FIXED POINTS OF AUTOMORPHISMS OF FINITELY GENERATED FREE GROUPS

SIMON THOMAS

(Communicated by Bhama Srinivasan)

In [1], Gersten proved that if $\alpha: F \rightarrow F$ is an automorphism of a finitely generated free group, then $\text{Fix}(\alpha) = \{w \in F \mid \alpha(w) = w\}$ is finitely generated. As Gersten pointed out, together with Howson's theorem [2], this implies that if H is a finitely generated subgroup of $\text{Aut } F$, then $\text{Fix}(H) = \bigcap_{\alpha \in H} \text{Fix}(\alpha)$ is finitely generated. However, it appears to have gone unnoticed that, using theorems of McCool and Stallings, a rather stronger result can be obtained.

THEOREM. *Let F be a finitely generated free group. If $H \leq \text{Aut } F$ is an arbitrary subgroup, then $\text{Fix}(H)$ is finitely generated.*

PROOF. Let $P = \text{Fix}(H)$. By 3.3 of [4], there exists a finitely generated subgroup $Q \leq P$ such that for all $\alpha \in \text{Aut } F$, $Q \leq \text{Fix}(\alpha)$ if and only if $P \leq \text{Fix}(\alpha)$. Let $K = \{\alpha \in \text{Aut } F \mid Q \leq \text{Fix}(\alpha)\}$. By [3], K is finitely generated. Clearly $P \leq \text{Fix}(K)$. Also, since $H \leq K$, $\text{Fix}(K) \leq P$ and so $P = \text{Fix}(K)$. Hence by Gersten's and Howson's theorems, P is finitely generated. $\square$

BIBLIOGRAPHY

1. S. M. Gersten, *Fixed points of automorphisms of free groups*, Adv. in Math. **64** (1987), 51–85.
2. A. G. Howson, *On the intersection of finitely generated free groups*, J. London Math. Soc. **29** (1954), 428–434.
3. J. McCool, *Some finitely presented subgroups of the automorphism group of a free group*, J. Algebra **35** (1975), 205–213.
4. J. R. Stallings, *Finiteness properties of matrix representations*, Ann. of Math. **124** (1986), 337–346.

DEPARTMENT OF MATHEMATICS, RUTGERS UNIVERSITY, NEW BRUNSWICK, NEW JERSEY 08903

Received by the editors July 28, 1987.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 20E05; Secondary 20F28.

Research supported by NSF Grant DMS-8703229.

©1988 American Mathematical Society
0002-9939/88 \$1.00 + \$.25 per page