda_{k,j}|_{\tau=\lambda_k}. \quad (4.6)$$

Since (4.3) holds and $r_1\Lambda_k = p_{m-1} - \sigma_{m-1}(R_0\Lambda_k^2)$, we have

$$\sum_{j=0}^n \left(r_1^{(j)}\Lambda_{k,j} - (p_{m-1} - \sigma_{m-1}(R_0\Lambda_k^2))^{(j)}_j / 2 \right) |_{\tau=\lambda_k} \equiv 0 \pmod{\lambda_k - \lambda_{m-N+k}}.$$

Therefore we have

$$\begin{aligned} r_2|_{\tau=\lambda_k} &\equiv p_{m-2} - \sum_{j=0}^n p_{m-1,j}^{(j)}/2 + \sum_{j=0}^n \sigma_{m-1}(R_0\Lambda_k^2)_j^{(j)}/2 \\ &\quad - \sigma_{m-2}(R_0\Lambda_k^2)|_{\tau=\lambda_k} \pmod{\lambda_k - \lambda_{m-N+k}}. \end{aligned} \quad (4.7)$$

Here

$$\sigma_{m-1}(R_0\Lambda_k^2) = \sum_{j=0}^n (r_0\Lambda_k^{(j)}\Lambda_{k,j} + 2r_0^{(j)}\Lambda_{k,j}\Lambda_k), \quad (4.8)$$

$$\begin{aligned} \sigma_{m-2}(R_0\Lambda_k^2)|_{\tau=\lambda_k} &\equiv \sum_{l,j=0}^n \left(r_0^{(l)}(\Lambda_{k,l}^{(j)}\Lambda_{k,j} + \Lambda_k^{(j)}\Lambda_{k,l}) + r_0^{(lj)}\Lambda_{k,l}\Lambda_{k,j} \right) |_{\tau=\lambda_k} \\ &\pmod{\lambda_k - \lambda_{m-N+k}}. \end{aligned} \quad (4.9)$$

By taking care of (4.8), (4.9) and (4.3), add $\sum_{l,j=0}^n p_{m,j}^{(lj)}/8|_{\tau=\lambda_k}$ to the right hand side of (4.7). Then we have that $r_2|_{\tau=\lambda_k}$ is equal to the left hand side of (4.4) $\pmod{\lambda_k - \lambda_{m-N+k}}$. This completes the proof of Proposition 4.1.

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