SHORTER NOTES

The purpose of this department is to publish very short papers of unusually polished character, for which there is no other outlet.

A REMARK ON WEAK CONVERGENCE IN THE DUAL OF A C^* -ALGEBRA

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In [J], the following remarkable result was obtained: *every reflexive quotient of a C^* -algebra is super-reflexive*. Along the way to proving this, Jarchow established the coincidence of the classes of weakly compact operators from a C^* -algebra A to any Banach space X and the uniformly convexifiable operators (introduced by B. Beauzamy [B]) from A to X . We take note of the following consequence of Jarchow's work.¹

COROLLARY. *In the dual A^* of a C^* -algebra A any weakly convergent sequence has a subsequence all of whose subsequences have norm convergent arithmetic means.*

One might view this Corollary as a noncommutative generalization of Szlenk's theorem [S] about weak convergence in $L_1[0, 1]$ in much the same way as Akemann's theorem [A] extends the classical Bartle-Dunford-Schwartz theorem [T, III.5.4].

Our method of proof is the classical method of theorem-quoting and relies heavily on the theory of Banach operator ideals for its success.

We consider a weakly compact subset K in A^* (we have in mind a weakly convergent sequence along with its limit) and apply the still-wonderful scheme of W. J. Davis, T. Figiel, W. B. Johnson, and A. Pełczyński [DFJP] to produce a reflexive Banach space R and a bounded linear injection $T: R \rightarrow A^*$ such that $T(B_R) \supseteq K$, where B_R is the closed unit ball of R . Naturally, $T^*: A^{**} \rightarrow R^*$ is a weakly compact linear operator on the C^* -algebra A^{**} [T]: Jarchow tells us that T^* is uniformly convexifiable and Beauzamy warns us that T must be, too. But uniformly convexifiable operators are Banach-Saks operators and so must take weakly convergent sequences to sequences having subsequences with norm convergent arithmetic means [BB]. All that remains is to apply the Ramsey theoretics of [EM].

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