EVERY FINITE ABELIAN GROUP IS THE BRAUER GROUP OF A RING

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ABSTRACT. Given an arbitrary finite abelian group G a ring R is constructed using cohomological techniques from algebraic geometry whose Brauer group is G . If G is a cyclic group, then R can be taken to be a three-dimensional noetherian integral domain. If G is not a cyclic group the ring R is a three-dimensional noetherian ring. At the expense of raising the dimension of R , R can be chosen to be a domain. We also calculate $B(R[x, 1/x])$ for R a commutative noetherian regular ring containing a field of characteristic zero.

0. Introduction. Let $A = \mathbb{C}[x, y]/(y^2 - x^2(x + 1))$ be the affine coordinate ring of the nodal cubic curve $y^2 - x^2(x + 1)$ over the complex number field. Let $n > 1$ be an integer, let $f_n = zy^{n-1} - x^n$ and let B_n be the subring of the localized ring $\mathbb{C}[x, y, z][1/f_n]$ consisting of all fractions g/f_n^r such that g is a homogeneous polynomial of degree $r \cdot n$, $r > 0$. We show that $B(A \otimes_{\mathbb{C}} B_n) = \mathbb{Z}_n$. Since B_n is the affine coordinate ring of a surface we see every finite cyclic group is the Brauer group of a three-dimensional noetherian integral domain. Since every finite abelian group G is of the form $\mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_r}$ we see that G is the Brauer group of the three-dimensional noetherian ring $A \otimes_{\mathbb{C}} (B_{n_1} \oplus \cdots \oplus B_{n_r})$. §4 uses excision techniques to calculate $B(R[x, 1/x])$ for certain rings R .

In this paper cohomology is always étale cohomology and a sheaf will always mean a sheaf in the étale topology. Throughout all unexplained notation and terminology is as in [10]. Tensor products and fibred products are always over the ground field k which is the field of complex numbers except in §4 where k is only required to have characteristic zero.

1. A Mayer-Vietoris sequence and an excision sequence. Let

$$\begin{array}{ccc} Y_0 & \leftarrow & Y_1 \\ \uparrow & & \uparrow \\ Y_2 & \leftarrow & Y_3 \end{array} \quad (1.1)$$

be a commutative diagram of schemes. Let F be a contravariant functor from the category of schemes over Y_0 to the category of abelian groups. Suppose F_{Y_i} (that is, F restricted to Y_i) is a sheaf on Y_i for each i . We will write FY_i to mean the direct image sheaf $f_*F_{Y_i}$ on Y_0 defined by $f_*F_{Y_i}(V) = F_{Y_i}(V \times_{Y_0} Y_i)$.

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DEFINITION 1.2. The diagram (1.1) is called a cartesian square with respect to the functor F if the sequence $0 \rightarrow F_{Y_0} \rightarrow FY_1 \oplus FY_2 \rightarrow FY_3 \rightarrow 0$ is an exact sequence of sheaves on Y_0 , where the maps are the diagonal map and the difference map, respectively.

PROPOSITION 1.3. *If (1.1) is a cartesian square with respect to the functor F then*

(a) *there is a long exact sequence of cohomology:*

$$\cdots \rightarrow H^n(Y_0, F_{Y_0}) \rightarrow H^n(Y_0, FY_1) \oplus H^n(Y_0, FY_2) \rightarrow H^n(Y_0, FY_3) \rightarrow \cdots;$$

(b) *if all the morphisms of (1.1) are finite morphisms, there is a long exact sequence:*

$$\cdots \rightarrow H^n(Y_0, F_{Y_0}) \rightarrow H^n(Y_1, F_{Y_1}) \oplus H^n(Y_2, F_{Y_2}) \rightarrow H^n(Y_3, F_{Y_3}) \rightarrow \cdots.$$

PROOF. (a) The proof follows from the definition of cohomology. To prove (b) we must show that $H^n(Y_0, FY_i) = H^n(Y_i, F_{Y_i})$. This follows from the Leray spectral sequence for each f in (1.1). Since the morphisms are finite $R^p f_* F_{Y_i} = 0$ for $p > 0$ [3, Arcata, Proposition II 3.6] so the result follows.

Let X be a locally noetherian regular scheme and Z a closed subscheme of X of codimension $d > 0$. Let F be a sheaf on X . Denote by $H_Z^0(X, F)$ the group of sections in $H_Z^0(X, F)$ with support in Z and by $H_Z^0(F)$ the subsheaf of F defined by the sections with support in Z . Define the groups (respectively, sheaves) of cohomology with support in Z , $H_Z^p(X, F)$ (respectively, $H_Z^p(F)$) to be the right derived functors of $H_Z^0(X, F)$ (respectively, $H_Z^0(F)$). The cohomology groups of X and $X - Z$ are related via the long exact sequence of excision:

$$\cdots \rightarrow H_Z^n(X, F) \rightarrow H^n(X, F) \rightarrow H^n(X - Z, F) \rightarrow H_Z^{n+1}(X, F) \rightarrow \cdots \quad (1.4)$$

and the spectral sequence $H_Z^{p+q}(X, F) \leftarrow H^p(Z, H_Z^q(F))$. For more details on cohomology with supports in a closed subscheme see [9, V, 6 and VIII, 6.6] and [7, III, 6].

2. The Brauer group of a ruled surface. In this section a ruled surface X is a nonsingular projective surface over the field k together with a surjective morphism $\pi: X \rightarrow C$ to a nonsingular projective curve C such that the fibre X_y is isomorphic to $\mathbf{P}^1$ for every closed point $y \in C$ and such that π admits a section. From [10, V, 2.2] we see that every ruled surface is birationally equivalent to $C \times \mathbf{P}^1$. Since the Brauer group is a birational invariant between complete nonsingular surfaces [7, III, Corollaire 7.2], to study the Brauer group of ruled surfaces we look at those of the form $C \times \mathbf{P}^1$.

PROPOSITION 2.1. *Let C be a (not necessarily projective) nonsingular integral complex curve and $X = C \times \mathbf{P}^1$. Then $B(X) = 0$.*

PROOF. Let $U = \text{Spec } R$ be an affine open subset of C . Let $V = \text{Spec}(k[x])$ be an open affine subset of $\mathbf{P}^1$ which is isomorphic to the affine line. Then $U \times V = \text{Spec}(R[x])$ is an open affine subset of X . Let K be the function field of X . The

natural map $B(X) \rightarrow B(K)$ is one-to-one and factors through $B(U \times V)$ [1, Theorem 7.2]. Since C is a complex curve $B(R) = 0$ [7, III, p. 89]; hence $B(U \times V) = B(R[x]) = B(R) = 0$ [1, Proposition 7.7]. So $B(X) = 0$.

3. Every finite cyclic group is a Brauer group. Let X be a noetherian integral scheme. Let $U = \text{Spec } A$ be an open affine subset of X . Let $\tilde{A}$ be the integral closure of A in its quotient field and let $\tilde{c}$ be the conductor ideal from $\tilde{A}$ to A , $\tilde{c} = \{r \in \tilde{A} | rx \in A \text{ for every } x \in \tilde{A}\}$. Glue together the $\tilde{U} = \text{Spec } \tilde{A}$ giving the normalization $\tilde{X}$ of X . We define the conductor $\tilde{c}$ from $\tilde{X}$ to X to be the quasicoherent sheaf of ideals $\tilde{c}$ on $\tilde{X}$. Define c to be the direct image sheaf $\pi_* \tilde{c}$ under the finite morphism $\pi: \tilde{X} \rightarrow X$. For each $U = \text{Spec } A$ we have $c(U) = \tilde{c}(\tilde{U})$. Let $(\tilde{Y}, \mathcal{O}_{\tilde{X}}/\tilde{c})$ and $(Y, \mathcal{O}_X/c)$ be the closed subschemes of $\tilde{X}$ and X corresponding to $\tilde{c}$ and c , respectively. There is a commutative diagram of schemes

$$\begin{array}{ccc} X & \leftarrow & \tilde{X} \\ \uparrow & & \uparrow \\ Y & \leftarrow & \tilde{Y} \end{array}$$

where the vertical arrows are closed immersions, hence, are finite. We wish to see that this diagram is a cartesian square with respect to the units functor G_m . That is, we wish to see that the following is an exact sequence of sheaves on X :

$$0 \rightarrow G_m \rightarrow G_m \tilde{X} \oplus G_m Y \rightarrow G_m \tilde{Y} \rightarrow 0. \quad (3.1)$$

Since exactness is a local property we may assume that X is affine, say $X = \text{Spec } A$. Now the diagram reduces to the diagram of rings;

$$\begin{array}{ccc} A & \rightarrow & \tilde{A} \\ \downarrow & & \downarrow \\ A/c & \rightarrow & \tilde{A}/c \end{array}$$

Since this diagram together with the units functor and the étale topology satisfy properties MV1 and MV2 of [2], the sequence (3.1) is locally exact, hence is exact. By Proposition 1.3 there is a long exact sequence $\cdots \rightarrow H^n(X, G_m) \rightarrow H^n(\tilde{X}, G_m) \oplus H^n(Y, G_m) \rightarrow H^n(\tilde{Y}, G_m) \rightarrow \cdots$.

Let C be a singular integral curve over k . Let X be a noetherian integral separated regular scheme over k . Let $\tilde{C}$ be the normalization of C , $\tilde{Y}$ and Y the closed subschemes of $\tilde{C}$ and C corresponding to the conductor ideal from $\tilde{C}$ to C . The diagram

$$\begin{array}{ccc} C & \leftarrow & \tilde{C} \\ \uparrow & & \uparrow \\ Y & \leftarrow & \tilde{Y} \end{array}$$

is a cartesian square with respect to the units functor G_m . We wish to see that

$$\begin{array}{ccc} C \times X & \leftarrow & \tilde{C} \times X \\ \uparrow & & \uparrow \\ Y \times X & \leftarrow & \tilde{Y} \times X \end{array}$$

is also a cartesian square with respect to the functor G_m . If $U = \text{Spec } A$ is an open affine subset of C and $V = \text{Spec } B$ is an open affine subset of X then the diagram

$$\begin{array}{ccc} A \otimes B & \rightarrow & \tilde{A} \otimes B \\ \downarrow & & \downarrow \\ A/c \otimes B & \rightarrow & \tilde{A}/c \otimes B \end{array}$$

is a cartesian square since we are tensoring over the field k . Since the functor G_m satisfies MV1 and MV2 of [2] for this diagram the sequence

$$0 \rightarrow G_m \rightarrow G_m(\tilde{C} \times X) \oplus G_m(Y \times X) \rightarrow G_m(\tilde{Y} \times X) \rightarrow 0$$

is an exact sequence of sheaves on $C \times X$. So we have the Mayer-Vietoris sequence:

$$\begin{aligned} \text{Pic}(C \times X) &\rightarrow \text{Pic}(\tilde{C} \times X) \oplus \text{Pic}(Y \times X) \rightarrow \text{Pic}(\tilde{Y} \times X) \rightarrow H^2(C \times X, G_m) \\ &\rightarrow H^2(\tilde{C} \times X, G_m) \oplus H^2(Y \times X, G_m) \rightarrow H^2(\tilde{Y} \times X, G_m) \rightarrow \cdots \end{aligned} \quad (3.2)$$

Now let C be the affine curve given by $y^2 = x^2(x+1)$. The affine coordinate ring of C is $A = k[x, y]/(y^2 - x^2(x+1))$ and $\tilde{C} = \text{Spec } \tilde{A}$ is isomorphic to the affine line since $\tilde{A} = k[u]$, the map $A \rightarrow \tilde{A}$ being induced by $y \mapsto u^3 - u$ and $x \mapsto u^2 - 1$. The conductor ideal c is the maximal ideal in A generated by $y + x$ and $y - x$, Y is the origin $P = (0, 0)$, $\tilde{Y}$ consists of the two closed points Q_1 and Q_2 lying over P and $\tilde{c}$ is the intersection of the two maximal ideals $(u-1)$ and $(u+1)$ in $\tilde{A}$. Therefore $\tilde{Y} \times X \cong X \amalg X$ and $Y \times X \cong X$. The sequence (3.2) becomes

$$\begin{aligned} \text{Pic}(C \times X) &\rightarrow \text{Pic}(\tilde{C} \times X) \oplus \text{Pic } X \xrightarrow{\phi} \text{Pic } X \oplus \text{Pic } X \rightarrow H^2(C \times X, G_m) \\ &\rightarrow H^2(\tilde{C} \times X, G_m) \oplus H^2(X, G_m) \rightarrow H^2(X, G_m) \oplus H^2(X, G_m) \rightarrow \cdots \end{aligned}$$

We wish to see that the image of ϕ is the diagonal of $\text{Pic } X \oplus \text{Pic } X$. It is clear that the image of $\text{Pic}(Y \times X)$ is the diagonal of $\text{Pic } X \oplus \text{Pic } X$ so we check now that the image of $\phi_1: \text{Pic}(\tilde{C} \times X) \rightarrow \text{Pic } X \oplus \text{Pic } X$ is the diagonal. Let $\mathcal{L}$ be an invertible sheaf on $\tilde{C} \times X$ in the Zariski topology. Let $f_1: Q_1 \times X \rightarrow C \times X$ and $f_2: Q_2 \times X \rightarrow C \times X$ be the closed immersions. The map ϕ_1 is induced by $\mathcal{L} \mapsto (f_1^* \mathcal{L}, f_2^* \mathcal{L})$ where we are viewing sheaves on $Q_1 \times X$ and $Q_2 \times X$ as sheaves on X . By [10, Proposition II 6.6], $\text{Pic } X \cong \text{Pic}(\tilde{C} \times X)$ by the map $\mathcal{N} \mapsto \pi^* \mathcal{N}$ where $\pi: \tilde{C} \times X \rightarrow X$ is the projection. Since $\pi f_1 = 1_X$ and $\pi f_2 = 1_X$ we see that f_1^* and f_2^* are both inverses for π^* . So $f_1^* = f_2^*$ and $f_1^* \mathcal{L}$ is isomorphic to $f_2^* \mathcal{L}$ as $\mathcal{O}_X$ modules. So we see that the image of ϕ is the diagonal of $\text{Pic } X \oplus \text{Pic } X$. The following sequence is exact:

$$\begin{aligned} 0 \rightarrow \text{Pic } X &\rightarrow H^2(C \times X, G_m) \rightarrow H^2(\tilde{C} \times X, G_m) \oplus H^2(X, G_m) \\ &\rightarrow H^2(X, G_m) \oplus H^2(X, G_m). \end{aligned} \quad (3.3)$$

We will need the following.

LEMMA 3.4. *Let X be a regular affine surface of finite type over the field k . Then $H^p(X, G_m) = 0$ for $p > 2$.*

PROOF. By [3, Arcata, Théorème IV 6.4], $H^p(X, F) = 0$ for $p > 2$ if F is a torsion sheaf. So if μ_n denotes the kernel of the n th power map in the exact sequence of sheaves

$$0 \rightarrow \mu_n \rightarrow G_m \rightarrow G_m \rightarrow 0, \quad (3.5)$$

we see that $H^p(X, \mu_n) = 0$ for all $p > 2$ and for every $n > 0$. From the long exact sequence of cohomology associated to (3.5), $H^p(X, G_m) \cong H^p(X, G_m)$ by the n th power map for all $n > 0$ and all $p > 2$. However, by [7, II, Proposition 1.4] the groups $H^p(X, G_m)$ are torsion groups for $p > 2$, so $H^p(X, G_m) = 0$ for $p > 2$.

Let X be the projective plane $\mathbf{P}^2$ and let Z be the curve whose homogeneous equation is $f_n = zy^{n-1} - x^n = 0$, $n \geq 2$. Then Z is a curve of degree n , genus $(n-1)(n-2)/2$. If $n > 2$, Z is singular and its desingularization is isomorphic to $\mathbf{P}^1$. Let $U = X - Z$. Then U is a regular affine surface whose coordinate ring B_n is the set of all homogeneous elements of degree zero in the localized ring $k[x, y, z][1/f_n]$. We will see that $\text{Pic}(B_n) = \mathbf{Z}_n$ and $B(B_n) = 0$.

To calculate the Picard group of U consider the exact sequence $0 \rightarrow \mathbf{Z} \rightarrow \text{Pic}(X) \rightarrow \text{Pic}(U) \rightarrow 0$ where the first map is induced by $1 \mapsto Z$ [10, Proposition II 6.5]. Since $\deg(Z) = n$ and $\text{Pic } X \cong \mathbf{Z}$ by the degree map, we see that $\text{Pic}(U) = \mathbf{Z}_n$.

Let Y be the line at infinity on X . The exact sequence (1.4) becomes

$$\cdots \rightarrow B(X - Y) \rightarrow H_Y^3(X, G_m) \rightarrow H^3(X, G_m) \rightarrow H^3(X - Y, G_m) \rightarrow \cdots \quad (3.6)$$

where we replace $H^2(X - Y, G_m)$ by $B(X - Y)$ since $X - Y$ is a regular surface [7, II, Corollaire 2.2]. Since $X - Y$ is just the affine plane, $B(X - Y) = B(k[x, y]) = B(k) = 0$. By Lemma 3.4, $H^3(X - Y, G_m) = 0$ so we have the isomorphism $H_Y^3(X, G_m) \cong H^3(X, G_m)$. Since X is smooth over the field k the couple (X, Y) satisfies the theorem of purity for the Brauer group [7, III, Théorème 6.1]. Therefore, using [7, III, Corollaire 6.2] and the fact that the groups are torsion we have $H_Y^3(X, G_m) = H^1(Y, \mathbf{Q}/\mathbf{Z})$. By [3, Arcata, II 2.1] we have $H^1(Y, \mathbf{Q}/\mathbf{Z}) = \text{Hom}(\Pi_1(Y), \mathbf{Q}/\mathbf{Z})$ where $\Pi_1(Y)$ is the algebraic fundamental group of Y . For any connected scheme S , $\Pi_1(S)$ is the inverse limit $\varprojlim \text{Gal}(T/S)$ over all étale Galois covers $T \rightarrow S$ (see [8]). Since Y is isomorphic to $\mathbf{P}^1$, $\Pi_1(Y) = 0$ [8, IX, Exemple 5.5]; hence $H^3(X, G_m) = 0$.

Now consider the excision sequence

$$0 \rightarrow B(X) \rightarrow B(U) \rightarrow H_Z^3(X, G_m) \rightarrow H^3(X, G_m) \rightarrow \cdots$$

Since X is a birationally ruled surface $B(X) = 0$ by Proposition 2.1. Since $H^3(X, G_m) = 0$ we have the isomorphism $B(U) \cong H_Z^3(X, G_m)$. Again applying [7, III, Théorème 6.1 and Corollaire 6.2] we have $H_Z^3(X, G_m) = H^1(Z, \mathbf{Q}/\mathbf{Z})$ and, by [3, Arcata, II 2.1], $H^1(Z, \mathbf{Q}/\mathbf{Z}) = \text{Hom}(\Pi_1(Z), \mathbf{Q}/\mathbf{Z})$. Since Z is a rational curve with at most one cusp singularity, by [8, IX, Exemple 5.5], $\Pi_1(Z) = 0$. Therefore, $H^1(Z, \mathbf{Q}/\mathbf{Z}) = 0$; hence $B(U) = B(B_n) = 0$.

Replacing X in the sequence (3.3) by $U = \text{Spec } B_n$ and using the values $\text{Pic } U = \mathbb{Z}_n$, $H^2(U, G_m) = B(U) = 0$ we have the exact sequence

$$0 \rightarrow \mathbb{Z}_n \rightarrow H^2(C \times U, G_m) \rightarrow H^2(\tilde{C} \times U, G_m) \rightarrow 0. \quad (3.7)$$

In his Ph.D. thesis [6, Theorem II 2], O. Gabber has proved the following result: if X is an arbitrary scheme such that $X = U_1 \cup U_2$ where U_1 and U_2 are affine schemes such that $U_1 \cap U_2$ is affine, then $B(X)$ is isomorphic to the subgroup of torsion elements in $H^2(X, G_m)$. Taking torsion subgroups in (3.7) yields the exact sequence $0 \rightarrow \mathbb{Z}_n \rightarrow B(C \times U) \rightarrow B(\tilde{C} \times U) \rightarrow 0$. Since $\tilde{C}$ is isomorphic to $\text{Spec}(k[u])$, $B(\tilde{C} \times U) = B(B_n[u]) = B(B_n) = 0$. So we have the desired result: $B(C \times U) = B(A \otimes B_n) = \mathbb{Z}_n$.

REMARK 3.8. One can readily construct the Azumaya algebra which generates $B(A \otimes B_n)$ using results from [11].

REMARK 3.9. Using arguments analogous to the above one can show that $\text{Pic}(B_{n_1} \otimes \cdots \otimes B_{n_r}) = \mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_r}$ and that $B(B_{n_1} \otimes \cdots \otimes B_{n_r}) = 0$. Therefore, $\mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_r}$ is the Brauer group of the noetherian integral domain $A \otimes B_{n_1} \otimes \cdots \otimes B_{n_r}$.

4. The Brauer group of $R[x, 1/x]$. The purpose of this section is to prove the following proposition. In [5] F. R. DeMeyer calculates $B(k[x, 1/x])$ for a perfect field k . We obtain analogous results for certain coordinate rings of affine varieties.

PROPOSITION 4.1. *Let R be a commutative noetherian regular ring which contains a field of characteristic zero such that $\text{Spec } R$ is irreducible. Let G be the Galois group of R . Then $B(R[x, 1/x]) = B(R) \oplus \text{Hom}(G, \mathbb{Q}/\mathbb{Z})$.*

PROOF. We can assume R is reduced (and hence integral since $\text{Spec } R$ is irreducible) by [4]. Let $X = \text{Proj } R[x_0, x_1]$ and let $X_0 = \text{Spec } R[x_0/x_1]$, $X_1 = \text{Spec } R[x_1/x_0]$ considered as open subsets of X . Let $Z = X - X_1$. Then Z is isomorphic to $\text{Spec } R$ and is contained in X_0 . Also $X_0 - Z = X_0 \cap X_1 = \text{Spec } R[x_0/x_1, x_1/x_0]$. We have two long exact sequences

$$\cdots \rightarrow H_Z^n(X, G_m) \rightarrow H^n(X, G_m) \rightarrow H^n(X_1, G_m) \rightarrow \cdots, \quad (4.2)$$

$$\cdots \rightarrow H_Z^n(X_0, G_m) \rightarrow H^n(X_0, G_m) \rightarrow H^n(X_0 \cap X_1, G_m) \rightarrow \cdots. \quad (4.3)$$

Since X_0 is an open neighborhood of Z we know that $H_Z^n(X, G_m) = H_Z^n(X_0, G_m)$. Since $X_0 \cong X_1$ by an isomorphism sending x_0/x_1 to x_1/x_0 we have a commutative diagram for each n :

$$\begin{array}{ccccc} H_Z^n(X, G_m) & \rightarrow & H^n(X, G_m) & \rightarrow & H^n(X_1, G_m) \\ & & \searrow & & \swarrow \cong \\ & & H^n(X_0, G_m) & & \end{array}$$

Therefore the map $H_Z^n(X_0, G_m) \rightarrow H^n(X_0, G_m)$ is the zero map for each n . The sequence (4.3) yields the short exact sequence

$$0 \rightarrow H^n(X_0, G_m) \rightarrow H^n(X_0 \cap X_1, G_m) \rightarrow H_Z^{n+1}(X_0, G_m) \rightarrow 0 \quad (4.4)$$

for each n . For $n = 2$ we have the exact sequence

$$0 \rightarrow H^2(X_0, G_m) \rightarrow H^2(X_0 \cap X_1, G_m) \rightarrow H_Z^3(X_0, G_m) \rightarrow 0.$$

By [7, II, Proposition 1.4] the H^2 groups are torsion groups and by Gabber's result [6, Theorem II 2] we have the exact sequence

$$0 \rightarrow B(R[x]) \rightarrow B(R[x, 1/x]) \rightarrow H_Z^3(X_0, G_m) \rightarrow 0.$$

Since X is regular over a perfect field k , it is smooth over k so we can replace $H_Z^3(X_0, G_m)$ with $\text{Hom}(G, \mathbf{Q}/\mathbf{Z})$ by [7, III, Corollaire (6.2)]. Hence the sequence becomes $0 \rightarrow B(R[x]) \rightarrow B(R[x, 1/x]) \rightarrow \text{Hom}(G, \mathbf{Q}/\mathbf{Z}) \rightarrow 0$. Since R is regular of characteristic zero, $B(R[x]) = B(R)$ [1, Proposition 7.7] and the map $B(R) \rightarrow B(R[x, 1/x])$ splits; hence

$$B(R[x, 1/x]) = B(R) \oplus \text{Hom}(G, \mathbf{Q}/\mathbf{Z}).$$

REMARK 4.5. In [12] the Brauer group of $k[x_1, \dots, x_d, x_1^{-1}, \dots, x_d^{-1}]$ is calculated to be $(\mathbf{Q}/\mathbf{Z})^n$ where $n = d(d-1)/2$ and k is an algebraically closed field with characteristic zero. Applying Proposition 4.1 inductively and using the fact that $H^1(k[x_1, \dots, x_d, x_1^{-1}, \dots, x_d^{-1}], \mathbf{Q}/\mathbf{Z}) = (\mathbf{Q}/\mathbf{Z})^d$ we obtain a more concise proof of this result.

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