UNIVERSAL SPACES FOR LOCALLY FINITE-DIMENSIONAL TYCHONOFF SPACES

M. G. CHARALAMBOUS

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ABSTRACT. We prove that the class of all locally finite-dimensional Tychonoff spaces and other related classes have universal elements.

1. INTRODUCTION AND DEFINITIONS

In this paper all spaces are at least Tychonoff, τ denotes an infinite cardinal, N the set of positive integers, I the unit interval $[0, 1]$, wX and βX the weight and Stone-Čech compactification of a topological space X , respectively, and $\dim X$ the covering dimension of X defined in terms of cozero covers. For standard results and notation in General Topology and Dimension Theory we refer to [5, 6, 12]. A space X is called locally finite dimensional if it has an open cover $\{G_\lambda: \lambda \in \Lambda\}$ with $\dim G_\lambda < \infty$ for each $\lambda \in \Lambda$ [7, 13]. In view of the known fact that $\dim G \leq \dim X$ whenever G is a cozero set of X [see, e.g., 3, Proposition 1], X is locally finite dimensional iff it has a cozero cover $\{G_\lambda: \lambda \in \Lambda\}$ with $\dim G_\lambda < \infty$ for each λ in Λ . We write $\text{loc dim } X \leq n$ if X has an open (or, equivalently, cozero) cover $\{G_\lambda: \lambda \in \Lambda\}$ with $\dim G_\lambda \leq n$ for each λ in Λ . If X is a normal space, since $\dim Z \leq \dim X$ if Z is either closed or cozero in X , then $\text{loc dim } X \leq n$ iff X has an open cover $\{G_\lambda: \lambda \in \Lambda\}$ with $\dim \overline{G_\lambda} \leq n$ for each λ in Λ , which is the original definition of loc dim [4]. Following [10], if $f: X \rightarrow Y$ is continuous, by $W(f)$ we denote the smallest cardinal α for which there exists a space Z of weight α and an embedding $g: X \rightarrow Y \times Z$ with $f = \pi \circ g$, where π denotes the canonical projection from $Y \times Z$ onto Y .

In this paper we establish the existence of universal spaces for several classes of locally finite-dimensional spaces, in particular, for (i) the class of all locally finite-dimensional spaces and (ii) the class of all locally finite-dimensional spaces that can be mapped by some continuous function f with $W(f) \leq \tau$ into a

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metrizable space of weight α . This seems to provide an affirmative answer to Pasynkov's Question 12 in [11].

The results on universal spaces follow from certain factorization theorems that are given in §2 and are corollaries of the factorization theorem for Dim , the covering dimension function for uniform spaces introduced in [2]. $\text{Dim } X$ is defined in terms of the uniformly open sets of a uniform space X , i.e., sets of the form $f^{-1}(0, 1]$ for some uniformly continuous $f: X \rightarrow I$. Thus, $\text{Dim } X \leq n$ iff every finite uniformly open cover of X has a finite uniformly open refinement of order $\leq n$. The collection of all uniformly open sets of X is a base for the open sets of X that is closed under finite intersections and countable unions. If X is Lindelöf or a metrizable uniform space or has the Stone-Čech uniformity, i.e., that inherited from βX , then every cozero set of X is uniformly open and hence $\text{Dim } X = \dim X$.

2. FACTORIZATION THEOREMS

In the following result, which is proved in [3], $W(X)$ denotes the weight of a uniform space X .

Theorem 1. *Let $f: X \rightarrow Y$ be a uniformly continuous function between uniform spaces and $\{X_\lambda: \lambda < \tau\}$ a collection of subspaces of X with $W(Y) \leq \tau$. Then there exists a uniformly continuous $g: X \rightarrow Y \times I^\tau$ such that $\pi \circ g = f$, where π is the projection of $Y \times I^\tau$ onto Y , and $\text{Dim } g(X_\lambda) \leq \text{Dim } X_\lambda$ for each $\lambda < \tau$.*

Proposition 1. *Let $f: X \rightarrow Y$ be a continuous function and $\{X_\lambda: \lambda < \tau\}$ a collection of Lindelöf subspaces of X with $wY \leq \tau$. Then there are continuous functions $g: X \rightarrow Z$ and $h: Z \rightarrow Y$, where $Z = Y \times I^\tau$, such that $h \circ g = f$ and $\dim g(X_\lambda) \leq \dim X_\lambda$ for each $\lambda < \tau$ (cf. [1, Theorem 2]).*

Proof. Let X be equipped with its Stone-Čech uniformity. Y can be embedded in I^τ and will be considered to be endowed with the subspace uniformity inherited from I^τ so that $W(Y) \leq \tau$. Now f is uniformly continuous and Theorem 1 supplies a uniformly continuous $g: X \rightarrow Z$ such that $\pi \circ g = f$, where π is the projection from $Z = Y \times I^\tau$ to Y , and $\text{Dim } g(X_\lambda) \leq \text{Dim } X_\lambda$ for each $\lambda < \tau$. The result follows if we let $h = \pi$ since each X_λ and $g(X_\lambda)$ is Lindelöf and $\dim Z = \text{Dim } Z$ if Z is Lindelöf.

A similar argument establishes the following result, which generalizes Theorem 3 of [1] and Theorem 1 of [9], where only normal spaces are considered. Recall that Y is said to be z -embedded in X if every cozero subset of Y is of the form $Y \cap G$ for some cozero set G of X . Closed subspaces of normal spaces and cozero or Lindelöf subspaces of arbitrary spaces are z -embedded.

Proposition 2. *Let $f: X \rightarrow Y$ be a continuous function into a metric space Y and $\{X_i: i \in N\}$ a countable collection of z -embedded subspaces of X . Then there exist a metric space Z and continuous functions $g: X \rightarrow Z$ and $h: Z \rightarrow Y$ such that $h \circ g = f$, $\dim g(X_i) \leq \dim X_i$ for each i in N and $wg(A) \leq wf(A)$ for every subspace A of X with $f(A)$ infinite [3, Proposition 5].*

We call a space strongly locally finite dimensional if it has a cozero cover $\{G_i: i \in N\}$ with $\dim G_i \leq i$. Every metric locally finite-dimensional space is strongly locally finite dimensional [7], but this is not always true in the class of normal spaces.

Example. Let M be a convergent subspace of $[0, \omega_1) \times I^N$, where ω_1 is the first uncountable ordinal. This means that for each x in I^N , there is an $\alpha < \omega_1$ such that $(\alpha, \omega_1) \times \{x\} \subset M$. Then M is normal with $\dim M = \dim I^N = \infty$, and we can choose M to have $\text{loc dim} = 0$ [12, Proposition 5.4.5]. Consider a cozero cover $\{G_i: i \in N\}$ of M and fix $x \in I^N$. Then there exist $\alpha < \omega_1$ and $i \in N$ with $(\alpha, \omega_1) \times \{x\} \subset G_i$ and since I^N is first countable and G_i open, there is an open neighborhood V of x such that $([\alpha, \omega_1) \times V) \cap M \subset G_i$. Now $([\alpha, \omega_1) \times V) \cap M$ is convergent in $[0, \omega_1) \times V$ and hence $\dim G_i = \dim V = \infty$.

Proposition 3. Let $f: X \rightarrow Y$ be a continuous function from a strongly locally finite-dimensional space X into a metric space Y . Then there exist a locally finite-dimensional metric space Z and continuous $g: X \rightarrow Z$ and $h: Z \rightarrow Y$ such that $f = h \circ g$ and $wZ \leq wY$.

Proof. Let $\{G_i: i \in N\}$ be a cozero cover of X with $\dim G_i \leq i$, and fix a continuous $g_i: X \rightarrow I$ with $G_i = g_i^{-1}(0, 1]$. By Proposition 2, there exist a metric space Z and continuous $g: X \rightarrow Z$ and $h: Z \rightarrow Y \times I^N$ such that $h \circ g = f \times \prod_{i=1}^{\infty} g_i$, $wg(X) \leq wY$ and $\dim g(G_i) \leq i$ for each $i \in N$. We may clearly take $Z = g(X)$ and the result follows as it can be verified that $g(G_i) = h^{-1}(\pi_i^{-1}(0, 1])$, where π_i denotes the projection of $Y \times I^N$ onto its $(i + 1)$ th factor, so that $\{g(G_i): i \in N\}$ is a cozero cover of Z with $\dim g(G_i) \leq i$.

3. UNIVERSAL SPACES

From Proposition 3 and a straightforward application of a method due to Pasynkov [8] follows a known result for locally finite-dimensional metric spaces. This was first established by different methods by Wenner [13] for separable metric spaces and by Luxemburg [7] for arbitrary metric spaces.

Proposition 4. The class of all metric locally finite-dimensional spaces of weight $\leq \tau$ has a universal element.

Proof. Let M be a universal space for all metric spaces weight $\leq \tau$. Let $\{X_\lambda: \lambda \in \Lambda\}$ be the collection of all locally finite-dimensional subspaces of M , X their topological sum and $f: X \rightarrow M$ the unique function from X to M whose restriction to each X_λ is the inclusion of X_λ into M . Then f is continuous, X is locally finite-dimensional and, by Proposition 3, there exist a locally finite-dimensional metric space Z and continuous $g: X \rightarrow Z$

and $h: Z \rightarrow M$ with $f = h \circ g$. Now the restriction of g to each X_λ is an embedding and Z is the required universal space.

The following result will prove useful. Recall that for an open set G of a space X , ExG denotes the biggest open set of βX whose intersection with X is G . $Ex(G_1 \cap G_2) = ExG_1 \cap ExG_2$ and, if G_1 and G_2 are cozero, $Ex(G_1 \cup G_2) = ExG_1 \cup ExG_2$ [5, Lemma 7.1.13].

Lemma 1. *For every cozero set G of a space X and every normal space H with $G \subset H \subset ExG$, $\dim G = \dim H$.*

Proof. Firstly, $\dim G \leq \dim H$ because G is z -embedded in H [e.g., 3, Proposition 1]. Conversely, suppose $\dim G = n$ and let $\{G_1, G_2, \dots, G_k\}$ be a cozero cover of H . Let $\{F_1, F_2, \dots, F_k\}$ and $\{H_1, H_2, \dots, H_k\}$ be, respectively, a zero and a cozero cover of H with $H_i \subset F_i \subset G_i$. Since $\dim G \leq n$, the cozero cover $\{G \cap H_1, G \cap H_2, \dots, G \cap H_k\}$ of G has a cozero shrinking $\{V_1, V_2, \dots, V_k\}$ of order $\leq n$. Then $\{H \cap ExV_1, H \cap ExV_2, \dots, H \cap ExV_k\}$ is an open cover of the normal space H and, therefore, it has a cozero shrinking $\{W_1, W_2, \dots, W_k\}$. It is readily verified that $H \cap ExV_i \subset F_i$ and hence $\{W_1, W_2, \dots, W_k\}$ is a cozero shrinking of the cozero cover $\{G_1, G_2, \dots, G_k\}$ of H of order $\leq n$. Thus, $\dim H \leq n = \dim G$, and the proof is complete.

Proposition 5. *The class of all locally finite-dimensional spaces of weight $\leq \tau$ has a universal element that is locally compact.*

Proof. Let $\{X_\lambda: \lambda \in \Lambda\}$ be the collection of all locally finite-dimensional subspaces of I^τ , X_0 their topological sum and $i: \beta X_0 \rightarrow I^\tau$ the unique continuous function whose restriction to each X_λ is the inclusion of X_λ into I^τ . Since clearly $wX_\lambda \leq \tau$, X_λ has a cozero cover $\{G_{\lambda\alpha}: \alpha < \tau\}$ with $\dim G_{\lambda\alpha} \leq n(\alpha) < \infty$. Now for each $\alpha < \tau$, $G_\alpha = \bigcup_{\lambda \in \Lambda} G_{\lambda\alpha}$ is a cozero set of X_0 with $\dim G_\alpha \leq n(\alpha) < \infty$. Let $g_\alpha: \beta X_0 \rightarrow I$ be a continuous function such that $G_\alpha = X_0 \cap H_\alpha$, where $H_\alpha = g_\alpha^{-1}(0, 1]$. Then X_0 is contained in $X = \bigcup (H_\alpha: \alpha < \tau)$ and, by Lemma 1, since H_α is Lindelöf, $\dim H_\alpha = \dim G_\alpha \leq n(\alpha) < \infty$. Now let $f: X \rightarrow I^\tau \times I^\tau$ be the restriction to X of $i \times \prod_{\alpha < \tau} g_\alpha$ and apply Proposition 1 to obtain a space Z of weight $\leq \tau$ and continuous functions $g: X \rightarrow Z$ and $h: Z \rightarrow I^\tau \times I^\tau$ with $f = h \circ g$ and $\dim g(H_\alpha) \leq n(\alpha) < \infty$ for $\alpha < \tau$. Note that for $\alpha < \tau$ there is a projection $\pi: I^\tau \times I^\tau \rightarrow I$ such that $g_\alpha = \pi \circ f$ and hence $g(H_\alpha) = h^{-1}(\pi^{-1}(0, 1]) \cap g(X)$ is a cozero set of $g(X)$, which, therefore, belongs to the class of locally finite-dimensional spaces of weight $\leq \tau$. Finally, $g(X)$ is a universal element of this class because the restriction of g to each X_λ is an embedding and I^τ is a universal space for all spaces of weight $\leq \tau$. Furthermore, if $H_{\alpha n} = g_\alpha^{-1}(1/n, 1]$ and $F_{\alpha n} = g_\alpha^{-1}[1/n, 1]$, $n \in \mathbb{N}$, $\alpha < \tau$, it is not difficult to see that $\overline{g(F_{\alpha n})}$ is compact and $\{g(H_{\alpha n}): n \in \mathbb{N}, \alpha < \tau\}$ is a cozero cover of $g(X)$ with $\overline{g(H_{\alpha n})}$ compact. Hence $g(X)$ is locally compact.

Only minor adjustments of the same proof suffice to establish the following two results. In the first case, $\{H_\alpha : \alpha < \tau\}$ can be taken to be countable and, in the second case, $n(\alpha)$ a fixed integer n .

Proposition 6. *The class of all strongly locally finite-dimensional spaces of weight $\leq \tau$ has a universal element that is σ -compact and locally compact.*

Proposition 7. *The class of all spaces with $\text{loc dim} \leq n$ and weight $\leq \tau$ has a universal element that is locally compact.*

We can, in fact, improve the last result as follows.

Proposition 8. *The class of all spaces with $\text{loc dim} \leq n$ and weight $\leq \tau$ has a universal element Z which is compact with $\dim Z \leq n$.*

Proof. Let Y be the universal element provided by Proposition 7 and let $Z = Y \cup \{\infty\}$ be the one-point compactification of Y . If F is a closed set of Z disjoint from ∞ , then F is a compact subspace of Y so that $\dim F \leq \text{loc dim } Y \leq n$. This and the normality of Z imply $\dim Z \leq n$ [4], and the result follows.

The same conclusion can be reached through the following result, which is of some interest, and the fact that the class of spaces of weight $\leq \tau$ and $\dim \leq n$ has a compact universal element [8].

Proposition 9. *An infinite space X with $\text{loc dim } X \leq n$ can be embedded in a space $Y = X \cup \{\infty\}$, where $\infty \notin X$, with $\dim Y \leq n$ and $wY = wX$.*

Proof. Let $\{G_\alpha : \alpha < \tau\}$ be a cozero cover of X with $\dim G_\alpha \leq n$ for $\alpha < \tau = wX$. Further, let $F_{\alpha i}$ be zero sets and $G_{\alpha i}$ be cozero sets of X , $i \in N$, with $F_{\alpha i} \subset G_{\alpha i} \subset F_{\alpha i+1}$ and $G_\alpha = \bigcup_{i=1}^\infty F_{\alpha i} = \bigcup_{i=1}^\infty G_{\alpha i}$. We define a subset G of Y to be open if either G is an open subset of X or $Y - G$ is a closed subset of $\bigcup (F_{\alpha k} : \alpha \in J)$ for some $k \in N$ and a finite subset J of τ . Clearly, $wY = \tau$. Consider a continuous $f : X \rightarrow I$ with $G_{\alpha i} = f^{-1}(0, 1]$ and $F_{\alpha i} = f^{-1}\{1\}$. This can be continuously extended to Y by setting $f(\infty) = 0$, for, in that case, the restrictions of f to both of the closed sets $F_{\alpha i+1}$ and $Y - G_{\alpha i}$ are continuous. Hence each $F_{\alpha i}$ is zero in Y and each $G_{\alpha i}$ and G_α cozero. Thus Y has a base consisting of cozero sets and is therefore Tychonoff.

Now consider a closed set F of βY disjoint from ∞ . Let V be an open neighborhood of F in βY with $\infty \notin \bar{V}$ and pick $\alpha_1, \dots, \alpha_k < \tau$ with $\bar{V} \cap Y \subset \bigcup_{i=1}^k G_{\alpha_i}$. Then F is contained in $\bigcup_{i=1}^k \text{Ex} G_{\alpha_i}$ and there exists a closed cover $\{F_i : i = 1, \dots, k\}$ of F shrinking $\{F \cap \text{Ex} G_{\alpha_i} : i = 1, \dots, k\}$. If H_i is a cozero set of βY with $F_i \subset H_i$ and $H_i \cap Y \subset G_{\alpha_i}$, by Lemma 1 $\dim H_i \leq n$ and hence $\dim F_i \leq n$. Now, by the countable sum theorem, $\dim F \leq n$. Hence $\dim Y = \dim \beta Y \leq n$, and this completes the proof.

For the rest of our results we need a lemma and a simplified version of [3, Theorem 9]. Recall that bwX is the smallest cardinal τ for which there exists a continuous $f : X \rightarrow M$ with M metrizable and $W(f) = \tau$ [10].

Lemma 2. *Let X be a locally finite-dimensional space with $bwX \leq \tau$. Then X has a cover of cardinality $\leq \tau$ consisting of cozero sets with $\dim < \infty$.*

Proof. We can consider X as a subspace of $M \times I^\tau$ for some metrizable M . Let $\{G_{\lambda i}: \lambda \in \Lambda, i \in N\}$ be a σ -discrete base of M and $\{U_\alpha: \alpha < \tau\}$ a base of I^τ consisting of cozero sets. For each i and n in N , let $V_{\alpha, i, n}$ be the union of all sets $(G_{\lambda i} \times U_\alpha) \cap X$ that are contained in some cozero set of X with $\dim \leq n$. Each $V_{\alpha, i, n}$ is the disjoint union of open sets with $\dim \leq n$. Hence $\dim V_{\alpha, i, n} \leq n$ and evidently $\{V_{\alpha, i, n}: \alpha < \tau, i, n \in N\}$ is the required cover of X .

Theorem 2. *Let $f: X \rightarrow Y$ be a perfect and uniformly continuous function between uniform spaces such that Y is paracompact and every cozero set of Y is uniformly open. Then $\dim X \leq \text{Dim } X$ [3, Theorem 9].*

Proposition 10. *The class $\mathcal{E}$ of all locally finite-dimensional spaces that can be mapped by some continuous function f with $W(f) \leq \tau$ into a metrizable space of weight θ has a universal element which is locally Čech-complete and paracompact.*

Proof. Let M be a universal element of the class of all metric spaces of weight $\leq \theta$ which is complete. Every member of $\mathcal{E}$ is homeomorphic with a locally finite-dimensional subspace of $M \times I^\tau$. Let $\{X_\lambda: \lambda \in \Lambda\}$ be the collection of all such subspaces, X_0 their topological sum and $i: \beta X_0 \rightarrow \beta(M \times I^\tau)$ the unique continuous function whose restriction to each X_λ is the inclusion of X_λ into $M \times I^\tau$. By Lemma 2, each X_λ has a cozero cover $\{G_{\lambda\alpha}: \alpha < \tau\}$ with $\dim G_{\lambda\alpha} \leq n(\alpha) < \infty$. Now each $G_\alpha = \bigcup_{\lambda \in \Lambda} G_{\lambda\alpha}$ is a cozero set of X_0 with $\dim G_\alpha \leq n(\alpha) < \infty$ and there exists a continuous $g_\alpha: \beta X_0 \rightarrow I$ such that $G_\alpha = g_\alpha^{-1}(0, 1] \cap X_0$. Let $Y = (M \times I^\tau) \times I^\tau$, $f = i \times \prod_{\alpha < \tau} g_\alpha: \beta X_0 \rightarrow \beta(M \times I^\tau) \times I^\tau$, $X = f^{-1}(Y)$ and $H_\alpha = g_\alpha^{-1}(0, 1] \cap X$, $\alpha < \tau$. Note that $f: X \rightarrow Y$ is perfect, X is a paracompact p -space, each H_α is paracompact as a cozero set of X and, by Lemma 1, $\dim H_\alpha = \dim G_\alpha \leq n(\alpha) < \infty$.

We now endow X with its finest uniformity, M with that induced by a compatible metric, I^τ with its unique uniformity and $M \times I^\tau$ and Y with the resulting product uniformities. Theorem 1 applied to $f: X \rightarrow Y$ yields a uniformly continuous $g: X \rightarrow Y \times I^\tau$ with $f = \pi \circ g$, where π denotes the projection of $Y \times I^\tau$ onto Y , and $\text{Dim } g(F_{\alpha n}) \leq \text{Dim } F_{\alpha n}$, where $F_{\alpha n} = g_\alpha^{-1}[\frac{1}{n}, 1] \cap X$ for each $\alpha < \tau$ and n in N . Note that $F_{\alpha n}$ is a z -embedded subspace of X and therefore $\text{Dim } g(F_{\alpha n}) \leq \text{Dim } F_{\alpha n} = \dim F_{\alpha n} \leq \dim H_\alpha \leq n(\alpha)$. Now, if $\pi_\alpha: Y \rightarrow I$ denotes the projection onto the α th factor of the second copy of I^τ in Y , then $g(H_\alpha) = \pi^{-1}(\pi_\alpha^{-1}(0, 1]) \cap g(X)$. It follows that, since $f: X \rightarrow Y$ is perfect, the same is true of $g: X \rightarrow g(X)$, $\pi: g(X) \rightarrow Y$ and $\pi: g(H_\alpha) \rightarrow \pi_\alpha^{-1}(0, 1]$. Hence $g(H_\alpha)$ is Čech-complete and paracompact. Finally, if $\sigma: Y \rightarrow M$ denotes natural projection, it is readily seen that the composite of $\pi: g(F_{\alpha n}) \rightarrow Y$ with σ is perfect and uniformly continuous and, by Theorem 2, $\dim g(F_{\alpha n}) \leq \text{Dim } g(F_{\alpha n}) \leq n(\alpha)$ for each n in N and

$\alpha < \tau$. By the countable sum theorem for $\dim$, $\dim g(H_\alpha) \leq n(\alpha)$, and it is now clear that $Z = \bigcup_{\alpha < \tau} g(H_\alpha)$ is a universal element of $\mathcal{C}$ which is locally Čech-complete and paracompact.

As with Proposition 5, two more results have almost the same proof. We first point out that a slightly stronger condition is satisfied by Z . Let $\alpha_1, \alpha_2, \dots < \tau$, $h = \sigma \times \prod_{i=1}^\infty \pi_{\alpha_i}: Y \rightarrow M \times I^N$ and $\sigma_i: M \times I^N \rightarrow I$ the projection onto the i th factor of I^N . Then h is perfect and so is its composite with $\pi: g(X) \rightarrow Y$. It follows that

$$\bigcup_{i=1}^\infty g(H_{\alpha_i}) = \pi^{-1} \left(h^{-1} \left(\bigcup_{i=1}^\infty \sigma_i^{-1}(0, 1] \right) \right)$$

is a Čech-complete paracompact p -space. This is exactly what is needed in Proposition 12.

Proposition 11. *The class of all spaces with $\text{loc dim} \leq n$ that can be mapped by some continuous function f with $W(f) \leq \tau$ into a metrizable space of weight θ has a universal element which is locally Čech-complete and paracompact.*

Proposition 12. *The class of all strongly locally finite-dimensional spaces that can be mapped by some continuous function f with $W(f) \leq \tau$ into a metrizable space of weight θ has a universal element which is a Čech-complete paracompact p -space.*

The last result can be rephrased as follows. Recall that $\mu w X$ is the supremum of all cardinals α for which there exists a map from X onto a metrizable space of weight α [10].

Proposition 13. *The class of all strongly locally finite-dimensional spaces with $\text{bw} \leq \tau$ and $\mu w \leq \theta$ has a universal element which is a Čech-complete paracompact p -space.*

Proof. It suffices to prove that for the universal space Z of Proposition 12, $\mu w Z \leq \theta$. This is true because Z can be mapped by a perfect map onto a metric space of weight $\leq \theta$ [10, Proposition 2].

Consider a locally finite-dimensional normal and countably paracompact space X such that $\dim F = \text{loc dim } F$ for every closed subset F of X . Let $\{H_i: i \in N\}$ be an open cover of X with $\text{loc dim } H_i \leq i$. This has a closed shrinking $\{F_i: i \in N\}$, and if G_i is a cozero set of X with $F_i \subset G_i \subset H_i$ then G_i is the countable union of closed sets of X with $\dim = \text{loc dim} \leq i$. By the countable sum theorem, $\dim G_i \leq i$, and hence X is strongly locally finite dimensional. Thus, Propositions 6 and 13 generalize the results of Bobkov quoted in [11].

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF NAIROBI, P. O. BOX 30197, NAIROBI, KENYA