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CLASS NUMBER RELATION BETWEEN CERTAIN SEXTIC NUMBER FIELDS

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ABSTRACT. The congruence relation modulo 7 between the class numbers of the real and imaginary sextic subfields of the extension of a quadratic number field obtained by adjoining a seventh root of unity is studied.

1. Introduction. The aim of this note is to study the congruence relation modulo 7 between the class numbers of the real and imaginary sextic subfields of the extension of a quadratic number field obtained by adjoining a seventh root of unity.

Let $m > 1$ be a square free rational integer, $\zeta = e^{2\pi\sqrt{-1}/7}$ a primitive seventh root of unity and Q the field of rational numbers. Let

$$K = Q(\sqrt{m}, \zeta),$$

which is of degree 12 over Q and has the following subfields:

$$K_0 = Q(\zeta), \quad K_1 = Q(\sqrt{m}, \zeta + \zeta^{-1}), \quad K_2 = Q(\sqrt{-7m}, \zeta + \zeta^{-1}),$$
$$F = Q(\zeta + \zeta^{-1}), \qquad k = Q(\sqrt{m}, \sqrt{-7}),$$
$$k_0 = Q(\sqrt{-7}), \quad k_1 = Q(\sqrt{m}), \quad k_2 = Q(\sqrt{-7m}).$$

K_0, K_1 and K_2 are cyclic sextic extensions of Q, and K_1 is the maximal real subfield of K. F is a cyclic cubic extension of Q and the maximal real subfield of K_0 and also of K_2. Denote the class numbers of K and K_i by h and h_i ($i = 1, 2$), respectively. In this note, we obtain a congruence relation modulo 7 between h_1 and h_2 by making use of the continuity of p-adic L-functions.

2. Class number relations. For any subfield Ω of K, U_Ω and W_Ω denote the unit group of Ω and its subgroup of all roots of unity in Ω, respectively. Put $Q_K = (U_K : U_{K_1}W_K)$ and $Q_2 = (U_{K_2} : U_F W_{K_2})$ (cf. [1]). That $Q_2 = 1$ can be proved by the argument similar to that in the proof of Theorem 1 in [4]. Hence one sees that $Q_K = (U_K : U_{K_0}U_{K_1}U_{K_2})$.

Let $\mathfrak{X}$ denote the set of Dirichlet characters attached to K. Let $\mathfrak{X}^+$ and $\mathfrak{X}^-$ be the set of even and odd characters in $\mathfrak{X}$, respectively. Furthermore, χ_0 denotes the principal character.

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Note that both of the class numbers of F and K_0 are equal to 1. Since $2h = Q_K h_1 h_2$ [6, Theorem (6.3)], it follows that

$$\frac{7}{w_K} h_2 \prod_{\chi \in \mathfrak{X}^-} (1 - \chi(7)) \equiv \frac{R_1 h_1}{\sqrt{D_1}} \prod_{\chi \in \mathfrak{X}^+ - \{\chi_0\}} \left(1 - \frac{\chi(7)}{7}\right) \pmod 7,$$

where w_K is the number of roots of unity in K, D_1 is the discriminant of K_1, and R_1 is the 7-adic regulator of K_1 [2, Theorem 1]. D_1 is given by

$$D_1 = \begin{cases} 7^4 m^3, & 7 \nmid m, \quad m \equiv 1 \pmod 4, \\ 7^4 (4m)^3, & 7 \nmid m, \quad m \equiv 2, 3 \pmod 4, \\ 7^2 m^3, & 7 | m, \quad m \equiv 1 \pmod 4, \\ 7^2 (4m)^3, & 7 | m, \quad m \equiv 2, 3 \pmod 4. \end{cases}$$

Therefore, if $7 \nmid m$, then $w_K = 14$ and

$$\text{(1)} \qquad \frac{1}{2} h_2 \equiv \frac{R_1}{7^2 m \sqrt{m}} h_1 \left(1 - \frac{1}{7}\left(\frac{m}{7}\right)\right) \pmod 7;$$

and if $7 | m$ and $m = 7m'$, then

$$\text{(2)} \qquad \frac{7}{w_K} h_2 \left(1 + \left(\frac{m'}{7}\right)\right) \equiv \frac{R_1}{7m\sqrt{m}} h_1 \pmod 7.$$

In order to calculate R_1, we must determine a system of fundamental units of K_1. For any $x \in F$, x' means the image of x under an automorphism of F which maps ζ to ζ^2. Let $\eta = \zeta + \zeta^{-1}$; then η and η' constitute a system of fundamental units of F. Let $\varepsilon = \frac{1}{2}(a + b\sqrt{m}) > 1$ be the fundamental unit of k_1. Furthermore, let $\xi = \frac{1}{2}(\alpha + \beta\sqrt{m})$, α, $\beta \in F$, be a unit of K_1 such that ξ and $\xi' = \frac{1}{2}(\alpha' - \beta'\sqrt{m})$ constitute a system of relative fundamental units of K_1, i.e., -1, ξ and ξ' generate a subgroup of all units in U_{K_1} whose relative norms to F and to k_1 are 1 and ± 1, respectively (cf. [3]). Note that α is an integer of F and that β or 7β is an integer of F according as $7 \nmid m$ or $7 | m$.

If the equation $x^2 = \pm\eta^r\eta'^s\xi$ for $(r, s) = (1, 0), (0, 1)$ or $(1, 1)$ has a solution in K_1, when the equation $y^2 = \pm\eta^{-s}\eta'^{r-s}\xi'$ also has a solution in K_1, let $\eta_0 = \sqrt{\pm\eta^r\eta'^s\xi}$ and $\eta'_0 = \sqrt{\pm\eta^{-s}\eta'^{r-s}\xi'}$; otherwise let $\eta_0 = \eta$ and $\eta'_0 = \eta'$. And, if the equation $z^3 = \varepsilon^{\pm 1}\xi\xi'$ has a solution in K_1, let $\xi_0 = \sqrt[3]{\varepsilon^{\pm 1}\xi\xi'}$; otherwise let $\xi_0 = \xi$. Then ε, η_0, η'_0, ξ_0 and ξ' constitute a system of fundamental units of K_1 [3]. Accordingly, if e is the index of the subgroup generated by -1, ε, η, η', ξ and ξ' in U_{K_1}, then $e = 1, 3, 4$ or 12. Hence, after easy calculation, one has

$$\text{(3)} \qquad R_1 = \frac{12}{e} \log \varepsilon R(\eta) R(\xi)$$

with

$$\text{(4)} \qquad R(\eta) = (\log \eta)^2 + (\log \eta)(\log \eta') + (\log \eta')^2$$

and

$$\text{(5)} \qquad R(\xi) = (\log \xi)^2 - (\log \xi)(\log \xi') + (\log \xi')^2,$$

where log is the 7-adic logarithm.

3. Calculating the 7-adic regulator. In this section, we calculate the 7-adic regulator R_1 of K_1. First, let $\pi = \zeta + \zeta^{-1} - 2$ and $\mathfrak{p} = (\pi)$; then $\mathfrak{p}$ is a unique prime ideal of F lying above (7): $\mathfrak{p}^3 = (7)$. It is easy to verify that

$$\pi^2 + \pi\pi' + \pi'^2 = -7(\pi + \pi' + 2) \equiv -14 \pmod{\mathfrak{p}^4}, \tag{6}$$

$$2\pi^3 + \pi^2\pi' + \pi\pi'^2 + 2\pi'^3 \equiv 3\pi^3 \equiv -21 \pmod{\mathfrak{p}^4}. \tag{7}$$

Since $\eta^6 \equiv 1 + 3\pi + 2\pi^2 \pmod{\mathfrak{p}^3}$, $\log\eta = \frac{1}{6}\log\eta^6 \equiv 4\pi - \pi^2 \pmod{\mathfrak{p}^3}$. Hence, by making use of (6), (7), it follows from (4) that

$$R(\eta) \equiv 2(\pi^2 + \pi\pi' + \pi'^2) - 4(2\pi^3 + \pi^2\pi' + \pi\pi'^2 + 2\pi'^3) \equiv 7 \pmod{\mathfrak{p}^4}.$$

As $R(\eta)$ is a rational 7-adic integer, this congruence holds modulo 7^2, that is,

$$R(\eta) \equiv 7 \pmod{7^2}. \tag{8}$$

We next consider $\log\varepsilon$. Here and hereafter $N_{\cdot/\cdot}$ means the norm with respect to the assigned field extension. It is easy to see that the following congruences hold:

$$\begin{array}{ll}
\varepsilon^2 \equiv 1 + 4ab\sqrt{m} \pmod{7} & \text{if } 7\mid m,\\
\varepsilon^4 \equiv 1 - \left(N_{k_1/Q}\varepsilon\right)ab\sqrt{m} \pmod{7^2} & \text{if } 7\mid a,\\
\varepsilon^2 \equiv 1 + 4ab\sqrt{m} \pmod{7^2} & \text{if } 7\mid b,\\
\varepsilon^6 \equiv 1 - (a^2 - 1)ab\sqrt{m} \pmod{7^2} & \text{if } N_{k_1/Q}\varepsilon = 1,\ a \equiv \pm 1 \pmod{7},\\
\varepsilon^8 \equiv 1 - (a^2 - 2)ab\sqrt{m} \pmod{7^2} & \text{if } N_{k_1/Q}\varepsilon = 1,\ a \equiv \pm 3 \pmod{7},\\
\varepsilon^{16} \equiv 1 - 4(a^2 - 8)ab\sqrt{m} \pmod{7^2} & \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 1 \pmod{7},\\
\varepsilon^6 \equiv 1 - (a^2 + 3)ab\sqrt{m} \pmod{7^2} & \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 2 \pmod{7},\\
\varepsilon^{16} \equiv 1 - 4(a^2 + 12)ab\sqrt{m} \pmod{7^2} & \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 3 \pmod{7}.
\end{array}$$

It then follows from these that

$$\log\varepsilon = \begin{cases}
\frac{1}{2}\log\varepsilon^2 \equiv 2ab\sqrt{m} \pmod{7} & \text{if } 7\mid m,\\
\frac{1}{4}\log\varepsilon^4 \equiv \left(N_{k_1/Q}\varepsilon\right)5ab\sqrt{m} \pmod{7^2} & \text{if } 7\mid a,\\
\frac{1}{2}\log\varepsilon^2 \equiv 2ab\sqrt{m} \pmod{7^2} & \text{if } 7\mid b,\\
\frac{1}{6}\log\varepsilon^6 \equiv (a^2 - 1)ab\sqrt{m} \pmod{7^2} & \\
\qquad \text{if } N_{k_1/Q}\varepsilon = 1,\ a \equiv \pm 1 \pmod{7}, & \\
\frac{1}{8}\log\varepsilon^8 \equiv 6(a^2 - 2)ab\sqrt{m} \pmod{7^2} & \\
\qquad \text{if } N_{k_1/Q}\varepsilon = 1,\ a \equiv \pm 3 \pmod{7}, & \\
\frac{1}{16}\log\varepsilon^{16} \equiv 5(a^2 - 8)ab\sqrt{m} \pmod{7^2} & \\
\qquad \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 1 \pmod{7}, & \\
\frac{1}{6}\log\varepsilon^6 \equiv (a^2 + 3)ab\sqrt{m} \pmod{7^2} & \\
\qquad \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 2 \pmod{7}, & \\
\frac{1}{16}\log\varepsilon^{16} \equiv 5(a^2 + 12)ab\sqrt{m} \pmod{7^2} & \\
\qquad \text{if } N_{k_1/Q}\varepsilon = -1,\ a \equiv \pm 3 \pmod{7}. &
\end{cases} \tag{9}$$

Lastly we treat $R(\xi)$. The fact that $N_{K_1/k_1}\xi = \pm 1$ implies that if $7 \nmid m$ then $\alpha^3 + 3\alpha\beta^2 m \equiv \pm 1 \pmod{\mathfrak{p}}$ and $3\alpha^2\beta + \beta^3 m \equiv 0 \pmod{\mathfrak{p}}$. On the other hand, $4\, N_{K_1/F}\xi = \alpha^2 - \beta^2 m = 4$. Hence, one has that if $7 \nmid m$, then $\alpha = \pm 1, \pm 2 \pmod{\mathfrak{p}}$, and if $7 | m$, then $\alpha \equiv \pm 2 \pmod{\mathfrak{p}}$ because $7\beta \equiv 0 \pmod{\mathfrak{p}^2}$. Then it is easy to see that the following congruences hold:

$$\begin{aligned} &\xi^6 \equiv 1 + 2(\alpha^2 - 1)^2 + 4(\alpha^2 - 1)(\alpha^2 - 3)\alpha\beta\sqrt{m} \pmod{\mathfrak{p}^3} && \text{if } \alpha \equiv \pm 1 \pmod{\mathfrak{p}},\\ &\xi^2 \equiv 1 + 4(\alpha^2 - 4 + \alpha\beta\sqrt{m}) \pmod{\mathfrak{p}^3} && \text{if } \alpha \equiv \pm 2 \pmod{\mathfrak{p}}. \end{aligned}$$

It follows from these that

$$\log \xi = \begin{cases} \frac{1}{6}\log \xi^6 \equiv 3(\alpha^2 - 1)(\alpha^2 - 3)\alpha\beta\sqrt{m} \pmod{\mathfrak{p}^3} \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } 7 \nmid m, \alpha \equiv \pm 1 \pmod{\mathfrak{p}}, \\ \frac{1}{2}\log \xi^2 \equiv 2\alpha\beta\sqrt{m} \pmod{\mathfrak{p}^3} \quad \text{if } 7 \nmid m, \alpha \equiv \pm 2 \pmod{\mathfrak{p}}, \\ \frac{1}{2}\log \xi^2 \equiv \alpha\beta(2 + 2\beta^2 m + \beta^4 m^2)\sqrt{m} \\ \qquad\qquad + \frac{1}{7}\alpha\beta^7 m^3(2 - 2\beta^2 m - 4\beta^4 m^2)\sqrt{m} \pmod{\mathfrak{p}^3} \quad \text{if } 7 | m. \end{cases}$$

We now assume that $7 \nmid m$ and put $\alpha\beta = c_0 + c_1\pi + c_2\pi^2$ with rational integers c_0, c_1, c_2. It is easy to verify that if $\alpha \equiv \pm 1 \pmod{\mathfrak{p}}$, then $c_0^2 m \equiv 4 \pmod 7$ and $(\alpha^2 - 1)(\alpha^2 - 3)\alpha\beta \equiv c_1\pi + (3c_0c_1m + c_2)\pi^2 \pmod{\mathfrak{p}^3}$, and if $\alpha \equiv \pm 2 \pmod{\mathfrak{p}}$, then $c_0 \equiv 0 \pmod 7$. Thus, by making use of (6), (7), it follows from (5) that

$$(10) \quad R(\xi) \equiv \begin{cases} 7c_1(3c_1 + 3c_0c_1^2 m + c_2)m \pmod{7^2} & \text{if } \alpha \equiv \pm 1 \pmod{\mathfrak{p}}, \\ 14c_1(3c_1 + c_2)m \pmod{7^2} & \text{if } \alpha \equiv \pm 2 \pmod{\mathfrak{p}}. \end{cases}$$

We next assume that $7 | m$ and put $7\alpha\beta = d_2\pi^2 + d_3\pi^3 + d_4\pi^4$ with rational integers d_2, d_3, d_4. In this case, $N_{K_1/k_1}\xi^2 = 1$ implies that $d_3 \equiv d_2 + 2d_2^3 m' \pmod 7$. Then, by easy calculation, it follows from (5) that

$$(11) \qquad R(\xi) \equiv 7d_2(d_2^5 - d_2^3 m'^2 - 2d_4 m')\left(1 + \left(\frac{m'}{7}\right)\right) \pmod{7^2}.$$

4. Theorems. The following theorem is obtained from (1)–(3) and (8)–(11):

THEOREM 1. *With the notation above, if* $7 \nmid m$, *then*

$$\frac{1}{2}h_2 \equiv -\left(\frac{m}{7}\right)\frac{12}{e}\frac{\log \varepsilon R(\xi)}{7^2 m\sqrt{m}}h_1 \pmod 7,$$

where $\log \varepsilon$ *and* $R(\xi)$ *are given by* (9) *and* (10), *respectively*, *and if* $7 | m$ *and* $m' = m/7 \equiv 1, 2$ *or* 4 (mod 7), *then*

$$\frac{7}{w_K}h_2 \equiv \frac{24}{e}abd_2(d_2^5 m'^2 - d_2^3 m' - 2d_4)h_1 \pmod 7.$$

As corollaries of this theorem, the following two theorems hold:

THEOREM 2. *With the notation above, assume that* $7 \nmid m$. *Then*, $7 | h_2$ *if and only if* $7 | h_1$ *or one of the following conditions is satisfied*:
(1) $N_{k_1/Q}\varepsilon = 1$ *and* $a \equiv 0, \pm 1, \pm 10 \pmod{7^2}$.
(2) $N_{k_1/Q}\varepsilon = -1$ *and* $a \equiv 0, \pm 12, \pm 20, \pm 24 \pmod{7^2}$.
(3) $b \equiv 0 \pmod{7^2}$.
(4) $c_1 \equiv 0 \pmod 7$ *or* $c_1 + c_0 c_1^2 m \equiv 2c_2 \pmod 7$.

THEOREM 3. *With the notation above, assume that* $7 | m$ *and* $m' \equiv 1, 2$ *or* $4 \pmod 7$. *Then*, $7 | h_2$ *if and only if* $7 | h_1$ *or one of the following conditions is satisfied*:
(1) $b \equiv 0 \pmod 7$.
(2) $d_2 \equiv 0 \pmod 7$ *or* $d_2^5 m'^2 - d_2^3 m' \equiv 2d_4 \pmod 7$.

Regarding the conditions in the above two theorems we give the following remark (cf. [**5**, Theorem 1]):

REMARK. (1) When $7 \nmid m$ and $\alpha \equiv \pm 1 \pmod{\mathfrak{p}}$, $c_1 \equiv 0 \pmod 7$ and $c_1 + c_0 c_1^2 m \equiv 2c_2 \pmod 7$ are necessary and sufficient conditions for $\xi^{12}\xi'^6 \equiv 1 \pmod{\mathfrak{p}^4}$ and $\xi^6 \xi'^{12} \equiv 1 \pmod{\mathfrak{p}^4}$, respectively.

(2) When $7 \nmid m$ and $\alpha \equiv \pm 2 \pmod{\mathfrak{p}}$, $c_1 \equiv 0 \pmod 7$ and $c_1 \equiv 2c_2 \pmod 7$ are necessary and sufficient conditions for $\xi^4 \xi'^2 \equiv 1 \pmod{\mathfrak{p}^4}$ and $\xi^2 \xi'^4 \equiv 1 \pmod{\mathfrak{p}^4}$, respectively.

(3) When $7 | m$, $d_2 \equiv 0 \pmod 7$ and $d_2^5 m'^2 - d_2^3 m' \equiv 2d_4 \pmod 7$ are necessary and sufficient conditions for $\xi^2 \xi'^4 \equiv 1 \pmod{\mathfrak{P}^7}$ and $\xi^4 \xi'^2 \equiv 1 \pmod{\mathfrak{P}^7}$, respectively. Herein, $\mathfrak{P}$ is a unique prime ideal of K_1 lying above (7). Therefore, it can be shown that the assertion of Theorem 3 holds without any restriction on m'.

Finally we give numerical examples for small m (cf. [**3**]).

The case where $7 \nmid m$.

$m = 2$, $\varepsilon = 1 + \sqrt{2}$, $\xi = 11 + 4\pi + (12 + 10\pi + 2\pi^2)\sqrt{2}$, $e = 4$, $(c_0, c_1, c_2) \equiv (3, 2, 3) \pmod 7$, $h_1 = 1$, $h_2 = 4$;

$m = 3$, $\varepsilon = 2 + \sqrt{3}$, $\xi = 293 + 210\pi + 36\pi^2 + (154 + 96\pi + 14\pi^2)\sqrt{3}$, $e = 12$, $(c_0, c_1, c_2) \equiv (0, 1, 0) \pmod 7$, $h_1 = 1$, $h_2 = 4$;

$m = 5$, $\varepsilon = \frac{1}{2}(1 + \sqrt{5})$, $\xi = \frac{1}{2}(107 + 77\pi + 10\pi^2 + (49 + 34\pi + 6\pi^2)\sqrt{5})$, $e = 12$, $(c_0, c_1, c_2) \equiv (0, 5, 2) \pmod 7$, $h_1 = 1$, $h_2 = 2$;

$m = 6$, $\varepsilon = 5 + 2\sqrt{6}$, $\xi = 673 + 588\pi + 108\pi^2 + (154 + 48\pi + 2\pi^2)\sqrt{6}$, $e = 12$, $(c_0, c_1, c_2) \equiv (0, 3, 1) \pmod 7$, $h_1 = 1$, $h_2 = 28$;

$m = 10$, $\varepsilon = 3 + \sqrt{10}$, $\xi = 10779 + 7140\pi + 1120\pi^2 + (3472 + 2366\pi + 386\pi^2)\sqrt{10}$, $e = 12$, $(c_0, c_1, c_2) \equiv (0, 0, 3) \pmod 7$, $h_1 = 2$, $h_2 = 28$.

The case where $7 | m$.

$m = 7$, $\varepsilon = 8 + 3\sqrt{7}$, $\xi = 15 + 12\pi + 2\pi^2 + \frac{1}{7}(28 + 14\pi + 2\pi^2)\sqrt{7}$, $e = 12$, $(d_2, d_3, d_4) \equiv (1, 3, 2) \pmod 7$, $h_1 = h_2 = 1$;

$m = 14$, $\varepsilon = 15 + 4\sqrt{14}$, $\xi = 83 + 64\pi + 12\pi^2 + \frac{1}{7}(126 + 70\pi + 8\pi^2)\sqrt{14}$, $e = 12$, $(d_2, d_3, d_4) \equiv (3, 6, 5) \pmod 7$, $h_1 = h_2 = 1$;

$m = 77$, $\varepsilon = \frac{1}{2}(9 + \sqrt{77})$, $\xi = \frac{1}{2}(2231 + 1430\pi + 220\pi^2 + \frac{1}{7}(1897 + 1330\pi + 220\pi^2)\sqrt{77})$, $e = 12$, $(d_2, d_3, d_4) \equiv (1, 2, 2) \pmod 7$, $h_1 = h_2 = 1$;

$m = 455$, $\varepsilon = 64 + 3\sqrt{455}$, $\xi = 992879 + 688410\pi + 115470\pi^2 + \frac{1}{7}(312774 + 203238\pi + 30804\pi^2)\sqrt{455}$, $e = 3$, $(d_2, d_3, d_4) \equiv (5,1,0) \pmod 7$, $h_1 = 4$, $h_2 = 224$;

$m = 497$, $\varepsilon = 1201887 + 53912\sqrt{497}$, $\xi = \frac{1}{2}(1762 + 1127\pi + 168\pi^2 + \frac{1}{7}(588 + 413\pi + 70\pi^2)\sqrt{497})$, $e = 3$, $(d_2, d_3, d_4) \equiv (0,0,6) \pmod 7$, $h_1 = 1$, $h_2 = 28$.

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