A NOTE ON AN EXAMPLE OF BIRMAN-MONTESINOS

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ABSTRACT. We will report the negative solution to a conjecture of Birman-Montesinos about their example of Heegaard diagram. This implies the negative answer to a question of Waldhausen related to minimal Heegaard splittings of 3-manifolds.

We follow the notation and terminology of [1]. Let $(M, F; v, w)$ be a Heegaard diagram and (M, F) be the underlying Heegaard splitting. In [3] and [2] Waldhausen posed the following two questions:

Question A [3]. Suppose that (M, F) is not minimal and that $(M, F; v, w)$ is pseudominimal (in the sense of [1]). Does $(M, F; v, w)$ have a cancelling pair of handles?

Question B [2]. Is there a manifold which admits two minimal Heegaard splittings of different genus?

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Birman and Montesinos gave a delicate example in [1] which relates to these questions: their example is a pseudominimal Heegaard diagram without any cancelling pair of handles which is an affirmative answer to Question B if the underlying Heegaard splitting is minimal, or a negative one to Question A if it is not minimal. They conjectured that the former is the case. However, in this note we will show that the latter is the case. Let $(M, F; v, w)$ be the Heegaard diagram of Birman-Montesinos, shown above (see Figure 10 of [1]) where $v = (1, 2, 3)$ and $w = (x, y, z)$.

Let m be a meridian disk in the handlebody V with the complete system v of meridian disks as shown in the above figure. It is easy to see that ∂m intersects ∂y transversally at one point. Therefore, the Heegaard splitting (M, F) is not minimal. Hence the Birman-Montesinos example gives a pseudominimal Heegaard diagram which has no cancelling pair of handles, although the underlying splitting (M, F) is not minimal. Thus it gives a negative answer to Question A.

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