PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 95, Number 2, October 1985

DEFORMATIONS OF COMPLEX SUPERMANIFOLDS

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ABSTRACT. The supermanifold analogue of the Kodaira-Nirenberg-Spencer existence theorem for deformations of complex structures is given. It is shown that every complex supermanifold is a deformation of a vector bundle.

0. Supermanifolds, first used by physicists for modelling quantum gravity, have emerged as objects of independent interest. This paper will concentrate on supermanifolds with a complex structure, though the results also yield a simple and transparent proof of the fact that any supermanifold with only its C^∞ structure is the sheaf of sections of a vector bundle [**1**].

Let X be a complex manifold, with sheaf of holomorphic functions $\mathcal{O}$. Let $\mathcal{E}$ be a locally free sheaf of $\mathcal{O}$-modules. Then $\Lambda\mathcal{E}$, the sheaf of exterior algebras of $\mathcal{E}$ over $\mathcal{O}$, is an example of a complex supermanifold. $\Lambda\mathcal{E}$ is, among other things, a sheaf of supercommutative algebras. This means that $\Lambda\mathcal{E}$ is $\mathbf{Z}_2$-graded and $ab = (-1)^{|b||a|}ba$ for a and b of definite parity. $\Lambda\mathcal{E}$ is also a sheaf of $\mathcal{O}$-modules, and a sheaf of $\mathbf{Z}$-graded algebras, but for supersymmetry one is concerned only with the $\mathbf{Z}_2$-grading. This leads to the following more general definition:

DEFINITION. A complex supermanifold of dimension (m, n) is a sheaf $(M, \mathcal{A})$ of supercommutative algebras over $\mathbf{C}$ such that

(1) $(M, \mathcal{A}/\mathcal{N})$ is an m-dimensional complex manifold. ($\mathcal{N}$ is the ideal of nilpotent elements of $\mathcal{A}$.)

(2) The sheaves $(M, \mathcal{A})$ and $(M, \Lambda\mathbf{C}^n \otimes \mathcal{A}/\mathcal{N})$ are locally isomorphic as sheaves of $\mathbf{Z}_2$-graded commutative algebras over $\mathbf{C}$.

Set $\mathcal{O} = \mathcal{A}/\mathcal{N}$ and $\mathcal{E} = \mathcal{N}/\mathcal{N}^2$. Then $\mathcal{E}$ is an $\mathcal{O}$-module, and it follows from (2) that $\mathcal{E}$ is locally free. That is, $\mathcal{E}$ is the sheaf of sections of a holomorphic vector bundle. By writing $\mathcal{A}/\mathcal{N}^2$ as the direct sum of its even and odd parts, one obtains an exact sequence of sheaves of vector spaces

$$(*) \qquad 0 \to \mathcal{N}^2 \to \mathcal{A} \to \mathcal{O} \oplus \mathcal{E} \to 0.$$

If there is a splitting

$$0 \to \mathcal{O} + \mathcal{E} \xrightarrow{\mu} \mathcal{A}$$

Received by the editors January 20, 1984 and, in revised form, December 27, 1984.
1980 *Mathematics Subject Classification*. Primary 32C35.

of $(*)$ such that $\mu(f\xi) = \mu(f)\mu(\xi)$ for all $f \in \mathcal{O}$ and $\xi \in \mathcal{O} \oplus \mathcal{E}$, then μ extends to an isomorphism $\Lambda\mathcal{E} \simeq \mathcal{A}$. On the other hand, for the sequence

$$0 \to \sum_{i \geqslant 2} \Lambda^i\mathcal{E} \to \Lambda\mathcal{E} \to \mathcal{O} \oplus \mathcal{E} \to 0,$$

such a splitting surely exists. Thus if $\mathcal{A} \simeq \Lambda\mathcal{E}$, we say that $\mathcal{A}$ is *split*, and if not we say that $\mathcal{A}$ is *nonsplit*.

The idea will be to regard a nonsplit supermanifold as a deformation of a split one. We define the notion of an analytic family of supermanifolds, and show that to any supermanifold $(M, \mathcal{A})$ there is associated an analytic one parameter family of supermanifolds $(M, \mathcal{A}(z))$, $z \in \mathbf{C}$, such that $\mathcal{A}(0) = \Lambda\mathcal{E}$ and $\mathcal{A}(1) = \mathcal{A}$. We also attach to $\mathcal{A}$ an integer $n(\mathcal{A})$ and an invariant $\Delta\mathcal{A}$, which measure the failure of $\mathcal{A}$ to split. Finally, we prove that under suitable conditions it is possible to construct a supermanifold with a prescribed invariant.

1. Let $\mathcal{A}ut\,\Lambda\mathcal{E}$ denote the sheaf of parity preserving $\mathbf{C}$-linear algebra automorphisms of $\Lambda\mathcal{E}$. Define $\Lambda^{(k)}\mathcal{E} = \sum_{j \geqslant k} \Lambda^j\mathcal{E}$. If $g\colon \Lambda\mathcal{E} \to \Lambda\mathcal{E}$ is an automorphism, then g induces an $\mathcal{O}$-linear automorphism $g'\colon \mathcal{E} \to \mathcal{E}$, by virtue of the identification $\mathcal{E} \simeq \Lambda^{(1)}\mathcal{E}/\Lambda^{(2)}\mathcal{E}$. Denote by $\mathcal{A}ut^+\,\Lambda\mathcal{E}$ the subsheaf of automorphisms for which $g' = id$.

For k an even integer, let $\mathcal{D}er_k\,\Lambda\mathcal{E}$ denote the sheaf of derivations which increase degree by k. Let $\mathcal{D}er^{(j)}\,\Lambda\mathcal{E} = \sum_{j \leqslant 2k \leqslant n} \mathcal{D}er_{2k}\,\Lambda\mathcal{E}$. For $Y \in \mathcal{D}er^{(0)}\,\Lambda\mathcal{E}$, let Y_k denote the $\mathcal{D}er_k\,\Lambda\mathcal{E}$ component of Y. Explicitly, if $z^1, \ldots, z^m$ are coordinates on M and $\pi^1, \ldots, \pi^n$ are a basis for $\mathcal{E}$ over $\mathcal{O}$, then any derivation is uniquely expressible in the form

$$Y = \sum f^i \frac{\partial}{\partial z^i} + g^j \frac{\partial}{\partial \pi^j},$$

where f^i and g^j are sections of $\Lambda\mathcal{E}$. Then Y lies in $\mathcal{D}er_k\,\Lambda\mathcal{E}$ if and only if, for all i and j, $\deg(f^i) = k$ and $\deg(g^j) = k + 1$.

Elements of $\mathcal{D}er^{(2)}\,\Lambda\mathcal{E}$ are nilpotent, so the power series $\exp\colon \mathcal{D}er^{(2)}\,\Lambda\mathcal{E} \to \mathcal{A}ut\,\Lambda\mathcal{E}$ is well defined.

PROPOSITION 1. *$\exp\colon \mathcal{D}er^{(2)}\,\Lambda\mathcal{E} \to \mathcal{A}ut^+\,\Lambda\mathcal{E}$ is bijective.*

PROOF. For $Y \in \mathcal{D}er^{(2)}\,\Lambda\mathcal{E}$, it is clear that $\exp Y$ induces the identity on $\mathcal{E}$. On the other hand, for $g \in \mathcal{A}ut^+\,\Lambda\mathcal{E}$, $1 - g$ is degree increasing and therefore nilpotent, so $\log g$ is well defined and lies in $\mathcal{D}er^{(2)}\,\Lambda\mathcal{E}$.

Define the *order* of $\mathcal{A}$, denoted $o(\mathcal{A})$, as the sup of integers $k \leqslant n + 1$ such that $\mathcal{A}/\mathcal{N}^k$ and $\Lambda\mathcal{E}/\Lambda^{(k)}\mathcal{E}$ are isomorphic. Assuming $n \geqslant 1$, $o(\mathcal{A})$ is at least 2, and is either even or equal to $n + 1$.

Let Ξ be an open cover of M such that, for all $u \in \Xi$, the isomorphism (2) of the definition exists and $\mathcal{E}$ is trivial over u. $\mathcal{A}$ is filtered by $\mathcal{N}$, and the associated graded sheaf is $\Lambda\mathcal{E}$. For each $u \in \Xi$, an isomorphism $T_u\colon \mathcal{A}|_u \to \Lambda\mathcal{E}|_u$ can be chosen in such a way that the associated map of $\mathbf{Z}$-graded algebras is the identity. The cocycle $\{T_u T_v^{-1} | u, v \in \Xi\}$ defines $\mathcal{A}$ up to isomorphism, and it follows that the isomorphism classes of supermanifolds $(X, \mathcal{A})$ with underlying $\mathcal{O}$-module $\mathcal{E}$ are in natural 1-1 correspondence with $H^1(M, \mathcal{A}ut^+\,\Lambda\mathcal{E})$.

PROPOSITION 2. *There exists a cocycle* $\exp(Y^{uv})$ *defining* $\mathscr{A}$ *such that, for all* $u, v \in \Xi$ *and all* $j < o(\mathscr{A})$, $Y_j^{uv} = 0$.

PROOF. Observe that any automorphism of $\Lambda\mathscr{E}/\Lambda^{(k)}\mathscr{E}$ is determined by its restriction to $\mathscr{O} + \mathscr{E}$. It follows that, for all $u \in \Xi$, the natural map $\mathscr{A}\mathit{ut}^+ \Lambda\mathscr{E}(u) \to \mathscr{A}\mathit{ut}^+(\Lambda\mathscr{E}/\Lambda^{(k)}\mathscr{E})(u)$ is surjective. Denote this map by π. Now choose a cocycle $\exp(Z^{uv})$ defining $\mathscr{A}$. Then the cocycle $\pi\exp(Z^{uv})$ defines $\mathscr{A}/\mathscr{N}^k$. If $\mathscr{A}/\mathscr{N}^k$ and $\Lambda\mathscr{E}/\Lambda^{(k)}\mathscr{E}$ are isomorphic, there exist automorphisms $\rho^u \in \mathscr{A}\mathit{ut}^+(\Lambda\mathscr{E}/\Lambda^{(k)}\mathscr{E})(u)$ such that

$$\pi\exp(Z^{uv}) = \rho^u(\rho^v)^{-1}.$$

ρ^u is of the form $\pi\exp(X^u)$, for some $X^u \in \mathscr{D}\mathit{er}^{(2)}\Lambda\mathscr{E}(u)$. Then the cocycle

$$\exp(Y^{uv}) = \exp(-X^u)\exp(Z^{uv})\exp(X^v)$$

defines $\mathscr{A}$ and satisfies $Y_j^{uv} = 0$ for $j \leqslant k$, as desired.

Let $\tau = \exp(Y^{uv})$ be a cocycle with coefficients in $\mathscr{A}\mathit{ut}^+\Lambda\mathscr{E}$. Call τ *reduced* if τ satisfies the property in Proposition 2.

For all u, v and $w \in \Xi$,

$$\exp(Y^{uw}) = \exp(Y^{uv})\exp(Y^{vw}) = \exp(Y^{uv} + Y^{vw} + \text{commutator terms}).$$

So if τ is reduced, then, for $j < 2o(\mathscr{A})$, Y_j is an *additive* cocycle. Thus Y determines a class $\omega(\tau) \in H^1(M, \mathscr{D}\mathit{er}^{(o(\mathscr{A}))}\Lambda\mathscr{E}/\mathscr{D}\mathit{er}^{(2o(\mathscr{A}))}\Lambda\mathscr{E})$.

Denote the group $H^0(M, \mathscr{A}\mathit{ut}^+\Lambda\mathscr{E})$ of global sections of $\mathscr{A}\mathit{ut}^+\Lambda\mathscr{E}$ by $G^+(\Lambda\mathscr{E})$. $G^+(\Lambda\mathscr{E})$ acts on $\mathscr{D}\mathit{er}^{(o(\mathscr{A}))}\Lambda\mathscr{E}/\mathscr{D}\mathit{er}^{(2o(\mathscr{A}))}\Lambda\mathscr{E}$ by conjugation, and one has

PROPOSITION 3. *The orbit of* $\omega(\tau)$ *under the action of* $G^+(\Lambda\mathscr{E})$ *is an invariant of* $(M, \mathscr{A})$.

PROOF. Let $\sigma = \exp(X^{uv})$ be another cocycle defining $\mathscr{A}$. Then there is a 0-cochain Z^u such that $\exp(X^{uv}) = \exp(Z^u)\exp(Y^{uv})\exp(-Z^v)$. If $X_j^{uv} = 0$ for all $j < o(\mathscr{A})$, then it follows by induction on j that $Z_j^u = Z_j^v$ for all u and $v \in \Xi$ and all $j < o(\mathscr{A})$. For $j \geqslant o(\mathscr{A})$ and $k < 2o(\mathscr{A})$, the Z_j^u terms have no effect on the cohomology class of Y_k. Thus $\omega(\sigma)$ and $\omega(\tau)$ are conjugate under $\exp(Z)$, where Z is defined by $Z|_u = \sum_{2 \leqslant j < o(\mathscr{A})} Z_j^u$.

DEFINITION. Let $\Delta\mathscr{A}$ denote the orbit of $\omega(\tau)$ under $G^+(\Lambda\mathscr{E})$.

THEOREM 1. $\mathscr{A}$ *splits if and only if* $\Delta\mathscr{A} = 0$.

PROOF. Let $\tau = \exp(Y)$ be a reduced cocycle representing $\mathscr{A}$. If $\mathscr{A}$ splits, then $\tau = 1$, so that $\omega(\tau) = 0$. On the other hand, assume $\omega(\tau) = 0$. Then there is a 0-cochain Z with coefficients in $\mathscr{D}\mathit{er}_{o(\mathscr{A})}\Lambda\mathscr{E}$ such that $Y_{o(\mathscr{A})}^{uv} = Z^u - Z^v$. Set $\sigma^{uv} = \exp(-Z^u)\tau^{uv}\exp(Z^v)$. Then $\sigma = \exp(W)$ for some 1-cochain W, σ defines $\mathscr{A}$, and $W_j^{uv} = 0$ for $j \leqslant o(\mathscr{A})$. Unless $o(\mathscr{A}) = n + 1$, this is a contradiction.

2. A *map* Φ: $(M, \mathscr{A}) \to (N, \mathscr{B})$ between two complex supermanifolds is a holomorphic map ϕ: $M \to N$ together with a sheaf morphism ϕ': $\phi^{-1}\mathscr{B} \to \mathscr{A}$. The notions of tangent space and differential map carry over directly to supermanifolds. For $p \in M$, Φ is called *submersive* at p if $d\Phi_p$ is surjective. If Φ is submersive and if the dimensions of $(M, \mathscr{A})$ and $(N, \mathscr{B})$ are (m_0, m_1) and (n_0, n_1), respectively, then we must have $n_i \leqslant m_i$ for $i = 0, 1$. Moreover, ϕ is submersive. (See [**6** and **7**] for general background.)

Assume Φ is everywhere submersive. If the odd dimension of $\mathscr{A}$ is 0, then for all $q \in N$, the structure sheaf of the fiber over q is the quotient of $\mathscr{O}_M|_{\phi^{-1}q}$ by $\mathscr{I}_q$, where $\mathscr{I}_q$ is the ideal whose members vanish along $\phi^{-1}(q)$. In general, however, the nilpotent elements of $\mathscr{A}$ take the value 0 at all points of M, and one does not want the fiber supermanifolds to be devoid of nilpotents. So the procedure for constructing $\mathscr{I}_q$ in general is to start by taking a derivation $X \in \mathscr{D}er\,\mathscr{A}|_p$ and saying X is *vertical* if X annihilates $\phi'(\phi^{-1}\mathscr{B})|_p$. Then let δ denote the quotient map

$$0 \to \mathscr{N} \to \mathscr{A} \xrightarrow{\delta} \mathscr{O} \to 0.$$

Finally, say f is in $\mathscr{I}_q|_p$ if and only if for all nonnegative integers k and all vertical derivations $X_1, \ldots, X_k$ at p, $\delta \circ X_1 \circ \cdots \circ X_k f$ vanishes along $\phi^{-1}(q)$.

An explicit description of $\mathscr{I}_q$ in terms of local coordinates adapted to Φ reveals that $\mathscr{A}|_{\phi^{-1}q}/\mathscr{I}_q$ is a supermanifold over $\phi^{-1}q$ with dimension $(m_0 - n_0, m_1 - n_1)$. Thus $(M, \mathscr{A})$ fibers over $(N, \mathscr{B})$, and we call $(M, \mathscr{A})$ an *analytic family of supermanifolds parameterized by* $(N, \mathscr{B})$.

As an example of such a family, consider the action of $\mathbf{C}$ on $\mathscr{D}er^{(0)}\,\Lambda\mathscr{E}$, given by $z \cdot Y = z^j Y$, $Y \in \mathscr{D}er_{2j}\Lambda\mathscr{E}$. Then $\mathbf{C}$ acts by homomorphisms on $\mathscr{A}ut^+\,\Lambda\mathscr{E}$, by $z\exp(Y) = \exp(z \cdot Y)$. This descends to an action of $\mathbf{C}$ on

$$H^1\big(M, \mathscr{D}er^{(k)}\,\Lambda\mathscr{E}/\mathscr{D}er^{(l)}\,\Lambda\mathscr{E}\big)/G^+\Lambda\mathscr{E}, \qquad k < l.$$

THEOREM 2. *Let $(M, \mathscr{A})$ be a supermanifold defined by some $\tau \in H^1(M, \mathscr{A}ut^+\,\Lambda\mathscr{E})$. Then the classes $z \cdot \tau$, $z \in \mathbf{C}$, determine an analytic family of supermanifolds parameterized by $\mathbf{C}$. If we denote the supermanifold on the zth fiber by $\mathscr{A}(z)$, then $\Delta\mathscr{A}(z) = z \cdot \Delta\mathscr{A}$.*

PROOF. Represent τ by a cocycle $\exp(Y^{uv})$ defined on an open cover Ξ of M. The $\mathscr{O}_M$ module $\mathscr{E}$ pulls back to an $\mathscr{O}_{M\times\mathbf{C}}$ module $\mathscr{E}'$ on $M \times \mathbf{C}$, and derivations of $\Lambda\mathscr{E}$ act on $\Lambda\mathscr{E}'$ by ignoring the z coordinate. Thus $z \cdot \tau$ can be regarded as lying in $H^1(M \times \mathbf{C}, \mathscr{A}ut^+\,\Lambda\mathscr{E}')$, and so determines a supermanifold $\mathscr{B}$ on $M \times \mathbf{C}$. The projection $\pi\colon M \times \mathbf{C} \to \mathbf{C}$ induces an injection $\pi^*\colon \pi^{-1}(\mathscr{O}_{\mathbf{C}}) \to \mathscr{O}_{M\times\mathbf{C}} \to \Lambda\mathscr{E}'$ on whose image the automorphisms $\exp(zY^{uv})$ act trivially. Thus π^* induces an injection $\pi'\colon \pi^{-1}(\mathscr{O}_{\mathbf{C}}) \to \mathscr{B}$ making $(M \times \mathbf{C}, \mathscr{B})$ an analytic family. That the fibers have the desired invariant is a direct verification.

The classical "no obstruction" theorem for deformations of complex structures [**5**] carries over to the supermanifold case.

PROPOSITION 4. *Let $\mathfrak{g} = \sum_{i\geqslant 1}\mathfrak{g}_i$ be a sheaf of $\mathbf{Z}$-graded Lie algebras over a space M. Fix positive integers j and k, with $j \leqslant k$. Let Y be a 1-cochain with coefficients in $\sum_{j\leqslant i\leqslant k}\mathfrak{g}_i$. Set $\exp(Y^{uv})\exp(Y^{vw}) = \exp(Y^{uw} + Z^{uvw})$, and assume that the 2-cochain Z has coefficients in $\sum_{i\geqslant(k+1)}\mathfrak{g}_i$. Then Z_{k+1} is a cocycle.*

PROOF. This is proved by induction on $k - j$. Note that Y_j is a cocycle. If $k = j$, then Z^{uvw} is the $(k+1)$st component of $\frac{1}{2}[Y^{uv}, Y^{vw}]$, which is easily seen to be a cocycle. Since the proposition is *local*, we may restrict our attention to a neighborhood on which $Y_j^{uv} = X^u - X^v$ for some 0-chain X. If we replace Y by Y', where Y' is defined by

$$\exp(Y'^{uv}) = \exp(-X^u)\exp(Y^{uv})\exp(X^v),$$

then the $(k+1)$st component of Z is unchanged, whereas $Y_j' = 0$. Thus the induction proceeds.

From this follows

THEOREM 3. *Let k be an even integer, $k \geqslant 2$. Let V be a finite-dimensional subspace of $H^1(M, \mathscr{Der}^{(k)}\Lambda\mathscr{E}/\mathscr{Der}^{(2k)}\Lambda\mathscr{E})$. Assume $H^2(M, \mathscr{Der}^{(2k)}\Lambda\mathscr{E}) = 0$. Then there is an analytic family of supermanifolds parameterized by V such that, for all $\omega \in V$,*

$$\Delta\mathscr{A}(\omega) = G^+(\Lambda\mathscr{E}) \cdot \omega.$$

PROOF. Fix a basis $\omega_1, \ldots, \omega_r$ for V. For $i = 1, \ldots, r$, let Y_j be a cocycle representing ω_j. Now consider the sheaf of $\mathbf{Z}$-graded Lie algebras $\mathfrak{g} = \mathbf{C}[z^1, \ldots, z^r] \otimes \mathscr{Der}^{(k)}\Lambda\mathscr{E}$, with grading inherited from the second factor. To prove the theorem we must find a 1-cochain Y with coefficients in $\mathfrak{g}$ such that

(i) $Y \equiv \sum z^j \otimes Y_j$ modulo $\mathscr{Der}^{(2k)}\Lambda\mathscr{E}$, and

(ii) $\exp(Y)$ is a cocycle.

Proposition 4 guarantees that the obstruction to finding this cochain lies in $H^2(M, \mathscr{Der}^{(2k)}\Lambda\mathscr{E})$, which vanishes. Note that the $\mathbf{Z}$-grading on $\mathscr{Der}^{(k)}\Lambda\mathscr{E}$ allows for a polynomial dependence on V, and thereby circumvents the problem of convergence.

3. The same considerations apply to C^∞ supermanifolds. In that case, $\mathscr{Der}^{(0)}\Lambda\mathscr{E}$ is a fine sheaf, so $\Delta\mathscr{A} = 0$. Therefore, Theorem 1 yields the theorem of Batchelor that all smooth supermanifolds are in fact vector bundles [**1**]. Also see [**3**] for the first proof of this result.

4. The conditions in Theorem 3 are easily achieved. For example, if $k > n/2$, then the hypothesis $H^2(M, \mathscr{Der}^{(2k)}\Lambda\mathscr{E}) = 0$ is vacuous. In fact, the ideal $\mathscr{Der}^{(k)}\Lambda\mathscr{E}$ is abelian, so any class $\omega \in H^1(M, \mathscr{Der}^{(k)}\Lambda\mathscr{E})$ can be exponentiated immediately to determine an isomorphism class of supermanifolds $\exp(\omega) \in H^1(M, \mathscr{Aut}^+\Lambda\mathscr{E})$. This is the sort of example given in [**4**]. As a further example, suppose $\mathscr{E}$ is free and has rank n. Then

$$\mathscr{Der}^{(0)}\Lambda\mathscr{E} = \Lambda^{\text{even}}\mathscr{E} \otimes \Theta + \Lambda^{\text{odd}}\mathscr{E} \otimes \mathscr{E}^* = \Theta^r + \mathscr{O}^s,$$

where $r = 2^{n-1}$ and $s = n2^{n-1}$ and where Θ is the sheaf of holomorphic vector fields. So if $H^1(M, \Theta + \mathscr{O}) \neq 0$ and $H^2(M, \Theta + \mathscr{O}) = 0$, for instance if M is a Riemann surface of positive genus, then M carries supermanifolds of any order.

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