SUSPENSIONS OF STRONG SHAPE EQUIVALENCES ARE CE EQUIVALENCES

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ABSTRACT. S. Ferry [7] constructed a compactum shape equivalent but not CE equivalent to the circle. We show that suspensions of shape equivalent spaces are CE equivalent.

A map $p: X \rightarrow Y$ is called CE if p is surjective and each point-inverse has trivial shape. If X and Y are ANR's, then p is a simple homotopy equivalence (T. A. Chapman [2, 3], J. West [8], cf. Ferry [6]). In contrast (Ferry [6]), homotopy equivalent compacta are CE equivalent, that is, connected by a sequence of CE maps

$$X \leftarrow X_1 \rightarrow X_2 \leftarrow X_3 \cdots \rightarrow Y.$$

The maps Ferry constructs even admit sections and have contractible point-inverses. However (Ferry [7]) shape equivalence need not imply CE equivalence: the circle is shape equivalent but not CE equivalent to the circle with an arc shown in Figure I.

FERRY'S CIRCLE WITH AN APPROACHING ARC

FIGURE I

We shall prove that the suspensions of shape equivalent compacta X and Y are CE equivalent by a simple geometric argument. By [5, Chapter 5], X and Y are strong shape equivalent, and by [1, Proposition 4.2] this equivalence is realized by inclusions $X \rightarrow Y' \leftarrow Y$ which are strong shape equivalences. We may therefore assume that X is a subspace of Y .

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THEOREM. Let $f: X \rightarrow Y$ be an inclusion and strong shape equivalence. Then $\Sigma f: \Sigma X \rightarrow \Sigma Y$ is homotopic to a CE equivalence.

PROOF. Let $Z = CX \cup_X CY$ and factor Σf as the composite of inclusions $\Sigma X \xrightarrow{i} Z \xrightarrow{j} \Sigma Y$ as shown in Figure II.

FIGURE II

There are CE maps $p: Z \rightarrow Z/CY (\cong \Sigma X)$ and $q: Z \rightarrow Z/(CX \cup_X Y) (\cong \Sigma Y)$ (since f is a strong shape equivalence, $CX \cup_X Y$ is shape equivalent to $CX (\simeq *)$ [1, Lemma 4.7], J. Dydak and J. Segal [4, Theorem 6.1]) with $pi \simeq \text{id}$ and $q \sim j$. The conclusion follows.

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