SECOND-DERIVATIVE ESTIMATES FOR SOLUTIONS OF TWO-DIMENSIONAL MONGE-AMPÈRE EQUATIONS

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ABSTRACT. Heinz–Lewy type a priori estimates are derived for the absolute values of the second derivatives of solutions $z(x, y) \in C^{1,1}(\Omega)$ of Monge–Ampère equations of the general form

$$Ar + 2Bs + Ct + (rt - s^2) = E$$

in the interior of the domain Ω . The coefficients A, B, C, E depend in particular on the gradient of $z(x, y)$ and satisfy certain structural conditions.

1. INTRODUCTION AND STATEMENT OF THE RESULT

Consider a solution $z = z(x, y) \in C^{1,1}(\Omega)$ of the elliptic Monge–Ampère equation

$$(1) \quad Ar + 2Bs + Ct + (rt - s^2) = E$$

or, equivalently,

$$(2) \quad (r + C)(t + A) - (s - B)^2 = \Delta,$$

where

$$\Delta = AC - B^2 + E > 0,$$

and assume without loss of generality that

$$t + A > 0.$$

The coefficients A, B, C and the nonhomogeneous term E depend on the five variables x, y, z, p, q . Here p, q, r, s, t represent the first and second derivatives of $z(x, y)$,

$$p = z_x, q = z_y; r = z_{xx}, s = z_{xy}, t = z_{yy}.$$

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Assumption (A). The solution $z(x, y)$ satisfies the estimate

$$|Dz| \leq K_1.$$

Assumption (B). Suppose that $A, B, C \in C^1(\Omega \times \mathbb{R}^3)$, and let

$$\Delta = K(x, y, z)D(x, y, z, p, q),$$

with $D \in C^1(\Omega \times \mathbb{R}^3)$ and $K \in C^\mu(\Omega \times \mathbb{R})$ for some μ , $0 < \mu < 1$, such that

$$\begin{aligned} |A|, |B|, |C|, |D|, |K| &\leq a, \\ |A_x|, \dots, |D_q| &\leq b, \\ [K]_\mu^{\Omega \times \mathbb{R}} &\leq b', \\ D, K &\geq \frac{1}{c}. \end{aligned}$$

Assumption (C). Suppose that the functions

$$\begin{aligned} \phi_1(x, y) &= A_p, \\ \phi_2(x, y) &= A_q + 2B_p, \\ \phi_3(x, y) &= 2B_q + C_p, \\ \phi_4(x, y) &= C_q \end{aligned}$$

belong to the Lipschitz class $C^{0,1}(\Omega)$ and satisfy the conditions

$$[\phi_1]_{0,1}^\Omega, \dots, [\phi_4]_{0,1}^\Omega \leq d.$$

The main result of this note then reads as follows:

Theorem 1. *The solution $z(x, y)$ belongs to the Hölder class $C_{\text{loc}}^{2,\mu}(\Omega)$. For each subset Ω' which is compactly contained in Ω , there is a second-derivative estimate of the form*

$$\|D^2 z\|_{C^\mu(\Omega')} \leq C,$$

where the constant C depends only on μ, a, b, b', c, d, K_1 , and $\text{dist}(\Omega', \partial\Omega)$.

The regularity part of the theorem and the Hölder estimate

$$[D^2 z]_\mu^{\Omega'} \leq C$$

have been derived in [7, 9] if the coefficients A, B, C, E are μ -Hölder continuous with respect to the variables x, y, z, p, q . This was shown by invoking the Legendre-like transformation

$$\begin{aligned} u &= x, \\ v &= q(x, y). \end{aligned}$$

The purpose of the present article is to provide sharp local estimates also for the absolute values of the second derivatives of $z(x, y)$. The present approach consists of introducing characteristic parameters; i.e., of constructing a conformal map $(u(x, y), v(x, y))$ with respect to the characteristic form

$$(r + C) dx^2 + 2(s - B) dx dy + (t + A) dy^2.$$

This approach is due to H. Lewy [6] and E. Heinz [1, 3], who also constructed a counterexample showing that Monge-Ampère equations of the form (1) in general do not admit second-derivative estimates for their solutions. Heinz [3] discovered the structural conditions of Assumption (C) and derived second-derivative estimates under additional regularity assumptions regarding both the solution $z(x, y)$ and the coefficients A, B, C, E .

The present estimates rest on sharp estimates for the Jacobian of a certain quasilinear elliptic second-order system which is satisfied by the inverse mapping $(x(u, v), y(u, v))$. These estimates were derived in [8], generalizing classical theorems of Lewy [5] and Heinz [2, 4]. An improved version is stated as Proposition 4.

The results of the present article are tailor-made for application to prescribed curvature equations, in particular to the prescribed Gauss curvature equation,

$$rt - s^2 = K(x, y, z)(1 + p^2 + q^2)^2,$$

and to the Darboux equation, which in terms of local parameters u, v for a convex surface takes the form

$$\begin{aligned} &(\rho_{uu} - \Gamma_{11}^1 \rho_u - \Gamma_{11}^2 \rho_v)(\rho_{vv} - \Gamma_{22}^1 \rho_u - \Gamma_{22}^2 \rho_v) - (\rho_{uv} - \Gamma_{12}^1 \rho_u - \Gamma_{12}^2 \rho_v)^2 \\ &= K(u, v)(EG - F^2 - G\rho_u^2 + 2F\rho_u\rho_v - E\rho_v^2). \end{aligned}$$

Here $\rho = \rho(u, v)$ represents any component of the radius vector of the surface. The expressions Γ_{ij}^k are the Christoffel symbols of the first fundamental form

$$E(u, v) du^2 + 2F(u, v) du dv + G(u, v) dv^2,$$

and $K(u, v)$ denotes the Gauss curvature.

2. CHARACTERISTIC PARAMETERS

Lemma 2. *Let A, B, C, D be functions of class $C^1(\Omega)$, and let $z \in C^{2,\mu}(\Omega)$ for some $\mu, 0 < \mu < 1$, be a solution of the Monge-Ampère equation (2) such that*

$$\Delta = K(x, y)D(x, y), \quad K, D > 0.$$

Let $\bar{B}_R = \bar{B}_R(x_0, y_0) = \{(x - x_0)^2 + (y - y_0)^2 \leq R^2\} \subset \Omega$. Then there exists a homeomorphism $(x, y) = (x(u, v), y(u, v))$ from $\bar{B} = \{u^2 + v^2 \leq 1\}$ onto $\bar{B}_R$

of class $C_{\text{loc}}^{1,\mu}(B)$ with $x(0, 0) = x_0$, $y(0, 0) = y_0$, which satisfies the system

$$(3) \quad L_K x = \frac{1}{\sqrt{D}} \left(A_x + B_y - (t + A) \frac{D_x}{2D} + (s - B) \frac{D_y}{2D} \right) Dx \wedge Dy,$$

$$(4) \quad L_K y = \frac{1}{\sqrt{D}} \left(B_x + C_y + (s - B) \frac{D_x}{2D} - (r + C) \frac{D_y}{2D} \right) Dx \wedge Dy.$$

Here

$$L_K = \frac{\partial}{\partial u} \left(\sqrt{K} \frac{\partial}{\partial u} \right) + \frac{\partial}{\partial v} \left(\sqrt{K} \frac{\partial}{\partial v} \right).$$

Furthermore,

$$Dx \wedge Dy = x_u y_v - x_v y_u \neq 0,$$

and

$$(5) \quad \frac{t + A}{\sqrt{\Delta}} = \frac{|Dx|^2}{Dx \wedge Dy},$$

$$(6) \quad -\frac{s - B}{\sqrt{\Delta}} = \frac{Dx \cdot Dy}{Dx \wedge Dy},$$

$$(7) \quad \frac{r + C}{\sqrt{\Delta}} = \frac{|Dy|^2}{Dx \wedge Dy}.$$

Proof. Consider the characteristic form

$$\begin{aligned} ds^2 &= (r + C) dx^2 + 2(s - B) dx dy + (t + A) dy^2 \\ &= a dx^2 + 2b dx dy + c dy^2 \end{aligned}$$

in the disc $\overline{B}_R$. Uniformizing parameters (u, v) in the disc $\overline{B} = \{u^2 + v^2 \leq 1\}$ can be introduced since the coefficients a, b, c of the metric ds^2 belong to the Hölder class $C^\mu(\Omega)$: There exists a homeomorphism $(u, v) = (u(x, y), v(x, y))$ from $\overline{B}_R$ onto $\overline{B}$ of class $C_{\text{loc}}^{1,\mu}(B_R)$ with $u(x_0, y_0) = 0$, $v(x_0, y_0) = 0$, which solves the Beltrami system

$$\sqrt{\Delta} u_x = -bv_x + av_y,$$

$$\sqrt{\Delta} u_y = -cv_x + bv_y.$$

The system for the inverse mapping $(x, y) = (x(u, v), y(u, v))$ reads as

$$(8) \quad \sqrt{\Delta} x_u = bx_v + cy_v,$$

$$(9) \quad \sqrt{\Delta} x_v = -bx_u - cy_u,$$

resp.,

$$(10) \quad \sqrt{\Delta} y_u = -ax_v - by_v,$$

$$(11) \quad \sqrt{\Delta} y_v = ax_u + by_u.$$

The equivalent conformality relations are

$$(12) \quad |Dx|^2 = \lambda c,$$

$$(13) \quad Dx \cdot Dy = -\lambda b,$$

$$(14) \quad |Dy|^2 = \lambda a,$$

where

$$\lambda = \frac{Dx \wedge Dy}{\sqrt{\Delta}}.$$

The relations (12, 13, 14) are identical to the statements (5, 6, 7).

In order to obtain the system (3, 4), one differentiates the systems (8, 9) and (10, 11):

$$\begin{aligned} L_K x &= \left(\frac{b}{\sqrt{D}} \right)_u x_v + \left(\frac{c}{\sqrt{D}} \right)_u y_v - \left(\frac{b}{\sqrt{D}} \right)_v x_u - \left(\frac{c}{\sqrt{D}} \right)_v y_u \\ &= \left(\left(\frac{c}{\sqrt{D}} \right)_x - \left(\frac{b}{\sqrt{D}} \right)_y \right) (x_u y_v - x_v y_u) \\ &= \frac{1}{\sqrt{D}} \left(A_x + B_y - (t + A) \frac{D_x}{2D} + (s - B) \frac{D_y}{2D} \right) Dx \wedge Dy, \end{aligned}$$

$$\begin{aligned} L_K y &= - \left(\frac{a}{\sqrt{D}} \right)_u x_v - \left(\frac{b}{\sqrt{D}} \right)_u y_v + \left(\frac{a}{\sqrt{D}} \right)_v x_u + \left(\frac{b}{\sqrt{D}} \right)_v y_u \\ &= \left(\left(\frac{a}{\sqrt{D}} \right)_y - \left(\frac{b}{\sqrt{D}} \right)_x \right) (x_u y_v - x_v y_u) \\ &= \frac{1}{\sqrt{D}} \left(B_x + C_y + (s - B) \frac{D_x}{2D} - (r + C) \frac{D_y}{2D} \right) Dx \wedge Dy. \end{aligned}$$

Here one first assumes that $z(x, y) \in C^3(\Omega)$ and then removes this restriction by approximation. This is possible because the Dirichlet integrals of the approximating maps are bounded in view of the conformality relations (5, 7). The system (3,4) and the system

$$\frac{\partial}{\partial x} \left(\frac{cu_x - bu_y}{\sqrt{\Delta}} \right) + \frac{\partial}{\partial y} \left(\frac{-bu_x + au_y}{\sqrt{\Delta}} \right) = 0,$$

$$\frac{\partial}{\partial x} \left(\frac{cv_x - bv_y}{\sqrt{\Delta}} \right) + \frac{\partial}{\partial y} \left(\frac{-bv_x + av_y}{\sqrt{\Delta}} \right) = 0$$

therefore each allow interior $C^{1,\mu}$ -estimates for their solutions.

Lemma 3. *Suppose that $A, B, C, D \in C^1(\Omega \times \mathbb{R}^3)$. The mapping $(x, y) = (x(u, v), y(u, v))$ from Lemma 2 then solves the system*

$$\begin{aligned} \frac{1}{\sqrt{K}} L_K x &= h_1 |Dx|^2 + h_2 Dx \cdot Dy + h_3 |Dy|^2 + h_4 Dx \wedge Dy, \\ \frac{1}{\sqrt{K}} L_K y &= \tilde{h}_1 |Dx|^2 + \tilde{h}_2 Dx \cdot Dy + \tilde{h}_3 |Dy|^2 + \tilde{h}_4 Dx \wedge Dy, \end{aligned}$$

where

$$\begin{aligned} h_1(x, y) &= B_q - \frac{1}{2D} (D_x + D_z p - D_p C + D_q B), \\ h_2(x, y) &= -A_q - B_p - \frac{1}{2D} (D_y + D_z q + D_p B - D_q A), \\ h_3(x, y) &= A_p, \\ h_4(x, y) &= \frac{1}{\sqrt{\Delta}} \left(A_x + A_z p + B_y + B_z q - A_p C + (A_q + B_p) B - B_q A - \frac{D_p K}{2} \right), \\ \tilde{h}_1(x, y) &= C_q, \\ \tilde{h}_2(x, y) &= -B_q - C_p - \frac{1}{2D} (D_x + D_z p - D_p C + D_q B), \\ \tilde{h}_3(x, y) &= B_p - \frac{1}{2D} (D_y + D_z q + D_p B - D_q A), \\ \tilde{h}_4(x, y) &= \frac{1}{\sqrt{\Delta}} \left(B_x + B_z p + C_y + C_z q - B_p C + (B_q + C_p) B - C_q A - \frac{D_q K}{2} \right). \end{aligned}$$

Proof. Compute

$$\begin{aligned} \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} - \frac{t + A}{2D} \frac{\partial D}{\partial x} + \frac{s - B}{2D} \frac{\partial D}{\partial y} &= A_x + A_z p + B_y + B_z q - A_p C + (A_q + B_p) B - B_q A \\ &\quad + A_p(r + C) + (A_q + B_p)(s - B) + B_q(t + A) \\ &\quad - \frac{t + A}{2D} (D_x + D_z p - D_p C + D_q B + D_p(r + C) + D_q(s - B)) \\ &\quad + \frac{s - B}{2D} (D_y + D_z q + D_p B - D_q A + D_p(s - B) + D_q(t + A)) \\ &= A_x + A_z p + B_y + B_z q - A_p C + (A_q + B_p) B - B_q A \\ &\quad + (t + A) \left(B_q - \frac{1}{2D} (D_x + D_z p - D_p C + D_q B) \right) \\ &\quad + (s - B) \left(A_q + B_p + \frac{1}{2D} (D_y + D_z q + D_p B - D_q A) \right) \\ &\quad + (r + C) A_p - \frac{D_p}{2D} ((t + A)(r + C) - (s - B)^2) \\ &= -\frac{\sqrt{\Delta}}{J(x, y)} (h_1 |Dx|^2 + h_2 Dx \cdot Dy + h_3 |Dy|^2 + h_4 Dx \wedge Dy). \end{aligned}$$

$$\begin{aligned}
& \frac{\partial B}{\partial x} + \frac{\partial C}{\partial y} + \frac{s-B}{2D} \frac{\partial D}{\partial x} - \frac{r+C}{2D} \frac{\partial D}{\partial y} \\
&= B_x + B_z p + C_y + C_z q - B_p C + (B_q + C_p)B - C_q A \\
&\quad + B_p(r+C) + (B_q + C_p)(s-B) + C_q(t+A) \\
&\quad + \frac{s-B}{2D} (D_x + D_z p - D_p C + D_q B + D_p(r+C) + D_q(s-B)) \\
&\quad - \frac{r+C}{2D} (D_y + D_z q + D_p B - D_q A + D_p(s-B) + D_q(t+A)) \\
&= B_x + B_z p + C_y + C_z q - B_p C + (B_q + C_p)B - C_q A \\
&\quad + (t+A)C_q \\
&\quad + (s-B) \left(B_q + C_p + \frac{1}{2D} (D_x + D_z p - D_p C + D_q B) \right) \\
&\quad + (r+C) \left(B_p - \frac{1}{2D} (D_y + D_z q + D_p B - D_q A) \right) \\
&\quad - \frac{D_q}{2D} ((r+C)(t+A) - (s-B)^2) \\
&= -\frac{\sqrt{\Delta}}{J(x, y)} (\tilde{h}_1 |Dx|^2 + \tilde{h}_2 Dx \cdot Dy + \tilde{h}_3 |Dy|^2 + \tilde{h}_4 Dx \wedge Dy).
\end{aligned}$$

3. UNIVALENT SOLUTIONS OF HEINZ-LEWY TYPE SYSTEMS

Consider a solution $(x, y) = (x(u, v), y(u, v))$ of class $W^{1,2}(B)$, $B = \{u^2 + v^2 < 1\}$, of the Heinz-Lewy type system:

(15)

$$Lx = h_1(x, y)|Dx|^2 + h_2(x, y)Dx \cdot Dy + h_3(x, y)|Dy|^2 + h_4(x, y)Dx \wedge Dy,$$

(16)

$$Ly = \tilde{h}_1(x, y)|Dx|^2 + \tilde{h}_2(x, y)Dx \cdot Dy + \tilde{h}_3(x, y)|Dy|^2 + \tilde{h}_4(x, y)Dx \wedge Dy,$$

where

$$L = \frac{1}{a(u, v, x, y)} \left(\frac{\partial}{\partial u} \left(a(u, v, x, y) \frac{\partial}{\partial u} \right) + \frac{\partial}{\partial v} \left(a(u, v, x, y) \frac{\partial}{\partial v} \right) \right).$$

The following assumptions are imposed on the coefficients $a, h_1, \dots, \tilde{h}_4$:

Assumption (D). The function $a = a(u, v, x, y)$ belongs to the Hölder class $C^\mu(B \times \mathbb{R}^2)$ for some μ , $0 < \mu < 1$, such that

$$0 < \lambda \leq a(u, v, x, y) \leq \Lambda < +\infty$$

for all $(u, v) \in B$, $(x, y) \in \mathbb{R}^2$, and

$$[a]_\mu^{B \times \mathbb{R}^2} \leq L.$$

Assumption (E). The coefficients $h_1, \dots, \tilde{h}_4$ are bounded, Borel-measurable functions on $\mathbb{R}^2$ such that

$$|h_1(x, y)|, \dots, |\tilde{h}_4(x, y)| \leq M_0$$

for all $(x, y) \in \mathbb{R}^2$, and the functions

$$\begin{aligned}\omega_1(x, y) &= \tilde{h}_1(x, y), \\ \omega_2(x, y) &= h_1(x, y) - \tilde{h}_2(x, y), \\ \omega_3(x, y) &= h_2(x, y) - \tilde{h}_3(x, y), \\ \omega_4(x, y) &= h_3(x, y)\end{aligned}$$

$((x, y) \in \mathbb{R}^2)$ are Lipschitz continuous with Lipschitz constant $\leq M_1$:

$$[\omega_1]_{0,1}^{\mathbb{R}^2}, \dots, [\omega_4]_{0,1}^{\mathbb{R}^2} \leq M_1.$$

Assumption (F). Suppose that the Dirichlet integral of the solution mapping (x, y) satisfies the estimate

$$\int_B (|Dx|^2 + |Dy|^2) du dv \leq N.$$

Proposition 4. Let $(x, y) = (x(u, v), y(u, v))$ be a homeomorphism from the closed unit disc $\bar{B}$ onto $\bar{B}_R(x_0, y_0) = \{(x - x_0)^2 + (y - y_0)^2 \leq R^2\}$ of class $W^{1,2}$ which solves the system (15, 16). Suppose that the Assumptions (D, E, F) are satisfied, and assume that $x(0, 0) = x_0$, $y(0, 0) = y_0$.

Then $(x, y) \in C_{\text{loc}}^{1,\mu}(B)$, and the Jacobian $Dx \wedge Dy$ does not vanish in B . The following estimates hold for all $B_\rho = \{u^2 + v^2 < \rho^2\}$, $0 < \rho < 1$:

$$(17) \quad \|x, y\|_{C^{1,\mu}(B_\rho)} \leq C,$$

$$(18) \quad Dx \wedge Dy \geq c > 0,$$

where the constants C, c depend only on $\mu, \lambda, \Lambda, L, M_0, M_1, N, \rho$, and R .

Proposition 4 was proved in [8] under the assumption that all coefficients $h_1, \dots, \tilde{h}_4$ are Lipschitz continuous. It was noted, however, at the end of [8] that the proof, which is based on [2], requires only the Lipschitz continuity of the combinations $\omega_1, \dots, \omega_4$.

4. PROOF OF THEOREM 1

The regularity $z(x, y) \in C^{2,\mu}(\Omega)$ follows from [9]. The mapping (x, y) from Lemma 2 therefore satisfies, by Lemma 3, a system of the form (15, 16).

In order to apply Proposition 4, estimate

$$\begin{aligned} \int_B (|Dx|^2 + |Dy|^2) du dv &= \int_{B_R(x_0, y_0)} \frac{t + A + r + C}{\sqrt{\Delta}} dx dy \\ &\leq CR^2 + \int_{\partial B_R} Dz \cdot \nu d\sigma \\ &\leq C(R^2 + R), \end{aligned}$$

which verifies Assumption (F). Assumption (E) follows from Assumption (C) via

$$\begin{aligned} \omega_1(x, y) &= \tilde{h}_1(x, y) = C_q = \phi_4, \\ \omega_2(x, y) &= h_1(x, y) - \tilde{h}_2(x, y) = 2B_q + C_p = \phi_3, \\ \omega_3(x, y) &= h_2(x, y) - \tilde{h}_3(x, y) = -A_q - 2B_p = -\phi_2, \\ \omega_4(x, y) &= h_3(x, y) = A_p = \phi_1. \end{aligned}$$

The estimates (17, 18) therefore hold in any disc B_ρ , $0 < \rho < 1$. In particular, by taking $\rho = 1/2$,

$$\begin{aligned} |Dx(0, 0)|, |Dy(0, 0)| &\leq C, \\ (Dx \wedge Dy)(0, 0) &\geq c. \end{aligned}$$

The conformality relations (5, 6, 7) translate this into the estimate

$$|D^2 z(x_0, y_0)| \leq C.$$

The Hölder estimate for the second derivatives follows from [7, 9] as required.

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