**A GENERALIZATION OF ODLYZKO'S CONJECTURE:
 THE COEFFICIENTS
 OF $(1 - q)^j / ((1 - q^{2^n}) \cdots (1 - q^{2^{n+2j}}))$ ALTERNATE IN SIGN**

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ABSTRACT. A positivity result is proved that generalizes a conjecture of Odlyzko, previously proved by Stanton and Zeilberger.

Let N, a, b, i, j, r, n denote arbitrary nonnegative integers, and let

$$(x)_N := \prod_{i=1}^{N-1} (1 - q^i x), \quad G(a, b) := \frac{(q)_{a+b}}{(q)_a (q)_b},$$

$$A := \{f(q); (-1)^i f^{(i)}(0) \geq 0, 0 \leq i < \infty\}.$$

Odlyzko conjectured that $(1 - q)^j / (q)_j \in A$. This was proved in [1], where the more general fact that $(1 - q)^j G(2j, r) \in A$ was proved. Here we show that this last result also implies the statement of the title, that it generalizes Odlyzko's conjecture (for even j) in a different direction. To wit: set $t = q^{2^n}$, $N = 2j$, in the q -binomial theorem $1/(t)_{N+1} = \sum_{r \geq 0} G(N, r) t^r$, and then multiply by $(1 - q)^j$. $\square$

REFERENCE

1. D. Stanton and D. Zeilberger, *The Odlyzko conjecture and O'Hara's unimodality proof*, Proc. Amer. Math. Soc. **107** (1989), 39–42.

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