NORMAL SPACES WITH PRESCRIBED STONE-ČECH REMAINDERS

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ABSTRACT. It is shown that if X is a locally compact Hausdorff space, then there is a normal space Y such that $\beta Y \setminus Y \cong X$. Examples are given of a countable nonsequential space, and a sequential nonlocally compact space, each of which is the Stone-Čech remainder of a normal space. A method of constructing normal almost compact spaces is presented.

1. INTRODUCTION

It is a well-known consequence of Glicksberg's theorem (see, for example, [PW, 4AG(7)]) concerning the Stone-Čech compactification of a pseudocompact product that given any Tychonoff space X , there exists a Tychonoff space Y such that $\beta Y \setminus Y = X$ (equations such as this will be used to abbreviate the more precise statement " $\beta Y \setminus Y$ is homeomorphic to X "). One can select the space Y to be the subspace $P \setminus (\{\kappa\} \times X)$ of the product $P = [0, \kappa] \times \beta X$, where κ is an ordinal of uncountable cofinality; then $P = \beta Y$. (Here $[0, \kappa)$ (resp. $[0, \kappa]$) denotes the set of all ordinals less than (resp. less than or equal to) the ordinal κ , equipped with the order topology.) It is clear that if X is not locally compact then Y will not be normal, for if $x \in X \cap \text{cl}_{\beta X}(\beta X \setminus X)$, then the point (κ, x) is in the P -closure of the disjoint closed sets $[0, \kappa) \times \{x\}$ and $\{\kappa\} \times (\beta X \setminus X)$ of Y . Hence if we are given a nonlocally compact space X and seek a normal space Y such that $X = \beta Y \setminus Y$, we will need a new technique.

Given the above result, it is natural to ask whether any Tychonoff space X is homeomorphic to $\beta T \setminus T$ for some normal space T . Perhaps surprisingly, the answer is "no." In [FL] Fleissner and Levy study normal spaces all of whose regular continuous images are normal, and in the course of their investigations obtain constraints on those spaces X that can be $\beta Y \setminus Y$ for some normal space Y . Specifically, they prove that if Y is normal and $\beta Y \setminus Y$ is first countable, then $\beta Y \setminus Y$ is locally compact (see [FL, Proposition 4]). Thus, in contrast to

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the situation described in the preceding paragraph, not every (Tychonoff) space X is homeomorphic to $\beta Y \setminus Y$ for some normal space Y . (The rationals will be such an X , for example.) This raises the question of characterizing those Tychonoff spaces that are homeomorphic to $\beta Y \setminus Y$ for some normal space Y . In particular, in Question 1 of [FL], Fleissner and Levy ask whether, if Y is normal, $\beta Y \setminus Y$ can be countable but not sequential, or sequential but not locally compact.

In this paper we show that if X is locally compact, then there is a normal space Y such that $X = \beta Y \setminus Y$. We also show that the answer to each of the questions of Fleissner and Levy quoted above is “yes,” although the assumption of the existence of a scale is necessary for our argument in the second case. In the process we develop a new general technique for constructing examples of normal spaces with interesting Stone-Čech compactifications. We conclude the paper with a result concerning normal almost compact spaces that is unrelated to the previous results.

All hypothesized topological spaces in this paper are assumed to be Tychonoff; thus “space” means “completely regular Hausdorff topological space.” The reader is referred to [PW] and [GJ] for unexplained notation and terminology. We will make use of the following result, which is Theorem 2.1 of [M].

Lemma 1. *Suppose that X is locally compact and K is Hausdorff. Then there exists a compactification αX of X such that $\alpha X \setminus X = K$ if and only if K is a continuous image of $\beta X \setminus X$.*

We will also use the following slight generalization of one direction of a theorem of Nogura [N], which essentially appears (with proof) as 3.19 of [P]. Recall that the *tightness* $t(X)$ of a space X is the smallest cardinal number κ such that for every point $x \in X$ and $S \subseteq X$, if $x \in \text{cl}(S)$ then $x \in \text{cl}(T)$ for some $T \subseteq S$ with $|T| \leq \kappa$ (see [H]). As usual, $\text{cf}(\kappa)$ is the cofinality of the cardinal κ .

Theorem 1. *If κ is a cardinal number and C is a compact space for which $t(C) < \text{cf}(\kappa)$, then the product space $[0, \kappa) \times C$ is normal.*

We can now present our first result.

Theorem 2. *Let X be a locally compact space. Then there exists a normal space Y such that $\beta Y \setminus Y = X$.*

Proof. Assume without loss of generality that X is infinite. Let $\kappa = |\beta X|^+$, $P = [0, \kappa] \times \beta X$, and $Y = P \setminus (\{\kappa\} \times X)$. Thus $[0, \kappa) \times \beta X \subseteq Y \subseteq P = \beta([0, \kappa) \times \beta X)$; the latter equality essentially follows from Glicksberg’s theorem (see [PW, 4AG(8)], for example). Thus $\beta Y = P$ (see [PW, 4.5(p)(3)]) and so $\beta Y \setminus Y = \{\kappa\} \times X \cong X$. Clearly, $t(\beta X) < \text{cf}(\kappa)$, so by Theorem 1, $H = [0, \kappa) \times \beta X$ is normal. But $Y = H \cup ([0, \kappa] \times (\beta X \setminus X))$ and $\beta X \setminus X$ is compact as X is locally compact, so by [GJ, 3D(5)], Y is normal. $\square$

2. THE BASIC CONSTRUCTION

Let S be a locally compact noncompact space whose pseudocompact closed subspaces are compact. (This last condition will be fulfilled if S is realcompact

(see [GJ, 8.10 and 8A(5)]), or metacompact (see [PW, 6N]).) Suppose that S has a compactification αS for which $\alpha S \setminus S = [0, \lambda]$, where λ is a cardinal such that $\text{cf}(\lambda) > t(S)$. In Theorem 4 below we will show that there exists a normal space Y for which $\beta Y \setminus Y = S \cup \{\lambda\}$. We will use this construction in §3 to answer the two questions of Fleissner and Levy posed in §1.

A central part of our construction of Y will be a generalization of a construction due to Vermeer [V]. The special case of Theorem 3 below in which $\lambda = \omega_1$ appears in 3.4 of [V]; the proof of Theorem 3 is virtually identical to Vermeer's proof of 3.4, and hence is not included. (See also [PW, 6W].)

Theorem 3. *Let λ be a cardinal of uncountable cofinality and let $P = [0, \lambda] \times [0, \lambda]$. If $\alpha < \lambda$, let $E_\alpha = \{(\lambda, \alpha), (\alpha, \alpha), (\alpha, \lambda)\}$ and $E = \bigcup \{E_\alpha : \alpha < \lambda\}$. Let $T = \{E_\alpha : \alpha < \lambda\} \cup \{p : p \in P \setminus E\}$ and let $f : P \rightarrow T$ map each point of P to the unique member of T to which it belongs. Give T the quotient topology induced by f . Then T is a compact Hausdorff space, $T \setminus \{(\lambda, \lambda)\} = Z$ is normal, and $\beta Z = T$.*

Theorem 4. *Let S be a locally compact noncompact space whose pseudocompact closed subsets are compact. Let S have a compactification αS for which $\alpha S \setminus S = [0, \lambda]$, where λ is a cardinal for which $\text{cf}(\lambda) > t(S)$. Then there exist a normal space Y for which $\beta Y \setminus Y = S \cup \{\lambda\}$ (viewed as a subspace of αS).*

Proof. Let $K = [0, \lambda] \times \alpha S$, and let T and Z be as in Theorem 3. Then K is a compactification of the locally compact space $L = [0, \lambda] \times S$, and $K \setminus L = [0, \lambda] \times [0, \lambda] = P$. Define a map $F : K \rightarrow L \cup T = H$ as follows: $F(x) = x$ if $x \in L$, and $F|P = f$ (as described in Theorem 3). Give H the quotient topology induced by F . By Lemma 1, H is a Hausdorff compactification of L and F is a continuous surjection; note that the subspace topologies inherited by L and T from H are the same as the original topologies on L and T . Observe that $F^{-1}(F(x)) = \{x\}$ if $x \in \{\lambda\} \times (S \cup \{\lambda\})$, so the subspace $(\{\lambda\} \times S) \cup \{(\lambda, \lambda)\}$, henceforth denoted as E , of H is homeomorphic to the subspace $S \cup \{\lambda\}$ of αS .

Denote the point (λ, λ) of H by a . Let $Y = H \setminus E$; thus $Y = ([0, \lambda] \times S) \cup Z$. Clearly Y is dense in H , as $F[[0, \lambda] \times \alpha S] \subseteq Y \subseteq H$. We will show that $H = \beta Y$ by showing that Y is C^* -embedded in H . We do this in two steps.

First, we show that $H \setminus \{a\}$ is C^* -embedded in H . To see this, observe that by [PW, 4AG(8)], $\beta([0, \lambda] \times \alpha S) = K$, and it quickly follows that $K = \beta(K \setminus \{(\lambda, \lambda)\})$. Now $F^{-1}[F((\lambda, \lambda))] = \{(\lambda, \lambda)\}$, so $F|K \setminus \{(\lambda, \lambda)\}$ is a perfect continuous surjection from the almost compact space $K \setminus \{(\lambda, \lambda)\}$ onto $H \setminus \{a\}$. It follows that $H \setminus \{a\}$ is almost compact and noncompact, and hence $H \setminus \{a\}$ is C^* -embedded in H . (See [GJ, 6J] for relevant information about almost compact spaces.)

Next we claim that Y is C^* -embedded in $H \setminus \{a\}$. By [GJ, 6H] it suffices to show that Y is C^* -embedded in $Y \cup \{(\lambda, s)\}$ for each $s \in S$. Let $g \in C^*(Y)$ and let M be a compact S -neighborhood of s . Then $[0, \lambda] \times M$ is C^* -embedded in $[0, \lambda] \times M$ (by Glicksberg's theorem yet again), so $g|[0, \lambda] \times M$ extends continuously to $g^* \in C^*([0, \lambda] \times M)$. Define $g^* : Y \cup ([0, \lambda] \times \text{int}_S M) \rightarrow \mathbb{R}$ by $g^*|Y = g$ and $g^*|[0, \lambda] \times \text{int}_S M = g^*|[0, \lambda] \times \text{int}_S M$. Then by [GJ, 1A(a)], g^* is continuous, so $g^*|Y \cup \{(\lambda, s)\}$ is the required continuous extension of g .

As Y is C^* -embedded in $H \setminus \{a\}$ and $H \setminus \{a\}$ is C^* -embedded in H , it

follows that $\beta Y = H$. Thus $\beta Y \setminus Y = E \cong S \cup \{\lambda\}$. It remains to show that Y is normal.

Observe that $K \setminus ((\lambda) \times (S \cup \{\lambda\})) = [P \setminus \{(\lambda, \lambda)\}] \cup [[0, \lambda) \times {}_\alpha S]$; hence it is the union of two countably compact subspaces and thus is itself countably compact. Clearly, $F[K \setminus ((\lambda) \times (S \cup \{\lambda\}))] = Y$, so Y is also countably compact.

Now we show that Y is normal. Let A and B be disjoint closed subsets of Y , and recall that $Y = ([0, \lambda) \times S) \cup Z$. As Z is normal and almost compact by Theorem 3, either $A \cap Z$ or $B \cap Z$ is compact. Suppose $A \cap Z$ is. Then there are disjoint open sets U and V of Y such that $A \cap Z \subseteq U$ and $B \subseteq V \subseteq X \setminus A$. Now A is countably compact as Y is, so $A \setminus U$ is a countably compact closed subset of $[0, \lambda) \times S$. If $\Pi_S: [0, \lambda) \times S \rightarrow S$ is the projection map, then $\Pi_S[A \setminus U]$ is countably compact and hence pseudocompact. Hence, $\text{cl}_S \Pi_S[A \setminus U]$ is pseudocompact closed subset of S , and hence by hypothesis is compact. Using the local compactness of S , we can find a compact subset F of S such that $\text{cl}_S \Pi_S[A \setminus U] \subseteq \text{int}_S F \subseteq F$. Thus $A \setminus U \subseteq [0, \lambda) \times \text{int}_S F \subseteq [0, \lambda) \times F$, and as $t(F) \leq t(S) < \text{cf}(\lambda)$, by Theorem 1, $[0, \lambda) \times F$ is normal. It follows that there are disjoint open sets R and Q in $[0, \lambda) \times F$ such that $A \setminus U \subseteq R$ and $B \cap [[0, \lambda) \times F] \subseteq Q$. Thus $A \subseteq U \cup [R \cap [[0, \lambda) \times \text{int}_S F]] = C$ and $B \subseteq V \cap [Q' \cup (Y \setminus ([0, \lambda) \times F))] = D$, where Q' is open in Y and $Q' \cap ([0, \lambda) \times F) = Q$. Then C and D are open in Y and are easily verified to be disjoint. Consequently Y is normal and the proof is complete. $\square$

3. APPLICATIONS

We began with a straightforward application of Theorem 4.

Example 1. Let $S = \{\alpha < \omega_1: \alpha \text{ is a successor ordinal}\}$, and let ${}_\alpha S = [0, \omega_1]$. Then ${}_a S \setminus S \cong [0, \omega_1]$ and $\text{cf}(\omega_1) = \omega_1 > t(S)$. As S is discrete, its pseudocompact closed subsets are compact. Hence, the hypotheses of Theorem 4 are satisfied, and so there exists a normal space Y such that $\beta Y \setminus Y \cong S \cup \{\omega_1\}$. From our construction of ${}_\alpha S$ it is clear that $S \cup \{\omega_1\}$ is the “one-point Lindelöf extension” of the discrete space of cardinality ω_1 ; hence this space is the Stone-Čech remainder of a normal space.

Example 2. Let $\mathbb{N}$ denote the discrete space of positive integers. As in [vD], $\mathbf{t}$ is defined to be the least cardinal κ for which there is a family $\{B_\alpha: \alpha < \kappa\} \subseteq [\mathbb{N}]^\omega$ such that $\{B_\alpha^*: \alpha < \kappa\}$ is a strictly decreasing chain in $\mathbb{N}^*$ whose intersection is nowhere dense in $\mathbb{N}^*$. As noted in [vD], $\aleph_0 < \mathbf{t} \leq 2^{\aleph_0}$ and $\mathbf{t}$ is a regular cardinal. Let $D = \bigcap \{B_\alpha^*: \alpha < \kappa\}$, $B_t = \emptyset$, $B_{-1} = \mathbb{N}$, and ${}_\alpha \mathbb{N} = \mathbb{N} \cup [0, \mathbf{t}]$. A subset $U \subseteq {}_\alpha \mathbb{N}$ is defined to be open in ${}_\alpha \mathbb{N}$ if $\alpha \in [0, \mathbf{t}] \cap U$ implies there is some $\beta < \alpha$ and a finite subset $F \subset \mathbb{N}$ such that $(\beta, \alpha] \cup B_\beta \setminus (B_\alpha \cup F) \subseteq U$.

It is straightforward to verify (see [FR, Examples 1.1 and 1.5]) that ${}_\alpha \mathbb{N}$ is a compactification of $\mathbb{N}$ and ${}_\alpha \mathbb{N} \setminus \mathbb{N} = [0, \mathbf{t}]$. By Theorem 4, there is a normal space M such that $\beta M \setminus M = \mathbb{N} \cup \{\mathbf{t}\}$. The countable space $\mathbb{N} \cup \{\mathbf{t}\}$ will provide an answer to the first question of Fleissner and Levy (quoted in §1 if $\mathbb{N} \cup \{\mathbf{t}\}$ is not sequential. To show $\mathbb{N} \cup \{\mathbf{t}\}$ is not sequential, it suffices to show that $\mathbb{N}$ is sequentially closed in $\mathbb{N} \cup \{\mathbf{t}\}$, i.e., no infinite subset of $\mathbb{N}$ converges to $\mathbf{t}$. Suppose A is an infinite subset of $\mathbb{N}$. Since D is nowhere dense, there is an infinite subset B of A such that $B^* \subseteq A^* \setminus D$. By the compactness of $\mathbb{N}^*$, it follows there is some $\alpha < \mathbf{t}$ such that $B^* \cap B_\alpha^* = \emptyset$. So $B \cap B_\alpha$ is a finite subset

of $\mathbb{N}$ and $(\alpha, t] \cup B_\alpha \setminus (B \cap B_\alpha)$ is an open set missing B . Hence, A does not converge to t .

Example 3. Let $S = (\mathbb{N} \times \mathbb{N}) \cup \{\omega(n) : n \in \mathbb{N}\}$; $U \subseteq S$ is defined to be open if $\omega(n) \in U$ implies there is some finite subset $F \subset \mathbb{N}$ such that $\{n\} \times (\mathbb{N} \setminus F) \subseteq U$. Thus, $S_n = (\{n\} \times \mathbb{N}) \cup \{\omega(n)\}$ is the one-point compactification of $\{n\} \times \mathbb{N}$, and S is the sum of the S_n 's. For a subset $F \subseteq \mathbb{N}$, let $S_F = \bigcup \{S_n : n \in F\}$. For $f_\alpha \in {}^\mathbb{N}\mathbb{N}$, let $T_\alpha = \bigcup \{\{n\} \times [1, f_\alpha(n)] : n \in \mathbb{N}\}$ and $T_d = S$.

Let us assume the existence of a scale, i.e., a cardinal $\mathfrak{d}$ and a cofinal family $\mathcal{F} = \{f_\alpha : \alpha < \mathfrak{d}\}$ of $({}^\mathbb{N}\mathbb{N}, \leq^*)$ such that $\mathcal{F}$ is well ordered by $\leq^*$. (Recall $f \leq^* g$ if $\{n \in \mathbb{N} : f(n) > g(n)\}$ is finite.) It is known that Martin's Axiom implies the existence of a scale. As noted in [vD], $\mathfrak{d}$ is a regular cardinal. Without loss of generality, we can assume that $f_0(n) > 1$ for all $n \in \mathbb{N}$ and for each $\alpha \leq \mathfrak{d}$, $T_\alpha \setminus (\bigcup_{\beta < \alpha} T_\beta \cup S_F)$ is infinite for all finite $F \subset \mathbb{N}$. Let $U(\alpha, \beta, F) = (T_\alpha \setminus (T_\beta \cup S_F)) \cup (\beta, \alpha]$, where $\beta < \alpha \leq \mathfrak{d}$ and $F \subseteq \mathbb{N}$.

Let $\alpha S = S \cup [0, \mathfrak{d}]$. A subset U of αS is defined to be open if (1) $U \cap S$ is open in S ; (2) $0 \in U$ implies for some finite subset $F \subset \mathbb{N}$, $T_0 \setminus S_F \subseteq U$; and (3) $\alpha \in (0, \mathfrak{d}] \cap U$ implies for some $\beta < \alpha$ and finite subset $F \subset \mathbb{N}$, $U(\alpha, \beta, F) \subseteq U$. It is easy to verify that αS is a Hausdorff extension of S . To show αS is compact, let $\mathcal{U}$ be an open cover of αS . Since $[0, \mathfrak{d}]$ is compact, there are $0 = \alpha_1 < \alpha_2 < \dots < \alpha_n = \mathfrak{d}$ and finite sets $F_1, \dots, F_n$ such that $U(\alpha_{i+1}, \alpha_i, F_i) \subseteq \text{some } U_i \in \mathcal{U}$. Let $F = F_1 \cup \dots \cup F_n$. Then $\alpha S \setminus S_F \subseteq U_1 \cup \dots \cup U_n$. Since S_F is compact, it follows that αS is compact. Thus, αS is a compactification of S such that $\alpha S \setminus S = [0, \mathfrak{d}]$. By Theorem 4, there is a normal space M such that $\beta M \setminus M = S \cup \{\mathfrak{d}\}$. Note that U is an open neighborhood of $\mathfrak{d}$ iff there is a finite subset $F \subset \mathbb{N}$ such that $\{\omega(n) : n \in \mathbb{N} \setminus F\} \subseteq U$ and $U \cap S_n$ is open in S_n for each n . In particular, $S \cup \{\mathfrak{d}\}$ is homeomorphic to the Arens Space (described in [F]) which is sequential but not locally compact. This provides an answer (assuming the existence of a scale) to the second question posed by Fleissner and Levy.

4. A RESULT ON NORMAL ALMOST COMPACT SPACES

Throughout this section X will denote a locally compact realcompact non-compact space, αX will be compactification of X , and $f : \beta X \rightarrow \alpha X$ will extend the identity map on X continuously.

Theorem 5. Assume that there exists $p \in \alpha X \setminus X$ such that:

- (1) $\beta((\alpha X \setminus X) \setminus \{p\}) = \alpha X \setminus X$.
- (2) $f^-(p)$ is nowhere dense in $\beta X \setminus X$.

Then

- (a) $\beta(\alpha X \setminus \{p\}) = \alpha X$.
- (b) The following are equivalent:
 - (i) $\alpha X \setminus \{p\}$ is normal.
 - (ii) $(\alpha X \setminus X) \setminus \{p\}$ is normal and $\alpha X \setminus \{p\}$ is countably compact.

Proof. (a) Evidently $\beta X \setminus f^-(p)$ is the union of the locally compact realcompact space X and the dense subspace $\beta X \setminus f^-(p)$ of $\beta X \setminus X$. Hence, by [PW, 5.11 (i)(2)] $\beta X \setminus f^-(p)$ is pseudocompact. Hence $f[\beta X \setminus f^-(p)] = \alpha X \setminus \{p\}$, henceforth denoted by L , is pseudocompact.

Let $Z, S \in \mathbf{Z}(L)$ with $Z \cap S = \emptyset$. Then there exists $g \in C^*(L)$ with $0 \leq g \leq 1$ such that $g[Z] = \{0\}$ and $g[S] = \{1\}$. Let $A = \text{cl}_L g^{-1}[[0, \frac{1}{3}]]$ and $B = \text{cl}_L g^{-1}[(\frac{2}{3}, 1]]$. Then $A \supseteq Z$ and $B \supseteq S$, so if we can show that one of A and B must be compact, then one of Z and S will be compact. As Z and S were arbitrarily chosen disjoint zero sets of L , this would prove (a) (see [GJ, 6J]). Now $A \setminus X$ and $B \setminus X$ are completely separated closed subsets of the space $L \setminus X$, which by (1) is almost compact, so at least one of them—say $A \setminus X$ —is compact. We will prove that A is compact.

There is an open set G in L containing $A \setminus X$ such that $\text{cl}_L G$ is compact. Then $H = \text{cl}_L(A \setminus \text{cl}_L G)$ is a regular closed subset of the pseudocompact space L and hence is pseudocompact (see [PW, 1Q(2),(4)]). But H is contained in A and disjoint from $A \setminus X$; so H is a (closed) subset of the realcompact space X and hence is realcompact. This shows that H is compact. As $A \subseteq H \cup \text{cl}_L G$, A is compact and (a) follows.

(b) (i) $\Rightarrow$ (ii): If $\alpha X \setminus \{p\}$ is normal, so is its closed subspace $(\alpha X \setminus X) \setminus \{p\}$. By (a), $\alpha X \setminus \{p\}$ is pseudocompact and hence (as it is normal) it is countably compact.

(ii) $\Rightarrow$ (i): Let (ii) hold and let A and B be disjoint closed subsets of $\alpha X \setminus \{p\}$ (which, recall, we denote by L). We will show that at least one is compact; hence L will be normal.

As $L \setminus X$ is normal and almost compact, at least one of $A \setminus X$ and $B \setminus X$ is compact; suppose $A \setminus X$ is. We show that A is compact. There is an open set G in L containing $A \setminus X$ such that $\text{cl}_L G$ is compact. The closed subset $A \setminus G$ of L is countably compact. Also, $A \setminus G$ is a closed subset of X (which is realcompact); so $A \setminus G$ is compact. As $A \subseteq (A \setminus G) \cup \text{cl}_L G$, A is compact. $\square$

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