THE STABILITY OF DE RHAM CURRENTS

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ABSTRACT. The purpose of this short note is to give a theorem concerning the action of the diffeomorphism group of a smooth manifold on the space of de Rham currents.

Let M be a paracompact, orientable n -dimensional, C^∞ -manifold. We shall denote by $\mathcal{D}'^p(M)$ [resp. $\mathcal{Z}'^p(M)$] the space of de Rham currents [resp. closed de Rham currents] on M , endowed with the uniform convergence topology [resp. induced topology] [3].

It is easy to see that the notions of local and global stability for differential forms [1], [2] can be extended, in natural way, for de Rham currents.

The following theorem shows that there exist no globally defined stable de Rham currents on M .

THEOREM. *Let $T \in \mathcal{D}'^p(M)$. Then for any neighborhood V_T of T there exists a p -current S in V_T such that for any smooth diffeomorphism $f: M \rightarrow M$ one has $f(S) \neq T$.*

THE SKETCH OF THE PROOF. We shall suppose that the theorem is not true. First one considers the case when T is a form-like de Rham current (i.e., a current induced by a form), and then the case when T is not a form-like de Rham current. Using the density of Dirac currents for the first case and the density of form-like currents for the second case, one obtains immediately a contradiction. Q.E.D.

So the notion of global stability for de Rham currents does not make sense.

REMARK. Assuming that M is a compact manifold without boundary, and taking into account the Hodge-Kodaira-de Rham decomposition theorem [3], the above theorem is also true if one replaces $\mathcal{D}'^p(M)$ with $\mathcal{Z}'^p(M)$.

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