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POINCARÉ AND SOBOLEV INEQUALITIES IN PRODUCT SPACES

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ABSTRACT. Some local Poincaré and Sobolev inequalities involving weights in product spaces are established.

Recently there has been some interest in considering local Poincaré and Sobolev inequalities involving weights; the purpose of this note is to establish these results in the context of product spaces.

Let $w(x, y)$ be a nonnegative locally integrable function, or weight, defined in the product space $R^n \times R^m$. We say that the weight w satisfies Muckenhoupt's $A_p(R^n \times R^m)$ condition, or that $w \in A_p(R^n \times R^m)$, $1 < p < \infty$, provided that

$$\left(\frac{1}{|P|}\int\int_P w(x, y)\,dx\,dy\right)\left(\frac{1}{|P|}\int\int_P w(x, y)^{-1/(p-1)}\,dx\,dy\right)^{(p-1)} \le c,$$

where P is the parallelepiped $P = I \times J$, and $I \subset R^n$ and $J \subset R^m$ are arbitrary open cubes with sides parallel to the coordinate axes. By the Lebesgue differentiation theorem it readily follows that if $w \in A_p(R^n \times R^m)$, then $w(x, \cdot) \in A_p(R^m)$ for almost every $x \in R^n$, with A_p constant independent of x; similarly for $w(\cdot, y)$.

Given a measurable set $E \subset R^n \times R^m$, we denote by $|E|$ its Lebesgue measure and $\mu(E) = \int\int_E w(x, y)\,dx\,dy$. It is also convenient to introduce the notation $\mu(x, A) = \int_A w(x, y)\,dy$ for measurable $A \subset R^m$; similarly for $B \subset R^n$ and $\mu(B, y)$.

We say that w, or μ, is doubling if

$$\mu(2P) \le c\mu(P), \qquad 2P = 2I \times 2J, \text{ all } P.$$

For $0 < \alpha, \beta, p, q < \infty$, we consider pairs of weights w, v, $d\nu(x, y) = v(x, y)\,dx\,dy$, which verify

$$\left(\frac{|I'|}{|I|}\right)^{\alpha/n}\left(\frac{|J'|}{|J|}\right)^{\beta/m}\left(\frac{\nu(P')}{\nu(P)}\right)^{1/q} \le c\left(\frac{\mu(P')}{\mu(P)}\right)^{1/p}, \tag{1}$$

where c is independent of $P' = I' \times J' \subseteq P$ and P. Again by the Lebesgue differentiation theorem it follows immediately that if w, v satisfy relation (1),

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then for almost every y in J,

$$\left(\frac{|I'|}{|I|}\right)^{\alpha/n}\left(\frac{\nu(I', y)}{\nu(I, y)}\right)^{1/q} \leq c\left(\frac{\mu(I', y)}{\mu(I, y)}\right)^{1/p} \tag{2}$$

and for almost every x in I,

$$\left(\frac{|J'|}{|J|}\right)^{\beta/m}\left(\frac{\nu(x, J')}{\nu(x, J)}\right)^{1/q} \leq c\left(\frac{\mu(x, J')}{\mu(x, J)}\right)^{1/p}. \tag{3}$$

Finally, if $f(x, y)$ is defined in an open subset of $R^n \times R^m$, we denote by $\nabla_1 f(x, y)$ the partial gradient of f containing the x-derivatives; similarly for $\nabla_2 f(x, y)$, the partial gradient of f containing the y-derivatives.

We may now state our results.

Theorem 1 (Poincaré's Inequality). *Assume f is a Lipschitz continuous function on a parallelepiped P, and suppose that the weights w, v satisfy the following conditions: v is doubling, $w \in A_p(R^n \times R^m)$, and (1) holds with $\alpha + \beta < 1$ and $1 < p \leq q < \infty$. If f_P denotes the average of f over P then*

$$\begin{aligned}\left(\frac{1}{\nu(P)}\int\int_P |f(x, y) - f_P|^q \, d\nu(x, y)\right)^{1/q} &\leq c|I|^{1/n}\left(\frac{1}{\mu(P)}\int\int_P |\nabla_1 f(x, y)|^p \, d\mu(x, y)\right)^{1/p} \\ &+ c|J|^{1/m}\left(\frac{1}{\mu(P)}\int\int_P |\nabla_2 f(x, y)|^p \, d\mu(x, y)\right)^{1/p},\end{aligned} \tag{4}$$

where c is independent of f and P.

Theorem 2 (Sobolev's Inequality). *Under the hypothesis of Theorem 1 and the additional assumption that f is supported in P, we have*

$$\begin{aligned}\left(\frac{1}{\nu(P)}\int\int_P |f(x, y)|^q \, d\nu(x, y)\right)^{1/q} &\leq c|I|^{1/n}\left(\frac{1}{\mu(P)}\int\int_P |\nabla_1 f(x, y)|^p \, d\mu(x, y)\right)^{1/p} \\ &+ c|J|^{1/m}\left(\frac{1}{\mu(P)}\int\int_P |\nabla_2 f(x, y)|^p \, d\mu(x, y)\right)^{1/p},\end{aligned} \tag{5}$$

where c is independent of f and P.

Theorem 3. *Inequality (5) holds under the hypothesis of Theorem 1 provided that f vanishes on a subset E of P with $|E| > \eta|P|$, and now the constant c depends also on η, $0 < \eta < 1$.*

We pass now to the proofs, beginning with some preliminary results.

Lemma 1. *Suppose f is a Lipschitz continuous function in $P = I \times J$ and $(x, y) \in P$. Then $|f(x, y) - f_P|$ does not exceed*

$$\begin{aligned}&\frac{c}{|P|}\int\int_P (|\nabla_1 f(u, z)|\,|u - x| + |\nabla_2 f(u, z)|\,|z - y|) \\ &\qquad \times \min\left(\frac{|I|^{1/n}}{|u - x|}, \frac{|J|^{1/m}}{|z - y|}\right)^{n+m} du\, dz,\end{aligned} \tag{6}$$

where c is a constant independent of f and P.

Proof. It is clear that $|f(x, y) - f_P| \le A + B$, say, where

$$A = \frac{1}{|P|} \int \int_P \int_0^1 |\nabla_1 f(x + t(u - x), y + t(z - y))| \, |u - x| \, dt \, du \, dz,$$

and the expression for B is obtained by replacing ∇_1 by ∇_2 above; since both integrals are handled in a similar fashion we only consider A. If χ_P denotes the characteristic function of P then we may rewrite A as

$$\frac{1}{|P|} \int \int_P |\nabla_1 f(u, z)| \, |u - x| \int_0^1 \chi_P \left(x + \frac{u - x}{t}, y + \frac{z - y}{t} \right) t^{-n-m-1} \, dt \, du \, dz.$$

Furthermore, since the integrand in the innermost integral above vanishes if either $|u - x| \ge t|I|^{1/n}$ or $|z - y| \ge t|J|^{1/m}$, it readily follows that

$$\begin{aligned} A &\le \frac{1}{|P|} \int \int_P |\nabla_1 f(u, z)| \, |u - x| \int_{\max(|u-x|/|I|^{1/n}, |z-y|/|J|^{1/m})}^{\infty} t^{-n-m-1} \, dt \, du \, dz \\ &\le \frac{1}{(n + m)} \frac{1}{|P|} \int \int_R |\nabla_1 f(u, z)| \, |u - x| \min \left(\frac{|I|^{1/n}}{|u - x|}, \frac{|J|^{1/m}}{|z - y|} \right)^{n+m} du \, dz, \end{aligned}$$

and the proof is complete. □

Corollary 1. *Let* $0 < \gamma, \lambda < 1$. *Under the hypothesis of Lemma* 1 *we also have*

$$\begin{aligned} |f(x, y) - f_P| &\le c \left(\frac{|I|^{1/n}}{|J|^{1/m}} \right)^{\gamma} \int \int_P \frac{|\nabla_1 f(u, z)|}{|u - x|^{n-(1-\gamma)} |z - y|^{m-\gamma}} \, du \, dz \\ &\quad + c \left(\frac{|I|^{1/n}}{|J|^{1/m}} \right)^{\lambda} \int \int_P \frac{|\nabla_2 f(u, z)|}{|u - x|^{n-\lambda} |z - y|^{m-(1-\lambda)}} \, du \, dz. \end{aligned} \tag{7}$$

Proof. Follows at once from estimate (6). □

Proof of Theorem 1. Fix $P = I \times J$, put $\lambda = 1 - \gamma$ in (7), and choose γ so that $\alpha < 1 - \gamma$ and $\beta < \gamma$; these choices are possible since $\alpha + \beta < I$. By Corollary 1 it suffices to estimate two terms, each corresponding to a summand on the right-hand side of (7). Since both terms are handled in a similar fashion, we only consider

$$\left(\frac{|I|^{1/n}}{|J|^{1/m}} \right)^{\gamma} \left(\frac{1}{\nu(P)} \int \int_P \left(\int \int_P \frac{|\nabla_1 f(u, z)|}{|u - x|^{n-(1-\gamma)} |z - y|^{m-\gamma}} \, du \, dz \right)^q \times d\nu(x, y) \right)^{1/q}. \tag{8}$$

In the first place note that if ψ is a nonnegative compactly supported smooth function on R^1 and $\psi_t(x) = t^{-n} \psi(|x|/t)$ and $\psi_s(y) = s^{-m} \psi(|y|/s)$, then

$$\begin{aligned} &\int_{R^n} \int_{R^m} \frac{g(u, z)}{|u - x|^{n-(1-\gamma)} |z - y| m - \gamma} \, du \, dz \\ &\quad = \int_0^\infty \int_0^\infty t^{(1-\gamma)-1} s^{\gamma-1} G(x, t, y, s) \, dt \, ds, \end{aligned}$$

where, if c denotes a constant that only depends on ψ,

$$G(x, t, y, s) = c \int_{R^n} \int_{R^m} g(u, z) \psi_s(x - u) \psi_s(y - z) \, du \, dz.$$

Thus, to estimate the innermost integral in (8), we set

$$g(u, z) = |\nabla_1 f(u, z)|\chi_R(u, z)$$

in the above expression, and breaking up the domain of integration into four parts, namely, $[0, |I|^{1/n}) \times [0, |J|^{1/m})$, $[0, |I|^{1/n}) \times [|J|^{1/m}, \infty)$, $[|I|^{1/n}, \infty) \times [0, |J|^{1/m})$, and $[|I|^{1/n}, \infty) \times [|J|^{1/m}, \infty)$, we obtain that (8) is bounded by the sum of four terms, $A_1 + A_2 + A_3 + A_4$, say, where A_1 is equal to

$$\left(\frac{|I|^{1/n}}{|J|^{1/m}}\right)^\gamma \left(\frac{1}{\nu(P)} \int\int_P \left(\int_0^{|J|^{1/m}} \int_0^{|I|^{1/n}} t^{(1-\gamma)-1} s^{\gamma-1} \right.\right. \tag{9}$$
$$\left.\left. \times G(x, t, y, s)\, dt\, ds\right)^q d\nu(x, y)\right)^{1/q},$$

and where A_2, A_3, and A_4 are defined similarly.

It is easy to estimate A_4; indeed, since $\psi_t(u) \le ct^{-n}$ and $\psi_s(z) \le cs^{-m}$, it readily follows that $A_4 \le c|I|^{1/n}\|g\|_1$, which, by the $A_p(R^n \times R^m)$ condition, is a bound of the right order.

We turn now to estimate A_1. For (x, t) a point in $I \times [0, |I|^{1/n})$, consider the integral

$$I(x, t) = \int_0^{|J|^{1/m}} s^{q\beta-1} \int_J G(x, t, y, s)^q v(x, y)\, dy\, ds,$$

and observe that if $d\nu_q(x, y, s) = v(x, y)s^{q\beta-1}\, dy\, ds$ and $\mathscr{U}(x, t, \lambda) = \{(y, s) \in J \times [0, |J|^{1/m}) : G(x, t, y, s) > \lambda\}$, then

$$I(x, t) = q\int_0^\infty \lambda^{q-1}\nu_q(\mathscr{U}(x, t, \lambda))\, d\lambda. \tag{10}$$

Now let

$$NG(x, t, y) = \sup_{|y-z|<s} G(x, t, z, s),$$

and for $\lambda > 0$, put

$$\mathscr{O} = \{y \in J : NG(x, t, y) > \lambda\}.$$

By the Whitney decomposition there is a sequence $\{J_k\}$ of nonoverlapping closed cubes, subcubes of J actually, such that $\mathscr{O} = \bigcup_k J_k$ and

$$\mathscr{U} \subseteq \bigcup_k (J_k \times [0, C|J_k|^{1/m})),$$

where C is a dimensional constant. Whence,

$$\begin{aligned}
\nu_q(\mathscr{U}(x, t, \lambda)) &\le \sum_k \nu_q(J_k \times [0, C|J_k|^{1/m})) \\
&= \sum_k \int_0^{C|J_k|^{1/m}} s^{q\beta-1}\, ds \int_{J_k} v(x, y)\, dy = c\sum_k |J_k|^{q\beta/m}\nu(x, J_k) \\
&\le c|J|^{q\beta/m}\nu(x, J)\sum_k \left(\frac{|J_k|}{|J|}\right)^{q\beta/m}\left(\frac{\nu(x, J_k)}{\nu(x, J)}\right).
\end{aligned}$$

We may now invoke the estimate in (3) and dominate the above expression by

$$\begin{aligned}(11)\qquad & c|J|^{q\beta/m}\nu(x, J)\sum_k\left(\frac{\mu(x, J_k)}{\mu(x, J)}\right)^{q/p} \\ & \leq c|J|^{q\beta/m}\nu(x, J)\mu(x, J)^{-q/p}\mu(x, \mathscr{O}(x, t, \lambda))^{q/p}.\end{aligned}$$

Substituting (11) into (10) gives

$$I(x, t) \leq c|J|^{q\beta/m}\nu(x, J)\mu(x, J)^{-q/p}\int_0^\infty \lambda^{q-1}\mu(x, \mathscr{O}(x, t, \lambda))^{q/p}\,d\lambda.$$

Next consider the integral

$$(12)\qquad B = \int_0^{|I|^{1/n}} t^{q\alpha-1}\int_I I(x, t)\,dx\,dt \leq c|J|^{q\beta/m}\int_0^\infty \lambda^{q-1}R(\lambda)\,d\lambda,$$

where

$$(13)\qquad R(\lambda) = \int_0^{|I|^{1/n}}\int_I\left(\frac{\mu(x, \mathscr{O}(x, t, \lambda))}{\mu(x, J)}\right)^{q/p}\nu(x, J)t^{q\alpha-1}\,dx\,dt.$$

Observe that if $F(x, t, \lambda) = \mu(x, \mathscr{O}(x, t, \lambda))/\mu(x, J)$ (≤ 1) and $d\mu_q(x, t) = \nu(x, J)t^{q\alpha-1}dx\,dt$, then we may write $R(\lambda)$ as

$$(14)\qquad \frac{q}{p}\int_0^1 \zeta^{q/p-1}\mu_q(\{(x, t) \in I \times [0, |I|^{1/n}) : F(x, t, \lambda) > \zeta\})\,d\zeta.$$

In order to estimate (14), once again we introduce appropriate maximal functions, namely,

$$NG(x, y) = \sup_{|x-u|<t,\ |y-z|<s} G(u, t, z, s)$$

and

$$NF(x, \lambda) = \sup_{|x-u|<t} \chi_I(u)F(u, t, \lambda) \qquad (\leq 1).$$

Let $\mathscr{O}'_\zeta$ be the open set $\{NF(x, \lambda) > \zeta\} \cap I$; $\mathscr{O}'_\zeta \neq \varnothing$ only for $\zeta \leq 1$. According to the Whitney decomposition there is a sequence $\{I_k\}$ of nonoverlapping cubes so that $\mathscr{O}'_\zeta = \bigcup_k I_k$, and if $\mathscr{U}(\lambda, \zeta) = \{(x, t) \in I \times [0, |I|^{1/n}) : F(x, t, \lambda) > \zeta\}$ then $\mathscr{U}(\lambda, \zeta) \subseteq \bigcup_k(I_k \times [0, C|I|^{1/n}))$. Thus, by (1),

$$\begin{aligned}\mu_q(\mathscr{U}(\lambda, \zeta) &\leq \sum_k \mu_q(I_k \times [0, C|I|^{1/n})) \\ (15)\qquad &\leq c|I|^{q\alpha/n}\nu(P)\sum_k\left(\frac{|I_k|}{|I|}\right)^{q\alpha/n}\left(\frac{\nu(I_k \times J)}{\nu(P)}\right) \\ &\leq c|I|^{q\alpha/n}\nu(P)\sum_k\left(\frac{\mu(I_k \times J)}{\mu(P)}\right)^{q/p}.\end{aligned}$$

Whence substituting (15) into (14), we immediately get

$$(16)\qquad R(\lambda) \leq c\frac{|I|^{q\alpha/n}\nu(P)}{\nu(P)^{q/p}}\int_0^1 \zeta^{q/p-1}\left(\sum_k \mu(I_k \times J)\right)^{q/p}d\zeta.$$

Next we estimate the integral in (16). The sum there does not exceed $\mu(\mathscr{O}'_\zeta \times J)$. Furthermore, since $\mathscr{O}(x, t, \lambda) \subseteq \{y \in J : NG(x, y) > \lambda\}$, it readily follows that

$$F(x, t, \lambda) \le \frac{\mu(x, \{y \in J : NG(x, y) > \lambda\})}{\mu(x, J)}.$$

Thus,

$$\mathscr{O}'_\zeta \subset \mathscr{U}(\lambda, \zeta) = \{x \in I : \mu(x, \{y \in J : NG(x, y) > \lambda\}) > \zeta\mu(x, J)\},$$

and the integral in (16) is bounded by

$$\int_0^1 \zeta^{q/p-1}\left(\int_J\int_I \chi_{\mathscr{U}(\lambda,\zeta)}(x)w(x, y)\,dx\,dy\right)^{q/p} d\zeta = \int_0^1 \zeta^{q/p-1} g(\zeta)^{q/p}\,d\zeta,$$

say. Moreover, since $g(\zeta)$ decreases with ζ, it is clear that the last integral above does not exceed

$$c\left(\int_0^1 g(\zeta)\,d\zeta\right)^{q/p} = c\left(\int_I\int_0^1 \chi_{\mathscr{U}(\lambda,\zeta)}(x)\mu(x, J)\,d\zeta\,dx\right)^{q/p}. \tag{17}$$

Setting $\zeta' = \zeta\mu(x, J)$, it readily follows that the innermost integral in (17) is bounded by $\mu(x, \{y \in J : NG(x, y) > \lambda\})$, and, consequently, the expression appearing in (17) does not exceed $c\mu(\{(x, y) \in P : NG(x, y) > \lambda\})^{q/p}$. Substituting this into (16) gives

$$R(\lambda) \le c|I|^{q\alpha/n}\nu(P)\left(\frac{\mu(\{(x, y) \in P : NG(x, y) > \lambda\})}{\mu(P)}\right)^{q/p},$$

which in turn implies that the integral B in (12) is less than or equal to

$$\begin{aligned} &c|I|^{q\alpha/n}|J|^{q\beta/m}\left(\frac{\nu(P)}{\mu(P)}\right)^{q/p}\int_0^1 \lambda^{q-1}\mu(\{(x, y) \in P : NG(x, y) > \lambda\})^{q/p}\,d\lambda \\ &\quad\le c|I|^{q\alpha/n}|J|^{q\beta/m}\nu(P)\left(\frac{1}{\mu(P)}\int\int_P NG(x, y)^p\,d\mu(x, y)\right)^{q/p}. \end{aligned} \tag{18}$$

Finally we are ready to estimate A_1. Let $0 < \varepsilon = \alpha/(1-\gamma)$, $\delta = \beta/\gamma < 1$, and observe that by Hölder's inequality the integral in (9) is bounded by

$$\begin{aligned} &\int\int_P \left(\int_0^{|J|^{1/m}}\int_0^{|I|^{1/n}} t^{(1-\gamma)(1-\varepsilon)q'-1}s^{\gamma(1-\delta)q'-1}\,dt\,ds\right)^{q/q'} \\ &\qquad\times\left(\int_0^{|J|^{1/m}}\int_0^{|I|^{1/n}} t^{(1-\gamma)\varepsilon q-1}s^{\gamma\delta q-1}G(x, t, y, s)^q\,dt\,ds\right)d\nu(x, y) \\ &\quad= c|I|^{q(1-\gamma-\alpha)/n}|J|^{q(\gamma-\beta)/m} \\ &\qquad\times\int\int_P\int_0^{|J|^{1/m}}\int_0^{|I|^{1/n}} t^{q\alpha-1}s^{q\beta-1}G(x, t, y, s)^q\,dt\,ds\,d\nu(x, y). \end{aligned}$$

Now, using estimate (18) for the above integral, be well-known properties of $A_p(R^n \times R^m)$ weights, it follows at once that

$$A_1 \le c|I|^{1/n}\left(\frac{1}{\mu(P)}\int\int_P NG(x, y)^p\, d\mu(x, y)\right)^{1/p}$$
$$\le c|I|^{1/n}\left(\frac{1}{\mu(P)}\int\int_{R^n\times R^m} \chi_P(x, y)|\nabla_1 f(x, y)|^p\, d\mu(x, y)\right)^{1/p},$$

which is a bound of the right order.

To handle A_2, let

$$H(x, t) = \int_J\int_I |\nabla_1 f(u, y)|\psi_t(x-u)\, du\, dy.$$

Clearly

$$(19)\quad A_2 \le c\left(\frac{|I|^{1/n}}{|I|^{\alpha/n}|J|}\right)\left(\frac{1}{\nu(P)}\int_0^{|I|^{1/n}}\int\int_P t^{q\alpha-1}H(x, t)^q\, d\nu(x, y)\, dt\right)^{1/q}.$$

In order to estimate the integral in (19), let $C(I) = \{(u, t) : u \in I,\ 0 < t < |I|^{1/n}\}$, define

$$NH(x) = \sup_{|x-u|<t} \chi_{C(I)}(u, t)H(x, t),$$

and put

$$\mathscr{V}(\lambda) = \{(u, t) \in C(I) : H(u, t) > \lambda\}.$$

If $d\nu_q(x, y, t) = v(x, y)t^{q\alpha-1}dx\,dy\,dt$ then the integral in (19) is dominated by $q\int_0^\infty \lambda^{q-1}\nu_q(\mathscr{V}(\lambda)\times J)d\lambda$, and, consequently, by a familiar argument we also have

$$A_2 \le c|I|^{1/n}|J|^{1/m}\left(\frac{1}{\mu(P)}\int\int_P\left(\int_J|\nabla_1 f(x, z)|\, dz\right)^p d\mu(x, y)\right)^{1/p}.$$

Now, since $w \in A_p(R^n \times R^m)$, this bound is also of the right order. A_3 is treated in an analogous fashion, and the proof is complete. □

Proof of Theorems 2 *and* 3. The proof of these results is similar to that of Theorem 1. In fact, if f is defined on P and vanishes on some subset E of P with $|E| > \eta|P|$, then for $(x, y) \in P$,

$$|f(x, y)| \le |f(x, y) - f_P| + \frac{1}{|E|}\int\int_E |f(u, z) - f_P|\, du\, dz$$
$$\le |f(x, y) - f_P| + \frac{1}{\eta|P|}\int\int_P |f(u, z) - f_P|\, du\, dz.$$

Whence, by Corollary 1, for $(x, y) \in P$, $|f(x, y)|$ is also dominated by the right-hand side in estimate (7), and Theorem 3 has been proved.

If, on the other hand, f is compactly supported and its support is contained in P, then we may extend f to be 0 off $2P$, say, and establish Theorem 2 from Theorem 3. □

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