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KANTOROVICH-RUBINSTEIN NORM AND ITS APPLICATION IN THE THEORY OF LIPSCHITZ SPACES

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ABSTRACT. We obtain necessary and sufficient conditions on a compact metric space (K, ρ) that provide a natural isometric isomorphism between completion of the space of Borel measures on K with the Kantorovich-Rubinstein norm and the space $(\mathrm{lip}(K, \rho))^*$ or equivalently between the spaces $\mathrm{Lip}(K, \rho)$ and $(\mathrm{lip}(K, \rho))^{**}$. Such metric spaces are studied and related properties of Lipschitz spaces are established.

1. NOTATION

Let (K, ρ) be a metric space and $M(K)$ be the set of all finite Borel measures on K. For a measure $\mu \in M(K)$, denote by μ_+, μ_- its positive and negative variations, respectively, and set $|\mu| = \mu_+ + \mu_-$, $\operatorname{Var} \mu = |\mu|(K)$.

The Lipschitz space $\mathrm{Lip}(K, \rho)$ is defined as the set of all functions f on K with the finite norm

$$\|f\|_{K, \rho} = \max\{\|f\|_K, |f|_{K, \rho}\},$$

where

$$\|f\|_K = \sup\{|f(x)|: x \in K\}$$

and

$$|f|_{K, \rho} = \sup\left\{\frac{|f(x) - f(y)|}{\rho(x, y)}: x, y \in K, x \neq y\right\}.$$

Functions $f \in \mathrm{Lip}(K, \rho)$ with the property

$$\lim_{\rho(x, y) \to 0} \frac{f(x) - f(y)}{\rho(x, y)} = 0$$

constitute the closed subspace $\mathrm{lip}(K, \rho)$ in $\mathrm{Lip}(K, \rho)$. The notation B and

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b will be used for closed unit balls in $\mathrm{Lip}(K, \rho)$ and $\mathrm{lip}(K, \rho)$, respectively. All spaces of measures and functions below are supposed to be real.

2. Introduction and background

Let (K, ρ) be a compact metric space. The total variation norm Var on $M(K)$ suffers from at least two grave shortcomings:

(i) it is very weakly connected with the metric since it is invariant for all metrics generating the same topology;
(ii) there is no transparent description of the conjugate space $(M(K))^*$.

This suggests the question: can another norm on $M(K)$ that is free from these defects be defined? Such norm has been actually invented in 1942 by Kantorovich [4] and has been studied in his works with Rubinstein [6, 7]. This norm is called the Kantorovich-Rubinstein (KR) norm.

For distinct points $x, y \in K$, the KR norm of the dipole $\delta_x - \delta_y$ is $\rho(x, y)$, while $\mathrm{Var}(\delta_x - \delta_y) = 2$. Thus, for each infinite set K, the space $M(K)$ with the KR norm is not complete.

The present work contains a theorem providing a description of its completion and connected results on Lipschitz spaces.

Let us recall the definition of the original KR norm (for this and other information on KR theory see [5, Chapter VIII, §4]). Let $M_0(K)$ be the set of measures $\mu \in M(K)$ such that $\mu(K) = 0$. With each measure $\mu \in M_0(K)$, we associate the family Ψ_μ of all nonnegative measures $\Psi \in M(K \times K)$ such that for every Borel set $e \subset K$, $\Psi(K, e) - \Psi(e, K) = \mu(e)$. The value $\Psi(e_1, e_2)$ can be interpreted as the mass carried from a set e_1 to a set e_2. Thus $\psi \in \Psi_\mu$ gives rise to a mass transfer on K with the given mass distribution μ_- and the required one μ_+. The classical KR norm of a measure $\mu \in M_0(K)$ is defined by

$$\|\mu\|_\rho^0 = \inf\left\{\int_{K\times K} \rho(x, y)\, d\psi(x, y) : \psi \in \Psi_\mu\right\}.$$

Its value together with the corresponding optimal transfer give the solution of the Monge-Kantorovich mass transfer problem [5, 10].

Now set

$$\|\mu\|_\rho = \inf\{\|\nu\|_\rho^0 + \mathrm{Var}(\mu - \nu) : \nu \in M_0(K)\} \qquad (\mu \in M(K)).$$

The functional $\|\cdot\|_\rho$ is a norm on $M(K)$. Though it differs from the norm on $M(K)$ introduced in [6], we preserve the name KR norm for it. Since

$$\|\mu\|_\rho^0 \le \frac{1}{2}\,\mathrm{diam}(K, \rho)\,\mathrm{Var}\,\mu \qquad (\mu \in M_0(K)),$$

the norms $\|\cdot\|_\rho$ and $\|\cdot\|_\rho^0$ on $M_0(K)$ are equivalent (if $\mathrm{diam}(K, \rho) \le 2$, they even coincide).

The following theorem (analogous to its counterparts in [5] and [7]) is the main result of the KR theory.

Theorem 0. *The duality*

$$\langle f, \mu\rangle = \int_K f\, d\mu \qquad (\mu \in M(K),\ f \in \mathrm{Lip}(K, \rho))$$

establishes an isometric isomorphism between the spaces $(M(K), \|\cdot\|_\rho)^*$ *and* $\mathrm{Lip}(K, \rho)$.

This result depends largely on the following basic property of the KR norm (compare with the corresponding statement in [5]).

Lemma 0. *The set of all measures with finite support is dense in* $M(K)$ *with respect to the KR norm.*

3. Statement of the results

Let (K, ρ) be a compact metric space. For every $\mu \in M(K)$, the formula

$$i(\mu)(f) = \int_K f\,d\mu \qquad (f \in \mathrm{lip}(K, \rho))$$

defines a bounded linear functional $i(\mu)$ on $\mathrm{lip}(K, \rho)$.

Lemma 1. *The set* $i(M(K))$ *is dense in* $(\mathrm{lip}(K, \rho))^*$ *in the norm topology.*

For $K = [0, 1]$, $\rho(x, y) = |x-y|^\alpha$, $0 < \alpha < 1$, the proof is contained in [8, Lemma 2.5]. In the more general case when (K, d) is a compact metric space and $\rho = d^\alpha$, $0 < \alpha < 1$, it is established in a similar way in [1, Lemma 3.1]. In the case of arbitrary compact metric space (K, ρ), the proof is the same.

By means of Theorem 0, we have

$$|i(\mu)(f)| \le \|f\|_{K,\rho}\|\mu\|_\rho \qquad (f \in \mathrm{lip}(K, \rho),\ \mu \in M(K)).$$

Hence $i\colon (M(K), \|\cdot\|_\rho) \to (\mathrm{lip}(K, \rho))^*$ is a linear map with the norm ≤ 1. Passing to the completion, we extend i to a map $i\colon (M(K), \|\cdot\|_\rho)^c \to (\mathrm{lip}(K, \rho))^*$ with the same properties (and notation).

The question we start with is the following: Under what conditions is the above map an isometric isomorphism?

Theorem 1. *The map* $i\colon (M(K), \|\cdot\|_\rho)^c \to (\mathrm{lip}(K, \rho))^*$ *is an isometric isomorphism if and only if the following condition is satisfied*:

(A) *for every finite set* $F \subset K$ *and every function* f *on* F, *for each* $C > 1$ *there exists a function* $g \in \mathrm{lip}(K, \rho)$ *such that* $g|F = f$ *and* $\|g\|_{K,\rho} \le C\|f\|_{F,\rho}$.

Remark. A closed interval with the Euclidean metric (or every metric space containing it as a subspace) fails to have property (A).

Taking the conjugate map i^* and applying Theorem 0 we get the natural linear map $j\colon (\mathrm{lip}(K, \rho))^{**} \to \mathrm{Lip}(K, \rho)$ with the norm ≤ 1. Then, from Theorem 1, we obtain the following result.

Theorem 2. *The map* $j\colon (\mathrm{lip}(K, \rho))^{**} \to \mathrm{Lip}(K, \rho)$ *is an isometric isomorphism if and only if the metric space* (K, ρ) *satisfies condition* (A).

The following definition is intended to provide a condition stronger than (A) but expressed in terms of metrics.

Definition 1. A metric ρ on a set K is called *noncritical* if there is a sequence $\{\rho_n\}_{n\in\mathbb{N}}$ of metrics on K such that

(i) $\lim_{n\to\infty} \rho_n(x, y) = \rho(x, y)$ for all $x, y \in K$;
(ii) $\lim_{n\to\infty} \sup\{\rho_n(x, y)/\rho(x, y)\colon x, y \in K,\ x \neq y\} = 1$;
(iii) $\lim_{\rho(x,y)\to 0} \rho_n(x, y)/\rho(x, y) = 0$ for all $n \in \mathbb{N}$.

Proposition 1. *Every noncritical (not necessarily compact) metric possesses property* (A).

Now we will introduce a more constructively defined subclass of noncritical metrics.

Definition 2. Let Ω be the set of all nondecreasing functions $\omega\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that $\omega(0) = 0$, $\lim_{t\to 0}\omega(t) = 0$, $\lim_{t\to 0}\omega(t)/t = +\infty$, and the function $\omega(t)/t$ is nonincreasing for $t > 0$.

The latter condition implies that every function $\omega \in \Omega$ is semiadditive, i.e., $\omega(s+t) \le \omega(s) + \omega(t)$ for all s, $t \ge 0$. Hence, if ρ is a (compact) metric then the same is $\omega(\rho)$ for all $\omega \in \Omega$.

Proposition 2. *If d is a metric on a set K then, for every $\omega \in \Omega$, $\rho = \omega(d)$ is a noncritical metric on K.*

Thus, for every compact noncritical metric (in particular, for a metric of the form $\rho = \omega(d)$, $\omega \in \Omega$), we have isometric isomorphisms indicated in Theorems 1 and 2. In the particular case $\omega(t) = t^\alpha$, $0 < \alpha < 1$, an isometric isomorphism between the spaces $(\mathrm{lip}(K, \omega(d)))^{**}$ and $\mathrm{Lip}(K, \omega(d))$ has been established earlier. For $K = [0, 1]$, $d(x, y) = |x - y|$, this result has been originally obtained in [8], and for arbitrary compact metric space (K, d) in [2, 3]. In the latter case, the above isomorphism has been also rediscovered in [1].

For metrics of the form $\rho = \omega(d)$, $\omega \in \Omega$, property (A) can be strengthened.

Proposition 3. *Let (K, d) be a metric space, and let $\rho = \omega(d)$ with $\omega \in \Omega$. Then for every finite set $F \subset K$ and every function f on F, for each $C > 1$ there exists a function $g \in \mathrm{Lip}(K, d)$ such that $g|F = f$ and $\|g\|_{K,\rho} \le C\|f\|_{F,\rho}$.*

The following Stone-Weierstrass type theorem for the spaces $\mathrm{lip}(K, \rho)$ is connected with Proposition 3.

Theorem 3. *Let (K, ρ) be a compact metric space. Suppose L is a linear subspace of $\mathrm{lip}(K, \rho)$ satisfying the following condition: there is a constant C such that for every finite set $F \subset K$ and for every function $f \in \mathrm{lip}(K, \rho)$ there exists a function $g \in L$ with the properties $g|F = f|F$ and $\|g\|_{K,\rho} \le C\|f\|_{K,\rho}$. Then L is dense in $\mathrm{lip}(K, \rho)$.*

From Theorem 3 and Proposition 3 we obtain

Proposition 4. *Suppose (K, d) is a compact metric space, and let $\rho = \omega(d)$, $\omega \in \Omega$. Then the space $\mathrm{Lip}(K, d)$ is dense in $\mathrm{lip}(K, \rho)$.*

For compact metric spaces (K, ρ) with $\rho = d^\alpha$, $0 < \alpha < 1$, Proposition 3 (with $C = \sqrt{2}$), Theorem 3, and Proposition 4 can be found in [1].

4. Proofs

Proof of Theorem 1. Necessity. We suppose that i is an isometric isomorphism, and we shall show that condition (A) is fulfilled. Let F be an n-point subset in K. Set $E = \{g|F\colon g \in \mathrm{lip}(K, \rho)\}$. Obviously, E can be identified with a linear subspace in $\mathbb{R}^n$. We claim that $E = \mathbb{R}^n$. To see this, take a functional annihilating E. This functional can be viewed as a measure μ on F (and thus on K). Then

$$\int_K g\,d\mu = 0 \qquad (g \in \operatorname{lip}(K, \rho)).$$

It means that $i(\mu) = 0$. Since the map i is supposed to be one-to-one, this implies that $\mu = 0$, and hence that $E = \mathbb{R}^n$.

Set $X = \{g \in \operatorname{lip}(K, \rho)\colon g|F = 0\}$ and observe that the set of functionals in $(\operatorname{lip}(K, \rho))^*$ annihilating X coincides with $i(M(F))$. Let f be a function on F. As was shown earlier, there is a function $g_0 \in \operatorname{lip}(K, \rho)$ such that $g_0|F = f$. Since i is an isometry, by Theorem 0 we have

$$\begin{aligned}\operatorname{dist}(g_0, x) &= \sup\{\lambda(g_0)\colon \lambda \in (\operatorname{lip}(K, \rho))^*,\ \lambda(X) = \{0\},\ \|\lambda\| = 1\}\\ &= \sup\left\{\int_F f\,d\mu\colon \mu \in M(F),\ \|\mu\|_\rho = 1\right\} = \|f\|_{F,\rho},\end{aligned}$$

where by $\|\cdot\|$ is denoted hereafter the usual norm on $(\operatorname{lip}(K, \rho))^*$. Hence for every $C > 1$ we can find a function $g_1 \in X$ such that $\|g_0 - g_1\|_{K,\rho} \le C\|f\|_{F,\rho}$. Then $g = g_0 - g_1$ is the function required.

Sufficiency. Suppose that condition (A) is satisfied. We need to show that the map i is an isometry. To see this, take $\mu \in M(K)$, $f \in B$, and $\varepsilon > 0$. By Lemma 0 there exists a measure ν on K with finite support F such that $\|\nu - \mu\|_\rho \le \varepsilon$. Condition (A) supplies us with a function $g \in \operatorname{lip}(K, \rho)$ with $g|F = f|F$ and $\|g\|_{K,\rho} \le 1 + \varepsilon$. Then via Theorem 0

$$\int_K f\,d\mu = \int_K f\,d(\mu - \nu) + \int_K g\,d(\nu - \mu) + \int_K g\,d\mu \le (2+\varepsilon)\varepsilon + \int_K g\,d\mu.$$

Hence again by means of Theorem 0 we obtain

$$\|\mu\|_\rho = \sup\left\{\int_K f\,d\mu\colon f \in B\right\} = \sup\left\{\int_K g\,d\mu\colon g \in b\right\} = \|i(\mu)\| \quad (\mu \in M(K)).$$

Proof of Proposition 1. Let ρ be a noncritical metric on K, and let $\{\rho_n\}_{n\in\mathbb{N}}$ be a sequence of metrics satisfying conditions (i)–(iii) of Definition 1. Take a finite set F in K and a function f on F. Denote

$$\begin{aligned}\alpha_n &= \max\left\{\frac{\rho(x, y)}{\rho_n(x, y)}\colon x, y \in F,\ x \ne y\right\},\\ \beta_n &= \sup\left\{\frac{\rho_n(x, y)}{\rho(x, y)}\colon x, y \in K,\ x \ne y\right\} \qquad (n \in \mathbb{N}).\end{aligned}$$

It follows from conditions (i) and (ii) that $\alpha_n \to 1$ and $\beta_n \to 1$ as $n \to \infty$. Note that $\|f\|_{F,\rho_n} \le \max\{\alpha_n, 1\}\|f\|_{F,\rho}$. The function f can be extended to a function $f_n \in \operatorname{Lip}(K, \rho_n)$ in such a way that $\|f_n\|_{K,\rho_n} = \|f\|_{F,\rho_n}$ [9]. Condition (iii) implies $f_n \in \operatorname{lip}(K, \rho)$. Further,

$$\|f_n\|_{K,\rho} \le \max\{\beta_n, 1\}\|f_n\|_{K,\rho_n} \le \max\{\beta_n, 1\}\max\{\alpha_n, 1\}\|f\|_{F,\rho} \qquad (n \in \mathbb{N}).$$

Then, for every $C > 1$, the function $g = f_n$, n being sufficiently large, meets condition (A).

Proof of Proposition 2. Suppose d is a metric on a set K, $\omega \in \Omega$, and $\rho = \omega(d)$. For every $n \in \mathbb{N}$, define the function

$$\varphi_n(t) = \begin{cases} 0 & \text{if } t = 0; \\ \left(n\omega\left(\frac{1}{n}\right)\frac{t}{\omega(t)}\right)^{1/2} & \text{if } 0 < t < \frac{1}{n}; \\ 1 & \text{if } t \geq \frac{1}{n}. \end{cases}$$

It can be seen easily that the functions $\omega_n = \varphi_n\omega$ belong to Ω for all n. The sequence of functions $\{\varphi_n\}_{n\in\mathbb{N}}$ possesses the following properties:

(i) $\lim_{n\to\infty}\varphi_n(t) = 1$ for all $t > 0$;
(ii) $\sup_{t>0}\varphi_n(t) = 1$ for all $n \in \mathbb{N}$;
(iii) $\lim_{t\to 0}\varphi_n(t) = 0$ for all $n \in \mathbb{N}$.

Then the corresponding sequence of metrics $\rho_n = \omega_n(\rho) = \varphi_n(\rho)\rho$, $(n \in \mathbb{N})$ satisfies Definition 1. Thus ρ is noncritical metric.

Proof of Proposition 3. Let d be a metric on a set K, and let $\rho = \omega(d)$ where $\omega \in \Omega$. Take a finite set $F \subset K$, a function f on F, and arbitrary $C > 1$. Denote $\delta_0 = \min\{\rho(x, y) : x, y \in F, x \neq y\}$, and choose $\delta \in (0, \delta_0)$ such that $\delta = \omega(\tau)$ for some $\tau > 0$ and

$$\max_{x, y\in F}\frac{\rho(x, y)}{\rho(x, y) - \delta} \leq C.$$

For $x, y \in K$, define $r(x, y) = \max\{\rho(x, y) - \delta, 0\}$.

Note that if there is a nonvoid family of functions $\{f_\alpha : \alpha \in \mathscr{A}\}$ defined on a metric space (X, σ) and satisfying for some N and for all $x, y \in X$ the Lipschitz condition

$$f_\alpha(x) - f_\alpha(y) \leq N\sigma(x, y) \qquad (\alpha \in \mathscr{A}),$$

then the same is true for the function $\sup\{f_\alpha(x) : \alpha \in \mathscr{A}\}$, $(x \in X)$ provided that the latter is not identically $+\infty$.

Applying this, we see that for every $y \in K$,

$$r(x, y) - r(z, y) \leq \rho(x, z) \qquad (x, z \in K). \tag{1}$$

Set $M = \|f\|_{F,\rho}$ and define the function

$$h(x) = \sup\{f(y) - CMr(x, y) : y \in F\} \qquad (x \in K).$$

We claim that $h|F = f$. Clearly, $h(x) \geq f(x)$, $(x \in F)$. Conversely, for given $x \in F$ we have, for all $y \in F \setminus \{x\}$,

$$f(y) - f(x) \leq M\rho(x, y) \leq CM(\rho(x, y) - \delta) = CMr(x, y),$$

and hence $f(x) \geq h(x)$.

It follows from (1) that, for every $y \in F$, the function $x \mapsto f(y) - CMr(x, y)$ satisfies the Lipschitz condition with respect to the metric ρ with the constant CM. Using the note mentioned above, we obtain $|h|_{K,\rho} \leq C\|f\|_{F,\rho}$.

Now we will show that there is a constant P such that for every $y \in K$,

$$r(x, y) - r(z, y) \leq P\,d(x, z) \qquad (x, z \in K). \tag{2}$$

It is sufficient to demonstrate this inequality for $\rho(x, y) \geq \delta$ and $r(z, y) \leq r(x, y)$. If $\rho(z, y) < \delta$ then $d(z, y) \leq \tau$, while $d(x, y) \geq \tau$. In this case, we have

$$\begin{aligned} r(x, y) - r(z, y) &= \rho(x, y) - \delta = \omega(d(x, y)) - \omega(\tau) \\ &\leq \frac{\omega(\tau)}{\tau}(d(x, y) - \tau) \leq \frac{\omega(\tau)}{\tau}(d(x, y) - d(z, y)) \\ &\leq \frac{\omega(\tau)}{\tau} d(x, z). \end{aligned}$$

If $\rho(z, y) \geq \delta$ then $d(z, y) \geq \tau$, and in this case, we have

$$\begin{aligned} r(x, y) - r(z, y) &= \rho(x, y) - \rho(z, y) \\ &\leq \frac{\omega(d(z, y))}{d(z, y)}(d(x, y) - d(z, y)) \leq \frac{\omega(\tau)}{\tau} d(x, z). \end{aligned}$$

Thus (2) is valid with $P = \omega(\tau)/\tau$. Again applying the "supremum argument" to the function h we obtain via (2) that $h \in \mathrm{Lip}(K, d)$. Finally,

$$g(x) = \begin{cases} M & \text{if } h(x) > M, \\ h(x) & \text{if } |h(x)| \leq M, \\ -M & \text{if } h(x) < -M \end{cases}$$

is the function desired.

Proof of Theorem 3. We have to show that for every functional $\varphi \in (\mathrm{lip}(K, \rho))^*$ annihilating L, $\varphi = 0$. Take $f \in \mathrm{lip}(K, \rho)$ and $\varepsilon > 0$. By Lemma 1 there is a measure $\mu \in M(K)$ such that $\|i(\mu\|i(\mu) - \varphi\| \leq \varepsilon) - \varphi\| \leq \varepsilon$, and by Lemma 0 we can find a measure ν on K with a finite support F such that $\|\nu - \mu\|_\rho \leq \varepsilon$. Hence $\|i(\nu) - \varphi\| \leq 2\varepsilon$ since the norm of i is not greater than 1. There exists a function $g \in L$ with $g|F = f|F$ and $\|g\|_{K,\rho} \leq C\|f\|_{K,\rho}$. We have $\varphi(f) = (\varphi - i(\nu))(f) + (i(\nu) - \varphi)(g)$; then $|\varphi(f)| \leq 2\varepsilon(1 + C)\|f\|_{K,\rho}$. This implies that $\varphi(f) = 0$ $(f \in \mathrm{lip}(K, \rho))$, i.e., $\varphi = 0$.

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References

1. W. G. Bade, P. C. Curtis, Jr., and H. G. Dales, *Amenability and weak amenability for Beurling and Lipschitz algebras*, Proc. London Math. Soc. (3) **55** (1987), 359–377.
2. T. M. Jenkins, *Banach spaces of Lipschitz functions on an abstract metric space*, Thesis, Yale Univ., New Haven, CT, 1967.
3. J. A. Johnson, *Banach spaces of Lipschitz functions and vector-valued Lipschitz functions* Bull. Amer. Math. Soc. **75** (1969), 1334–1338.
4. L. V. Kantorovich, *On mass transfer*, Dokl. Akad. Nauk SSSR **37** (1942), 227–229. (Russian)
5. L. V. Kantorovich and G. P. Akilov, *Functional analysis*, 2nd ed., New York, 1982.

6. L. V. Kantorovich and G. Sh. Rubinstein, *On a functional space and certain extremal problems*, Dokl. Akad. Nauk SSSR **115** (1957), 1058–1061. (Russian)

7. ——, *On a space of completely additive functions*, Vestnik Leningrad Univ. Math. **13** (1958), 52–59. (Russian)

8. K. de Leeuw, *Banach spaces of Lipschitz functions*, Studia Math. **21** (1961), 55–66.

9. E. J. McShane, *Extension of range of functions*, Bull. Amer. Math. Soc. **40** (1934), 837–842.

10. S. T. Rachev, *The Monge-Kantorovich mass transference problem and its stochastic applications*, Teor. Veroyatnost. i Primenen. **29** (1984), 625–653 (Russian); English transl. in Theor. Probab. Appl. **29** (1984), 647–676.

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