SPECTRAL RADIUS FORMULAE IN QUOTIENT C^* -ALGEBRAS

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ABSTRACT. If I is a closed two-sided ideal in C^* -algebra A , we prove spectral radius formulae for the coset $x + I$ ($x \in A$) in the quotient algebra A/I . Then, as a corollary we get the main result of Mau-Hsiang Shin (Proc. Amer. Math. Soc. **100** (1987), 137–139).

Let A be a unital C^* -algebra and $\text{Inv}(A)$ the set of invertible elements of A . If I is a closed two-sided ideal in A , let $x + I$ denote the coset in the quotient algebra A/I containing x . For $x \in A$ denote by $r(x)$ ($r(x + I)$) the spectral radius of the element x ($x + I$). G. J. Murphy and T. T. West [1, Proposition 4, Corollary] have proved that

$$(1) \quad r(x) = \inf_{a=a^* \in A} \|e^a x e^{-a}\| = \inf_{y \in \text{Inv}(A)} \|y x y^{-1}\| \quad (x \in A)$$

and

$$(2) \quad r(x + I) = \inf_{y \in I} r(x + y) \quad (x \in A).$$

Now we prove:

Theorem 1. *Let A be a unital C^* -algebra and I a closed two-sided ideal in A . Then*

$$r(x + I) = \inf_{s \in \text{Inv}(A)} \|s^{-1} x s + I\| \quad (x \in A).$$

Proof. From the proof of (2) we have

$$r(x + I) = \inf_{a \in A} \|e^a x e^{-a} + I\|.$$

Further, by (1) and (2) we have

$$\begin{aligned} r(x + I) &\geq \inf_{s \in \text{Inv}(A)} \|s^{-1} x s + I\| \\ &= \inf_{s \in \text{Inv}(A)} \inf_{y \in I} \|s^{-1} (x + y) s\| \\ &= \inf_{y \in I} \inf_{s \in \text{Inv}(A)} \|s^{-1} (x + y) s\| \\ &= \inf_{y \in I} r(x + y) = r(x + I). \end{aligned}$$

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(Note that $s^{-1}xs + y = s^{-1}(x + sys^{-1})s$, and $y \in I$ if and only if $sys^{-1} \in I$.) This completes the proof.

Now as a corollary, we get the main result of Mau-Hsiang Shin [3, Theorem 1].

Let H be an infinite-dimensional complex Hilbert space and denote the set of bounded (compact) linear operators on H by $B(H)$ ($K(H)$). Let $r_e(T) = r(T + K(H))$ denote the essential spectral radius of an element T of $B(H)$. If Q is a bounded subset of H , we define $\alpha(Q)$ to be $\inf\{d > 0: \text{there exists a finite number of sets } S_1, \dots, S_n \text{ such that } \text{diameter}(S_i) \leq d \text{ and } Q \subset \bigcup_{i=1}^n S_i\}$. The function α is called Kuratowski's measure of noncompactness. For $T \in B(H)$ let $\alpha(T) = \inf\{k \geq 0: \alpha(T(Q)) \leq k\alpha(Q), \text{ for every bounded set } Q \subset H\}$.

Corollary [3, Theorem 1]. *Let $T \in B(H)$. Then*

$$r_e(T) = \inf_{S \in \text{Inv}(B(H))} \alpha(S^{-1}TS).$$

Proof. By [2, Lemma 1] and [4, Lemma], we have

$$\alpha(T) = \|T + K(H)\| \quad (T \in B(H)).$$

Hence, the result follows by Theorem 1.

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