THE SYNTACTIC MONOID OF AN INFIX CODE

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ABSTRACT. Necessary and sufficient conditions on a monoid M are found in order that M be isomorphic to the syntactic monoid of a language L over an alphabet X having one of the following properties. In the first theorem L is a P_L -class and $P_{W(L)} \subseteq P_L$ where P_L is the syntactic congruence of L and $W(L)$ is the residue of L . In the second theorem L is an infix code; that is, satisfies $u, uvw \in L$ implying $u = w = 1$. In the third theorem L is an infix code satisfying a condition which amounts to the requirement that M be a nilmonoid. Various refinements of these conditions are also considered.

1. INTRODUCTION AND SUMMARY

For any set X , a language L is any subset of X^* , the free monoid on X . In the study of properties and classification of languages, L is usually specialized to satisfy some strong conditions that make it amenable to various kinds of treatments. One of the most common restrictions is the requirement that L be a code; that is, a set of free generators for the submonoid of X^* generated by L . Further specialization leads to the rich family of codes, among which the infix codes figure prominently. A language L is an infix code if it is an antichain for the partial order on X^* called the infix order.

Going back to general languages, one of the most successful tools of studying them is the syntactic monoid. For a language L over X , one first constructs the syntactic congruence P_L (the greatest congruence on X^* saturating L) and the quotient monoid, $Syn(L) = X^*/P_L$, is the syntactic monoid of the language L . Monoids isomorphic to syntactic monoids of languages are easily characterized as monoids containing a disjunctive subset D (that is, the obvious analogue of the syntactic congruence for D is the equality relation). In various special cases, deeper relations between the properties of the language and the properties of its syntactic monoid can be discovered.

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It should now be clear that if we combine the subjects of the preceding two paragraphs, we are naturally led to a study of the syntactic monoid of an infix code. As should be expected, such a syntactic monoid will have some strong properties. In the first place, since an infix code L always has a nonempty residue $W(L)$, which is then a P_L -class, we get that $\text{Syn}(L)$ has a zero. Furthermore, since L itself is a P_L -class we know that $\text{Syn}(L)$ has a disjunctive element. Finally, since $P_L = P_{W(L)}$, it follows that the zero is a disjunctive element of $\text{Syn}(L)$. Continuing in this vein, we get necessary and sufficient conditions on a monoid M to be isomorphic to the syntactic monoid of an infix code. Further specializations of either the infix code L directly or of its syntactic monoid $\text{Syn}(L)$ are of particular interest. For precedences of this study, see [2, 4].

Section 2 contains necessary preliminaries. A characterization of $\text{Syn}(L)$ when L is a P_L -class and $P_{W(L)} \subseteq P_L$ is the essence of §3. Several characterizations of $\text{Syn}(L)$ when L is an infix code are given in §4. Finally, several consequences of this result are treated in §5.

2. PRELIMINARIES

A language L over an alphabet X is any subset of X^* , the free monoid on X consisting of words over X with juxtaposition as product. The *syntactic congruence* P_L of L defined on X^* is

$$u \equiv v \ (P_L) \text{ if } xuy \in L \Leftrightarrow xvy \in L \text{ for all } x, y \in X^*.$$

The quotient monoid $\text{Syn}(L) = X^*/P_L$ is the *syntactic monoid* of L . Further $\bar{L}$ denotes the complement of L in X^* . The identity of X^* is denoted by 1 ; the symbol $\emptyset$ stands for the empty set except when it is the identity of X^* . The complement of 1 in X^* is the *free semigroup* X^+ on X . For $w \in X^*$, $w^* = \{w^n | n \geq 1\} \cup \{1\}$. A language L over X is an *infix code* if it is nonempty, does not contain 1 , and for any $u, v, w \in X^*$, $v, uvw \in L$ implies $u = w = 1$. Hence it can be characterized as an antichain in X^+ under the infix order. For $u, v \in X^*$, $u \leq v$ in the *embedding order* if $u = x_1x_2 \dots x_n$, $v = y_1x_1y_2x_2 \dots x_ny_{n+1}$ for some $x_i, y_i \in X^*$. A nonempty antichain, not containing 1 , in this order is a *hypercode*. A hypercode is obviously an infix code. It is well known that if L is a hypercode over a finite alphabet X , then L is itself finite.

The above definition of P_L is used for any subset L of any semigroup S . If $L = \{a\}$, we write P_a instead of $P_{\{a\}}$. If P_L is the equality relation, L is a *disjunctive subset* of S ; if $L = \{a\}$, we say that a is a *disjunctive element* of S . For any $a \in S$, $J(a)$ denotes the *principal ideal* of S generated by a . If S has a zero, the intersection of all nonzero ideals, if different from zero, is the *core* of S , denoted by $\text{core}(S)$; the set

$$\mathcal{A}(S) = \{a \in S | as = sa = 0 \text{ for all } s \in S\}$$

is the *annihilator* of S . A trivial semigroup has a core by definition.

For any sets A and B , $A \setminus B = \{a \in A \mid a \notin B\}$. If φ is a mapping defined on A , then $\bar{\varphi}$ denotes the equivalence relation on A induced by φ .

3. THE FIRST THEOREM

The first theorem provides necessary and sufficient conditions on a monoid to be isomorphic to the syntactic monoid of a language L such that L is a P_L -class and $P_{W(L)} \subseteq P_L$. The following lemma is well known.

Lemma 1. *Let M be a monoid. Let $X = M$ and define a mapping φ on X^* :*

$$\varphi: x_1 x_2 \dots x_n \longrightarrow x_1 x_2 \dots x_n \in M.$$

Then φ is a homomorphism of X^ onto M .*

Theorem 1. *A monoid M is isomorphic to the syntactic monoid of a language L such that L is a P_L -class and $P_{W(L)} \subseteq P_L$ if and only if M has a disjunctive zero and a core.*

Proof.

Necessity. First assume that $W(L) = \emptyset$. Then $P_{W(L)}$ is the universal relation and the hypothesis implies also that P_L is the universal relation. But then $L = X^*$ and M is the trivial monoid that evidently satisfies the desired conclusion.

Suppose next that $W(L) \neq \emptyset$. Then $W(L)$ is a P_L -class yielding $P_L \subseteq P_{W(L)}$ that together with the hypothesis gives $P_{W(L)} = P_L$. In view of the fact that $W(L)$ is the zero of $\text{Syn}(L)$, we conclude that the zero of M is disjunctive. Since L is a P_L -class, we have $W(L) \neq X^*$. Let $v \in X^* \setminus W(L)$. Then $X^* v X^* \cap L \neq \emptyset$ so that $uvw \in L$ for some $u, w \in X^*$. Letting c be the image of L in $\text{Syn}(L)$ and for any $x \in X^*$, and $[x]$ the image of x in $\text{Syn}(L)$, we obtain $c = [u][v][w]$. This means that c is contained in the ideal of $\text{Syn}(L)$ generated by $[v]$. Therefore c is in the core of $\text{Syn}(L)$, and since $\text{Syn}(L)$ has a core, so does M .

Sufficiency. Let $X = M$. If M is trivial, we may let $L = X^*$ and all the required conditions are satisfied. By Lemma 1, we have a homomorphism φ of X^* onto M . Let c be a nonzero element of the core of M and let $L = c\varphi^{-1}$ and $W = 0\varphi^{-1}$. By hypothesis, 0 is disjunctive which easily gives that $P_W = \bar{\varphi}$.

Let $u, v \in X^*$ be such that $u\varphi = v\varphi$. Assume that $xuy \in L$ for some $x, y \in X^*$. Then $(x\varphi)(u\varphi)(y\varphi) = c$ and hence $(x\varphi)(v\varphi)(y\varphi) = c$ giving $xvy \in L$. By symmetry, we conclude that $u \equiv v \ (P_L)$. Therefore $P_W = \bar{\varphi} \subseteq P_L$, as required.

We show next that $W = W(L)$. If $w \in W$, then $w\varphi = 0$ and hence $(u\varphi)(w\varphi)(v\varphi) \neq c$ for all $u, v \in X^*$ and thus $X^* w X^* \cap L = \emptyset$; that is, $w \in W(L)$. Conversely, let $w \in W(L)$. Then $X^* w X^* \cap L = \emptyset$ and thus $c \notin J(w\varphi)$. Since c is contained in all nonzero ideals of M , we must have

$J(w\varphi) = 0$. Therefore $w\varphi = 0$ so that $w \in W$. Consequently $W = W(L) \neq \emptyset$ and hence W is a P_L -class which implies that $P_L \subseteq P_W$. In conclusion, $P_L = P_W = \emptyset$, and since $L = c\varphi^{-1}$, it follows that L is a P_L -class, as required.

Remarks. The condition that L is a P_L -class is equivalent to

$$u, v \in X^*, \quad uLv \cap L \neq \emptyset \Rightarrow uLv \subseteq L,$$

see [1, Theorem 10.6]. The condition $P_{W(L)} \subseteq P_L$ is equivalent to $P_L = P_{W(L)}$ since if $W(L) = \emptyset$, then $P_{W(L)}$ is the universal relation, hence $P_L \subseteq P_{W(L)}$; in addition, if $W(L) \neq \emptyset$, then it is a P_L -class, and again $P_L \subseteq P_{W(L)}$.

Example 1 below shows that a language L can be a P_L -class but $P_{W(L)} \not\subseteq P_L$. In the next section Example 2 will exhibit a language L that is not a P_L -class but $P_{W(L)} \subseteq P_L$. The two conditions therefore are independent. These examples will also show that the conditions of 0 being disjunctive and the existence of a core are independent.

Example 1. Let $L = \{a^m b a^n \mid m, n \geq 1\}$ with $X = \{a, b\}$. For any $m, n \geq 1$ and $u, v \in X^*$, we have $ua^m b a^n v \in L$ if and only if $u, v \in a^*$, implying that L is a P_L -class. Also

$$uabav \notin W(L) \Leftrightarrow u, v \in a^* \Leftrightarrow ubv \notin W(L)$$

which implies that $aba \equiv b \ (P_{W(L)})$. Since $aba \not\equiv b \ (P_L)$, it follows that $P_{W(L)} \not\subseteq P_L$.

In the proof of necessity of Theorem 1, we see that L being a P_L -class alone implies that $Syn(L)$ has a core. Hence this is true in our example as well. If 0 was disjunctive in this example, Theorem 1 would imply that $P_{W(L)} \subseteq P_L$, which is not the case. Therefore 0 is not disjunctive for this L .

4. THE SECOND THEOREM

It contains three characterizations of the syntactic monoid of an infix code and represents the main result of the paper.

Lemma 2. *Let L be a language over an alphabet X . If one of the P_L -classes is an infix code, then the P_L -class of 1 is trivial.*

Proof. Let C be a P_L -class, an infix code, and let $u \equiv 1 \ (P_L)$. For any $v \in C$, we then get $uv \equiv v \ (P_L)$. But then $v, uv \in C$ and hence $u = 1$ since C is infix. Thus the P_L -class of 1 is trivial.

Lemma 3. *Let M be a monoid with identity e and assume that $M \setminus \{e\}$ is a subsemigroup of M . Let $X = M \setminus \{e\}$ and define a mapping on X^* :*

$$\varphi: x_1 x_2 \dots x_n \longrightarrow x_1 x_2 \dots x_n \in M \setminus \{e\}$$

if $x_1, x_2, \dots, x_n \in M \setminus \{e\}$ and

$$\varphi: 1 \longrightarrow e.$$

Then φ is a homomorphism of X^* onto M such that $w\varphi = e$ if and only if $w = 1$. If c is a disjunctive element of M , then for $L = c\varphi^{-1}$, we have $\bar{\varphi} = P_L$.

Proof. Standard.

Theorem 2. *The following conditions on a monoid M with identity e are equivalent.*

- (i) M is isomorphic to the syntactic monoid of an infix code L .
- (ii) (α) $M \setminus \{e\}$ is a subsemigroup of M ;
 (β) M has a zero;
 (γ) M has a disjunctive element c such that $c \notin \{e, 0\}$ and $c = xcy$ implies $x = y = e$.
- (iii) (α);
 (δ) M has a disjunctive zero;
 (ε) $\text{core}(M) = \{c, 0\}$ with $c \in \mathcal{A}(M \setminus \{e\})$.
- (iv) (α), (δ);
 (ζ) there exists $0 \neq c \in \text{core}(M) \cap \mathcal{A}(M \setminus \{e\})$.

Proof. (i) $\Rightarrow$ (ii). Since L is an infix code, [2, Proposition 1] gives that L is P_L -class and hence Lemma 2 yields that the P_L -class of 1 is trivial. Hence (α) holds for $\text{Syn}(L)$ and thus also for M . If $u \in L$ and $v \in X^+$, then $X^*uvX^* \cap L = \emptyset$ since L is infix and hence $uv \in W(L)$. Thus $\text{Syn}(L)$ has a zero and therefore (β) holds. Let c be the image of L in $\text{Syn}(L)$. Since $L \subseteq X^+$ by hypothesis, we have $c \neq e$, and since $L \cap W(L) = \emptyset$, we also have $c \neq 0$. It follows easily that c is disjunctive. For any $w \in X^*$, let $[w]$ be its image in $\text{Syn}(L)$, and let $u \in L$. Assume that for $x, y \in X^*$ we have $c = [x]c[y]$. Then $u, xuy \in L$ implies that $x = y = 1$ whence $[x] = [y] = e$. Therefore M satisfies (γ) as well.

(ii) $\Rightarrow$ (iii). Let $a \in M \setminus \{0\}$. Since c is disjunctive, there exist $x, y \in M$ such that either $xay = c$ and $x0y \neq c$ or $xay \neq c$ and $x0y = c$. Since the second alternative is impossible, we have $xay = c$. It follows that c is contained in every nonzero ideal of M so that $c \in \text{core}(M)$.

Since P_c is the equality relation, we have $P_c \subseteq P_0$. For the opposite inclusion, it suffices to show that $\{c\}$ is a P_0 -class. Let $d \equiv c (P_0)$. Then $d \neq 0$ since 0 is a P_0 -class. As above, there exist $x, y \in M$ such that $c = xdy$. In addition, $xdy \equiv xcy (P_0)$ and hence $c \equiv xcy (P_0)$. It follows that $xcy \neq 0$ so that, as above, there exist $u, v \in M$ such that $uxcyv = c$. Then (γ) gives $ux = yv = e$ which by (α) yields $x = y = e$ so that $c = d$. Therefore $P_0 \subseteq P_c$ and equality prevails. Therefore 0 is disjunctive and (δ) holds.

Next let $x \in M \setminus \{0\}$. If $xc \neq 0$, then $c = uxcv$ for some $u, v \in M$, as above. But then (γ) and (α) imply that $x = e$. Similarly $cx \neq 0$ implies that $x = e$. It follows that $xc = cx = 0$ for all $x \in M \setminus \{e\}$ and, therefore, $c \in \mathcal{A}(M \setminus \{e\})$.

As a consequence, we have that $I = \{c, 0\}$ is a nonzero ideal of M whence $\text{core}(M) \subseteq I$. We have seen above that $c \in \text{core}(M)$ so that $I \subseteq \text{core}(M)$ and equality prevails. Consequently (ϵ) holds.

(iii) $\Rightarrow$ (iv) trivially.

(iv) $\Rightarrow$ (i). We adopt the result and the notation of Lemma 3 so that $\text{Syn}(L) \cong M$. It remains to show that L is infix. Let $v, uvw \in L$. Then $v\varphi = (u\varphi)(v\varphi)(w\varphi)$ and we have $c = (u\varphi)c(v\varphi)$. It follows that $(u\varphi)c \neq 0$ and $c(v\varphi) \neq 0$ which by (ζ) yields $u\varphi = v\varphi = e$. By Lemma 3, we conclude that $u = v = 1$ so that L is indeed an infix code.

Remarks. In the presence of condition (δ) , according to [3, Theorem 3.7], condition (ϵ) is equivalent to M being subdirectly irreducible. Even though this latter notion is a more familiar one, the existence of a core is a condition easier to verify.

The following example shows that in condition (γ) the implication $c = xcy \Rightarrow x = y = e$ can not be omitted.

Example 2. Let $L = \{a^m b^n | m, n \geq 0\}$ with $X = \{a, b\}$. Then P_L has the following nonresidue classes:

$$e = \{1\}, \quad f = a^*, \quad g = b^*, \quad c = \{a^m b^n | m, n \geq 1\}$$

and $\text{Syn}(L)$ is isomorphic to the monoid $M = \{e, f, g, c, 0\}$ with the following multiplication table.

	e	f	g	c	0
e	e	f	g	c	0
f	f	f	c	c	0
g	g	0	g	0	0
c	c	0	c	0	0
0	0	0	0	0	0

Simple verification shows that c is a disjunctive element of M . Here $c = cg$ with $g \neq 1$. Therefore M satisfies conditions (α) , (β) , and (γ) , except for the implication $c = xcy \Rightarrow x = y = e$, in Theorem 2. The last condition should also be added to [2, Corollary 7]. Clearly L is not an infix code.

In this example we also have that L is not a P_L -class but, since $W(L) = \overline{L}$, we have $P_{W(L)} = P_L$. Since c is a disjunctive element of M , this shows that 0 is a disjunctive element too. In addition, M has no core. This example was promised before Example 1 in connection with Theorem 1.

Lemma 4. Let L be a hypercode over an alphabet X . Then every nonresidue P_L -class is a hypercode.

Proof. Let $u, v \in X^*$ be such that $u \leq v$ in the embedding order, $u \equiv v (P_L)$ and $u \notin W(L)$. Hence $X^* u W^* \neq \emptyset$ so that there exist $s, t \in X^*$ such that $sut \in L$. Since $sut \equiv svt (P_L)$, it follows that $svt \in L$. Now $sut \leq svt$ implies that $sut = svt$ since L is a hypercode and thus $u = v$.

Lemma 4 does not hold if we substitute hypercode by infix. For example, let $L = \{ab^n a | n \geq 1\}$. Then L is infix but the P_L -class $\{b^n | n \geq 1\}$ is not.

Corollary 1. *The following conditions on a monoid M with identity e are equivalent.*

- (i) M is isomorphic to the syntactic monoid of a hypercode.
- (ii) (α) and β from Theorem 2;
 (γ') M has a disjunctive element c such that $c \notin \{e, 0\}$;
 (δ') if $a_1 a_2 \dots a_n = x_1 a_1 x_2 a_2 \dots x_n a_n x_{n+1} \neq 0$, then $x_1 = x_2 = \dots = x_{n+1} = e$.
- (iii) (α) and β from Theorem 2;
 (γ'') M has a disjunctive element c such that $c \notin \{e, 0\}$ and if $c = a_1 a_2 \dots a_n = x_1 a_1 x_2 a_2 \dots x_n a_n x_{n+1}$, then $x_1 = x_2 = \dots = x_{n+1} = e$.

Proof. (i) $\Rightarrow$ (ii). Since a hypercode is an infix code, conditions α , β , and γ of Theorem 2 are valid here. Assume the antecedent of (δ') in $\text{Syn}(L)$. Let $u_i, v_i \in X^*$ be such that $[u_i] = a_i$ and $[v_i] = x_i$ for all relevant indices i . Then

$$u_1 u_2 \dots u_n \equiv v_1 u_1 v_2 u_2 \dots v_n u_n v_{n+1} \quad (P_L)$$

and $u_1 u_2 \dots u_n \notin W(L)$. Now $u_1 u_2 \dots u_n \leq v_1 u_1 v_2 u_2 \dots v_n u_n v_{n+1}$ in the embedding order which by Lemma 4 yields that $v_1 = v_2 = \dots = v_{n+1} = 1$. But then $x_1 = x_2 = \dots = x_{n+1} = e$, and (δ') holds.

(ii) $\Rightarrow$ (iii) trivially.

(iii) $\Rightarrow$ (i). By Theorem 2, $M \cong \text{Syn}(L)$ for an infix code L . Now let

$$u_1 u_2 \dots u_n, v_1 u_1 v_2 u_2 \dots v_n u_n v_{n+1} \in L$$

for some $u_i, v_i \in X^*$. Denoting $a_i = [u_i]$ and $x_i = [v_i]$ for all relevant indices i and letting c stand for L , we get $c = a_1 a_2 \dots a_n = x_1 a_1 x_2 a_2 \dots x_n a_n x_{n+1}$. By (γ'') , we have $x_1 = x_2 = \dots = x_{n+1} = e$. In view of Lemma 2, we conclude that $v_1 = v_2 = \dots = v_{n+1} = 1$ which finally shows that L is a hypercode.

5. THE THIRD THEOREM

It deals with a special case of Theorem 2. A monoid M with identity e and zero 0 is said to be *nil* if for every $a \in M \setminus \{e\}$ there exists a positive integer n such that $a^n = 0$. Following [4], we call a monoid with zero *nontrivial* if it has at least three elements.

Theorem 3. *A monoid M is isomorphic to the syntactic monoid of an infix code L over an alphabet X with the property that for every $w \in X^+$ there exists $n \geq 1$ such that $w^n \in W(L)$ if and only if M is a subdirectly irreducible nontrivial nilmonoid.*

Proof.

Necessity. Let e be the identity of M . By Theorem 2 (α) , we know that $M \setminus \{e\}$ is a subsemigroup of M . Since $W(L)$ is the zero of $\text{Syn}(L)$, the

hypothesis evidently implies that $M \setminus \{e\}$ is a nil semigroup and hence M is a nilmonoid. By Theorem 2 (γ), M has a disjunctive element c different from e and 0 and thus M is nontrivial. In addition, $M \setminus \{e\}$ is a nilsemigroup having c as a nonzero disjunctive element, which by [3, Theorem 4.5] implies that $M \setminus \{e\}$ is a subdirectly irreducible semigroup. Now [3, Theorem 3.6] yields that M is a subdirectly irreducible monoid.

Sufficiency. By [5, Théorème I], subdirect irreducibility implies that M has a core. Since M is a nilmonoid, $M \setminus \{e\}$ is a nilsemigroup which evidently has a core, in fact equal to the core of $M \setminus \{e\}$ since $M \setminus \{e\}$ is a nonzero proper ideal of M . Now [3, Lemma 4.2] gives that $\text{core}(M \setminus \{e\}) = \mathcal{A}(M \setminus \{e\})$ and has exactly two elements. By [3, Theorem 3.6], subdirect irreducibility of M implies that of $M \setminus \{e\}$. But then [3, Theorem 4.5] yields that $M \setminus \{e\}$ contains a nonzero disjunctive element. Now [3, Lemma 4.4] gives that 0 is a disjunctive element of $M \setminus \{e\}$. If $a \in M \setminus \{0, e\}$, then there exists $n \geq 1$ such that $a^n \neq 0$ and $a^{n+1} = 0$, since $M \setminus \{e\}$ is a nilsemigroup, and thus $aa^n e = 0$ and $ae = a \neq 0$. Therefore 0 is disjunctive in M .

We have proved that M satisfies conditions (α) , (δ) , and (ϵ) of Theorem 2 and thus $M \cong \text{Syn}(L)$ for an infix code L over an alphabet X . Let $w \in X^+$ and let $[w]$ be the corresponding element of $\text{Syn}(L)$. By Lemma 2, $[w] \neq [1]$ and hence there exists $n \geq 1$ such that $[w]^n = 0$ since M is a nilmonoid. This means that $w^n \in W(L)$, as required.

A monoid M with identity e and zero 0 is said to be *nilpotent* if there exists a positive integer n such that $a^n = 0$ for all $a \in M \setminus \{e\}$.

Corollary 1. *A monoid M is isomorphic to the syntactic monoid of an infix code L over an alphabet X with the property that there exists $n \geq 1$ such that $w^n \in W(L)$ for every $w \in X^+$ if and only if M is a subdirectly irreducible nontrivial nilpotent monoid.*

Proof.

Necessity. By Theorem 3, M must be subdirectly irreducible and nontrivial. The additional hypothesis here evidently implies that for any $a \in M \setminus \{e\}$, we have $a^n = 0$ and thus M is a nilpotent monoid.

Sufficiency. By Theorem 3, we have $M \cong \text{Syn}(L)$ for some infix code L over an alphabet X . Let $w \in X^+$. By Lemma 2, $[w] \neq [1]$ where these are P_L -classes. The hypothesis implies that $[w]^n = 0$ and hence $w^n \in W(L)$.

Corollary 2 [2, Corollary 8 and Proposition 9]. *A monoid M is isomorphic to the syntactic monoid of a finite infix code over a finite alphabet if and only if M is a finite subdirectly irreducible nontrivial nil monoid.*

Following [4], we call a monoid *strict* if it satisfies condition (δ') in Corollary 1 to Theorem 2.

Corollary 3 [4, Proposition 3]. *A monoid M is isomorphic to the syntactic monoid of a hypercode L over a finite alphabet X if and only if M is a finite subdirectly irreducible nontrivial strict monoid.*

Proof. This follows easily from Corollary 1 to Theorem 2, Corollary 2 to Theorem 3, [4, Corollary to Proposition 1] and the well-known statement that a hypercode over a finite alphabet must be finite.

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