PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 95, Number 4, December 1985

FOR ANY X, THE PRODUCT $X \times Y$ HAS REMOTE POINTS FOR SOME Y

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ABSTRACT. Any space with a σ-locally finite π-base will be called a σ-π space. The work of Chae and Smith can be extended to show that every nonpseudocompact σ-π space has remote points.[2] Sufficient conditions for a product to be a σ-π space are developed. It is shown that, for each space, if α is a cardinal with the discrete topology, where α is not less than π-weight of X, then $X \times \alpha^\omega$ has remote points. Cardinal function criteria are developed for the existence of σ-π spaces. An example is given of a pathological product which is a σ-π space even though none of its finite partial products is a σ-π space.

1. Introduction, motivation and basic preliminaries. All spaces considered are taken to be completely regular, Hausdorff. This work exhibits products which are σ-π spaces with remote points. Such a product may have each of its factors be of uncountable π-weight.[3] Typically, these products are nowhere locally compact and, thus, it can be explicitly demonstrated why their remainders are nonhomogeneous [**VW**]. Some examples of pathological products in this class are also exhibited.

Any terminology and notation not defined below is standard or may be found in the texts [$\mathbf{C_4}$, **GJ**, or **W**]. Let X be a space. The notation $|X|$ denotes the cardinality of X; πX denotes the π-weight of X; $\tau^*(X)$ denotes the family of nonempty open subsets of X. The subspace $\beta X \setminus X$ will be abbreviated as X^*. The Greek letters α, γ, and λ will always denote infinite cardinals. The Greek letter κ will be used to denote

Received by the editors November 15, 1984 and, in revised form, February 4, 1985.

1980 *Mathematics Subject Classification*. Primary 54D40, 54D35, 54B10; Secondary 54A25, 54B25, 54G20.

Key words and phrases. σ-π space, products, G-space, σ-locally finite π-base, remote point, Stone-Čech compactification, remainder, homogeneity, pseudo-γ-compact.

[1] Many of these results are from the author's doctoral dissertation, which was written under the supervision of W. W. Comfort. His insight, guidance, and encouragement—given deftly and graciously—have delightfully seasoned this work. S. B. Chae first made the suggestion to focus particular attention upon the class of σ-π spaces, and he so named that class. G. C. Baloglou, A. Dow, A. W. Hager, and S. W. Williams provided enlightening direction and appreciated encouragement. The referee's comments were most helpful, particularly with respect to the inclusion of Example 3.8 and Theorem 3.9. Theorem 3.9 is a generous contribution of the referee. The author extends his thanks to these mathematicians, as well as to those who read an earlier, much longer version of this manuscript. The author gratefully acknowledges the financial support of Wesleyan University and Computervision Corporation.

[2] Chae and Smith's work utilizes normality. However, the author has learned that the more general result stated above is true. The author gratefully acknowledges the mathematical stimulus which led to that conclusion. (For a proof of the assertion, see [**HP**].)

[3] Independently, but concurrently, Dow [$\mathbf{D_1}$] exhibited other products with remote points. A typical example from [$\mathbf{D_1}$] fails to be a σ-π space and each of its factors has countable π-weight.

a cardinal, either finite or infinite. The notation $\mathrm{cf}(\alpha)$ denotes the cofinality of α. Cardinals are represented by initial ordinals. The space ω always has the discrete topology. The symbol $\mathbb{N}$ denotes the set $\{1, 2, 3, \dots\}$. The symbol $\mathbb{R}$ denotes the real line with the usual topology. The symbol $\oplus$ denotes disjoint topological union. For an arbitrary set S and a cardinal κ, let $[S]^{\kappa} = \{A\colon A \subset S \text{ and } |A| = \kappa\}$ and let $[S]^{<\kappa} = \{A\colon A \subset S \text{ and } |A| < \kappa\}$.

1.1 Definition. Let X be a space. A point $p \in X^*$ is a *remote point of* X if there is no nowhere dense subset A of X such that $p \in \mathrm{cl}_{\beta X} A$. Let TX denote the set of remote points of X.

1.2 Definition. A space X is pseudo-γ-compact if every locally finite family of nonvoid open subsets of X has cardinality less than γ.

2. Products which are σ-π spaces. In this section sufficient conditions for products to be σ-π spaces are given. The following theorem is recorded here but its easy proof is omitted.

2.1 Theorem. *The class of σ-π spaces is countably productive.*

In contrast to Theorem 2.1, see Example 3.7.

Uncountable products which are σ-π spaces are now considered. Later, (3.10), (3.12) it will be shown that an uncountable product of σ-π spaces need not be a σ-π space.

2.2 Theorem. *Let $\{X_\xi\}_{\xi<\alpha}$ be a family of σ-π spaces. If there exists a countably infinite subfamily of spaces which are not pseudo-α-compact, then $X = \prod_{\xi<\alpha} X_\xi$ is a σ-π space.*

Proof. Without loss of generality, assume that $\alpha > \omega$ and that the countably infinite subfamily of spaces which are not pseudo-α-compact is $\{X_\xi\}_{\xi<\omega}$. Then, for each $\xi < \omega$ there exists $\mathbf{V}_\xi \subset \tau^*(X_\xi)$ such that $|\mathbf{V}_\xi| = \alpha$ and $\mathbf{V}_\xi$ is locally finite.

For each $\xi < \alpha$, let $\mathbf{B}_\xi = \bigcup_{i<\omega} \mathbf{B}_{\xi,i}$ be a σ-locally finite π-base for X_ξ, where for each $\xi < \alpha$ and each $i < \omega$, $\mathbf{B}_{\xi,i}$ is locally finite. Without loss of generality assume that $\mathbf{B}_{\xi,i} \subset \mathbf{B}_{\xi,i+1}$.

For each $A \in [\alpha]^{<\omega}$ and each $i < \omega$, let

$$\mathbf{B}_{A,i} = \left\{ \prod_{\xi<\alpha} B_\xi\colon \quad \begin{array}{l} B_\xi \in \mathbf{B}_{\xi,i} \text{ if } \xi \in A \text{ and} \\ B_\xi = X_\xi \text{ otherwise} \end{array} \right\}.$$

If $\prod_{\xi<\alpha} H_\xi$ is a basic open subset of X, then for each $A \in [\alpha]^{<\omega}$, let $H_A = \prod_{\xi\in A} H_\xi$.

Now, for each $n \in \mathbb{N}$, a locally finite family $\mathbf{U}_n$ of nonempty open subsets of X will be defined such that $\mathbf{U} = \bigcup_{n=1}^{\infty} \mathbf{U}_n$ is a σ-locally finite π-base for X. Let $n \in \mathbb{N}$ and consider $[\alpha]^n$. For each $k < n+1$ there exists a one-to-one function f_k: $[\alpha]^n \to \mathbf{V}_k$. For each $A \in [\alpha]^n$, there exists a unique least $k(A) < n+1$ such that $k(A) \notin A$. For each $A \in [\alpha]^n$, define $\mathbf{U}(A)$ to be a family of basic open subsets of X

as follows. Let

$$\mathbf{U}(A) = \left\{ \prod_{\xi<\alpha} U_\xi: \quad \begin{array}{l} U_A \in \mathbf{B}_{A,n},\ U_{k(A)} = f_{k(A)}(A) \text{ and} \\ U_\xi = X_\xi \text{ otherwise} \end{array} \right\}.$$

Let $\mathbf{U}_n = \bigcup\{\mathbf{U}(A): A \in [\alpha]^n\}$. It will now be shown that $\mathbf{U}_n$ is locally finite. Let $x = (x_\xi)_{\xi<\alpha}$ be a point of X. For each $\xi < n + 1$, there exists a neighborhood W_ξ of x_ξ such that W_ξ intersects only finitely many members of $\mathbf{B}_{\xi,n} \cup \mathbf{V}_\xi$. Then

$$\prod_{\xi<n+1} W_\xi \times \prod_{n+1\leqslant\xi<\alpha} X_\xi$$

is a neighborhood of x which intersects some element of $\mathbf{U}(A)$, for only finitely many A's where $A \in [\alpha]^n$. This condition is because, for each $A \in [\alpha]^n$, it is clear that $k(A) < n + 1$ and if $\prod_{\xi<\alpha} U_\xi \in \mathbf{U}(A)$ then $U_{k(A)} = f_{k(A)}(A) \in \mathbf{V}_{k(A)}$ and $\mathbf{V}_{k(A)}$ is locally finite.

Let $\{A_l\}_{l<N}$ be the collection of these finitely many A's for some $N \in \omega$. Let $F = \bigcup_{l<N} A_l$ and note that $|F| < \omega$. For each $\xi \in F \setminus n + 1$, there exists a neighborhood W_ξ of x_ξ such that W_ξ intersects only finitely many members of $\mathbf{B}_{\xi,n}$. For each $\xi \in \alpha \setminus (n + 1 \cup F)$, let $W_\xi = X_\xi$. Let $W = \prod_{\xi<\alpha} W_\xi$. Then W is a neighborhood of x which intersects only finitely many members of $\mathbf{U}_n$.

Let $\mathbf{U} = \bigcup_{n=1}^{\infty} \mathbf{U}_n$.

It will now be shown that $\mathbf{U}$ is a π-base for X. Let H be a basic open subset of X whose nonempty restriction set is $R(H)$, that is, $H = \prod_{\xi<\alpha} H_\xi$, where each $H_\xi \in \tau^*(X_\xi)$, and $0 < |R(H)| = m < \omega$. Let $R(H) = \{\xi_0, \ldots, \xi_{m-1}\}$. For each $j < m$, there exists i_j such that $\mathbf{B}_{\xi_j, i_j}$ contains a subset of H_{ξ_j}. Let $M = \max\{m, i_0, \ldots, i_{m-1}\}$. Then $\mathbf{U}_M$ contains a subset of H and, thus, $\mathbf{U}$ is a π-base for X.

2.3 COROLLARY. *With γ discrete, if $\gamma \geqslant \alpha$, then γ^α is a σ-π space, with remote points.*

For a result similar to, but more general than Corollary 2.3, see Theorem 3.12.

The next theorem has the peculiarity that it allows for the possibility of a product being a σ-π space even though none of its factors is a σ-π space. Later (3.7), an example is given of a countable product which is a σ-π space, even though no finite partial product is a σ-π space. (For a related example, see [$\mathbf{P}_2$, 6.3].)

If the hypotheses of Theorem 2.2 above are compared to the hypotheses of Theorem 2.4 below, it is easy to observe that the hypotheses of Theorem 2.4 may require the existence of larger locally finite families of open sets than is needed for Theorem 2.2. Theorem 2.4 could be proved by modifying the arguments of 2.2 (see [$\mathbf{P}_1$, 3.7]), but a much shorter and simpler proof is given in [$\mathbf{P}_3$, 6.2].

2.4 THEOREM. *Let $\{X_\xi\}_{\xi<\alpha}$ be a family of spaces. Let $\lambda = \sup_{\xi<\alpha}\{\pi X_\xi\} + \alpha$. If there exists a countably infinite subfamily of spaces which are not pseudo-λ-compact, then $X = \prod_{\xi<\alpha} X_\xi$ is a σ-π space.*

2.5 COROLLARY. *For each space X, there exists a space Y such that $X \times Y$ is a nonpseudocompact σ-π space with remote points.*

PROOF. Let $Y = \alpha^\omega$, where α has the discrete topology and $\alpha \geqslant \pi X$.

As many of the examples given in this paper can be shown to be σ-π spaces via Theorem 2.4, an example is now given which can be shown to be a σ-π space via Theorem 2.2, but for which Theorem 2.4 is inapplicable.

2.6 EXAMPLE. Let $\{X_\xi\}_{\xi<\omega_1}$ be the family of discrete spaces, where $X_\xi = \omega_{\xi+1}$ for each $\xi < \omega_1$. For each $\xi < \omega_1$, X_ξ is not pseudo-ω_1-compact and $X = \prod_{\xi<\omega_1} X_\xi$ is a σ-π space by Theorem 2.2.

To see that Theorem 2.4 does not apply, note that if A is any uncountable subset of ω_1, then $\sup_{\xi\in A}\{\pi X_\xi\} = \omega_{\omega_1}$, but it is clear that, for each $\xi < \omega_1$, X_ξ is pseudo-ω_{ω_1}-compact.

The interested reader can also construct examples which are σ-π spaces via Theorem 2.4 but for which Theorem 2.2 is inapplicable (see also (3.7)).

In each of Theorems 2.2 and 2.4, hypotheses were stated in terms of locally finite families of either cardinality α or λ. Each of the theorems has an analogue, with slightly weaker hypotheses, when the cardinal α or λ has countable cofinality. This analogue of Theorem 2.2 is stated below as Theorem 2.7. The statement of the analogue of Theorem 2.4 is left to the reader. Proofs are omitted. It is easy to construct spaces which satisfy the hypotheses of Theorem 2.7, but which fail to satisfy the hypotheses of Theorem 2.2. Similar remarks apply to the analogue of Theorem 2.4.

2.7 THEOREM. *Let $\alpha \geqslant \omega_1$, where $\mathrm{cf}(\alpha) = \omega$, and let $\{\alpha_n\}_{n<\omega}$ be a strictly increasing sequence of infinite cardinals such that $\sum_{n<\omega}\alpha_n = \alpha$. Let $\{X_\xi\}_{\xi<\alpha}$ be a family of σ-π spaces. If there exists a countably infinite subfamily $\{X_{\xi_j}\}_{j<\omega}$ such that, for each $j < \omega$ and each $n < \omega$, X_{ξ_j} is not pseudo-α_n-compact, then $X = \prod_{\xi<\alpha} X_\xi$ is a σ-π space.*

Like other product theorems, the following theorem has hypotheses about countably infinite subfamilies satisfying certain conditions. The advantage of this theorem is that the conditions on the countably infinite subfamilies are determined by the π-weights of its members. In general, this situation is more natural and easier to recognize than the previously stated conditions. Note that the family consists entirely of σ-π spaces.

2.8 THEOREM. *Let $\{X_\xi\}_{\xi<\alpha}$ be a family of σ-π spaces. If there exists a countably infinite subfamily of spaces, each having π-weight at least α, then $X = \prod_{\xi<\alpha} X_\xi$ is a σ-π space.*

PROOF. If $\alpha = \omega$, the result follows from Theorem 2.1. If $\mathrm{cf}(\alpha) > \omega$, the result follows from 3.5(a) (below) and Theorem 2.2. If $\mathrm{cf}(\alpha) = \omega$, the result follows from 3.5(b) (below) and Theorems 2.2 and 2.7.

In Theorem 2.8 the hypotheses concerning the existence of σ-π spaces can not be entirely removed. For if $\alpha = \omega_1$ and $X_\xi = \beta\omega_1$ for each $\xi < \omega_1$ (where ω_1 is discrete), then $(\beta\omega_1)^{\omega_1}$ is not a σ-π space because it is compact and has uncountable π-weight [see (3.6)].

3. Some pathological products within the class of σ-π spaces. Throughout this section, if a space is assumed to be a σ-π space, it is to be understood that it has a π-base $\mathbf{B} = \bigcup_{n<\omega} \mathbf{B}_n$, where each $\mathbf{B}_n$ is locally finite and cellular [**Po**] (cf. [**T**, **Wh**, **Wi**]). The easy proofs of the following lemmas are left to the reader.

3.1 LEMMA. *If X is a σ-π space and Y is an open subspace of X, then Y and* $\mathrm{cl}_X Y$ *are σ-π spaces.*

3.2 LEMMA. *If X is a σ-π space and Y is a dense subspace of X, then Y is a σ-π space.*

3.3 REMARK. In contrast to Lemma 3.2, let Y be ω_1, with the discrete topology, and let X be ω_1 with the well-ordered topology. Then Y is a σ-π space, and Y is (a homeomorph of) a dense open subspace of X. However, it will be shown (3.6) that X is not a σ-π space.

3.4 LEMMA. *Let $\{X_\xi\}_{\xi<\alpha}$ be a family of spaces. Then $\bigoplus_{\xi<\alpha} X_\xi$ is a σ-π space if and only if X_ξ is a σ-π space for each $\xi < \alpha$.*

Cardinal functions can be used to demonstrate when a space does not have a σ-locally finite π-base.

3.5 THEOREM. *Let X be a space. If*
(a) $\mathrm{cf}(\pi X) > \omega$ *and X is pseudo-πX-compact, or*
(b) *there exists $\lambda < \pi X$ such that X is pseudo-λ^+-compact,*
then X is not a σ-π space.

PROOF. Suppose X is a σ-π space.
In case (a), the pseudo-πX-compactness implies that, for each $n < \omega$, $|\mathbf{B}_n| < \pi X$.
In case (b), the pseudo-λ^+-compactness implies that, for each $n < \omega$, $|\mathbf{B}_n| \leqslant \lambda$.
Each case implies that $|\mathbf{B}| < \pi X$, which is a contradiction.

3.6 COROLLARY. *A pseudocompact space is a σ-π space if and only if it has countable π-weight.*

The promised surprising product pathology can now be presented. That is, an example is given of a product which is a σ-π space, even though none of its finite partial products is a σ-π space.

3.7 EXAMPLE. With ω_1 discrete, let $X = \omega_1 \times \beta\omega_1$. Then for each $n < \omega$, X^n is not a σ-π space (3.4), (3.6), but X^ω is a σ-π space (2.4). (For another example, see [$\mathbf{P_2}$, 6.3].)

The next example [$\mathbf{C_5}$] demonstrates that if the hypothesis concerning the cofinality of an uncountable π-weight is eliminated from 3.5(a), then the conclusion of 3.5(a) need not hold.

3.8 EXAMPLE. Let $\gamma > \omega$ and let $\mathrm{cf}(\gamma) = \omega$. Let $X = \gamma + 1$, topologized as follows:

If $x \in X$ and $x \neq \gamma$, then $\{x\}$ is open.

If $U \subset X$ and $\gamma \in U$, then U is open if and only if $|X \setminus U| < \gamma$.

The space X is a pseudo-γ-compact, σ-π space, where $\pi X = \gamma$. (It is easy to exhibit an explicit σ-locally finite π-base for such a space X. For each $n < \omega$, let γ_n be a cardinal such that $\gamma_n < \gamma_{n+1} < \gamma$, where $\sup\{\gamma_n \colon n < \omega\} = \gamma$. For each $n < \omega$, let $\mathbf{B}_n = \{\{x\} \colon x < \gamma_n\}$. Then $B = \bigcup_{n<\omega} \mathbf{B}_n$ is the desired π-base.)

However, Example 3.8 has the property that each element of the π-base has small π-weight. Such a situation would occur rarely in the context of infinite products. The following variant of 3.5(a) may be more appropriate in the context of infinite products.

3.9 THEOREM. *Let X be a space, where $\pi X > \omega$ and X is pseudo-πX-compact. If for each $U \in \tau^*(X)$, $\pi U = \pi X$, then X is not a σ-π space.*

PROOF. Because of 3.5(a), it may be assumed, without loss of generality, that $\mathrm{cf}(\pi X) = \omega$.

Suppose X is a σ-π space. Let $\mathbf{B} = \bigcup_{n<\omega} \mathbf{B}_n$ be a π-base for X, where for each $n < \omega$, $\mathbf{B}_n$ is locally finite. There exists $n' < \omega$ such that $\mathbf{B}_{n'}$ is infinite. Let $\{U_k \colon k < \omega\}$ be an infinite subset of $\mathbf{B}_{n'}$. For each $k < \omega$, there exists a cardinal $\lambda_k < \pi X$ such that $\sup\{\lambda_k \colon k < \omega\} = \pi X$.

The supposition concerning X implies that, for each $k < \omega$, U_k is a σ-π space (3.1). Therefore, for each $k < \omega$, there exists a locally finite family $\mathbf{W}_k \subset \tau^*(U_k)$, such that $|\mathbf{W}_k| = \lambda_k^+$ (3.5(b)). Let $\mathbf{W} = \bigcup_{k<\omega} \mathbf{W}_k$. Then $\mathbf{W}$ is locally finite, $\mathbf{W} \subset \tau^*(X)$ and $|\mathbf{W}| = \pi X$, which contradicts the pseudo-πX-compactness of X.

The next example demonstrates that even though the class of σ-π spaces is countably productive, it is not productive.

3.10 EXAMPLE. The space $\mathbb{R}^{\omega_1}$, which has the countable chain condition [**RS**], is pseudo-ω_1-compact [$\mathbf{C_1}$]. Thus, $\mathbb{R}^{\omega_1}$ is not a σ-π space (3.5(a)), even while each of its countable partial products is a metric space.

It is known that if X and Y are spaces and $f \colon X \to Y$ is a perfect, irreducible surjection, then X is a σ-π space if and only if Y is a σ-π space [**Po**]. However, open continuous images of σ-π spaces need not be σ-π spaces. Also, even though open subspaces of σ-π spaces are σ-π spaces (3.1), arbitrary subspaces of σ-π spaces need not be σ-π spaces. The next example demonstrates both of these phenomena.

3.11 EXAMPLE. With ω_1 discrete, let $X = \omega_1^\omega \times \mathbb{R}^{\omega_1}$, which is a σ-π space (2.2). Let $Y = \mathbb{R}^{\omega_1}$ and let $\pi \colon X \to Y$ be the natural projection. Then Y is an open continuous image of X, and Y is (homeomorphic to) a subspace of X, but Y is not a σ-π space (3.10).

It is possible to specify exactly when an infinite power of an infinite discrete space is a σ-π space.

3.12 THEOREM. *With γ discrete, the space γ^α is a σ-π space if and only if $\gamma \geqslant \alpha$.*

PROOF. ($\Leftarrow$) Corollary 2.3.

($\Rightarrow$) Let $X = \gamma^\alpha$ and suppose $\gamma < \alpha$. Note that X has calibre γ^+ [$\mathbf{C_1}$], and is, thus, pseudo-γ^+-compact [$\mathbf{C_4}$, 2.1(e)]. Hence, it is clear that X is pseudo-α-compact. Also, note that $\pi X = \alpha$ [**J**, 4.3]. Therefore, X is not a σ-π space (3.9).

4. Questions.[4]

4.1. If $\gamma \geqslant \omega_1$ is discrete and $\gamma < \alpha$, must γ^α have remote points? (2.3, 3.12), **[$\mathbf{D}_1$]**

4.2. Do there exists spaces X and Y such that neither X nor Y is a σ-π space but $X \times Y$ is a σ-π space? (3.7), **[To]**.

4.3. For each X, does there exist a discrete cardinal α, such that $\alpha \times X$ has remote points? (2.5, 3.4), **[vDvM, $\mathbf{D}_2$]**.

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[4] These three questions form the focus of current research by A. Dow and T. J. Peters. This research yields (in ZFC) affirmative answers to 4.1 and 4.2 and also yields (under assumptions concerning the existence of measurable cardinals) an affirmative answer to 4.3.

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