SHORTER NOTES

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ON MONOIDAL TRANSFORMATIONS OF COHERENT SHEAVES

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ABSTRACT. We prove that the monoidal transformation of a coherent sheaf S is torsion-free if and only if S is locally-free.

Rossi, in [4], introduces the notion of the monoidal transformation of a reduced complex space X , with respect to a coherent sheaf S over X . This is a pair $(\hat{X}, \pi)$ where $\hat{X}$ is a reduced complex space and $\pi: \hat{X} \rightarrow X$ is a proper modification mapping such that $\pi^*S/\text{Tor}(\pi^*S)$ is locally-free. Riemenschneider, in [3], characterizes these transforms by a universal property. In [2], Grauert and Riemenschneider exhibit an example to show that π^*S may have torsion even if S itself is torsion-free. The purpose of this note is to prove:

PROPOSITION. *Let S be a coherent sheaf over a reduced complex space X . Let $(\hat{X}, \pi)$ be the monoidal transformation of X with respect to S . Then π^*S is torsion-free if and only if S is locally-free.*

PROOF. One direction is trivial. If S is locally-free, then $\hat{X} \cong X$ and π is the identity map. Thus $\pi^*S = S$ which is locally-free and so, a fortiori, is torsion-free. Suppose that S is not locally-free. Let $E = V(S)$ be the linear space associated to S in the sense of Grauert (see e.g. [1, p. 52]). If E were a vector bundle then S would be locally-free; thus E has varying fiber dimensions. Let π^*E be the inverse image of E . π^*E is the fiber product of $\hat{X}$ and E over X (see [1, p. 53]). Since π is surjective, π^*E does not have constant fiber dimension (see [1, Corollary 0.32, p. 29]), so π^*E is not a vector bundle. Now, by definition, $\pi^*S/\text{Tor}(\pi^*S)$ is locally-free. If π^*S were torsion-free, it would follow that π^*S is locally-free. But if π^*S were locally-free, then by [1, p. 53], we would have $\pi^*E = \pi^*(V(S)) = V(\pi^*S)$ is a vector bundle. Thus $\text{Tor}(\pi^*S)$ is nonzero; that is, π^*S has torsion.

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