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SOME CONSEQUENCES OF HARISH-CHANDRA'S SUBMERSION PRINCIPLE

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ABSTRACT. Let G be a reductive $\mathfrak{p}$-adic group, K a good maximal compact subgroup, $K_1 \subset K$ any open subgroup, and π an admissible representation of G of finite type. In *A submersion principle and its applications*, Harish-Chandra proves the theorem that $\int_K \pi(kgk^{-1})\,dk$ is a finite-rank operator for g in the regular set G' in order to show that the character $\Theta_\pi(g)$ is a locally constant class function on G'. From this, the authors derive the formula $\theta(1)\Theta_\pi(g) = d(\pi)\int_{G/Z}\int_{K_1}\theta(xkgk^{-1}x^{-1})\,dk\,d\dot{x} \quad (g \in G')$ for any K-finite matrix coefficient θ of a discrete series representation π with formal degree $d(\pi)$. They use another technical result of the paper to prove that invariant integrals of Schwartz space functions converge absolutely. None of these results depends upon a characteristic zero assumption.

1. INTRODUCTION

Let $\mathbb{F}$ be a commutative p-field, G the group of $\mathbb{F}$-points of a connected reductive $\mathbb{F}$-group $\mathbf{G}$, and G' the set of regular (semisimple) points of G.

There is a well-known integral formula, proved originally by Harish-Chandra [4, pp. 60, 94] and rederived and used by Kutzko [6] (cf. also [7, Theorems 1.7 and 1.9]), which allows one in principle to compute the values of the character of a supercuspidal representation of G on G', either by integrating a matrix coefficient of the representation or the character of a K-type which induces the representation. One purpose of this paper is to point out that this integral formula is actually valid for any discrete series representation of G and, in this context, to give a new and simple proof of the integral formula based on Harish-Chandra's elegant paper [3].

In order not to obtain an unnecessarily restrictive special case of the integral formula for discrete series, the authors found it necessary to sharpen the statement of [3, Theorem 2]. The restatement appears here in §2 as Theorem 1; the modification to Harish-Chandra's argument is given in §4. Theorem 2 of §2 presents the integral formula for the character of a discrete series representation; the derivation of this formula from Theorem 1 is also given in §2.

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Although he does not present the details, Harish-Chandra mentions that [3, Theorem 3] can be used to give a characteristic-free proof of the absolute convergence of invariant integrals for Schwartz space functions. In §3, Theorem 4, we use [3, Theorem 4] to prove this absolute convergence. Our proof, while reminiscent of the argument presented in [9, p. 244] for real reductive Lie groups, simplifies Wallach's approach through the use of the "numerical function" of Geometric Invariant Theory from Kempf [5] and Mumford. In Corollary 5 we use Theorem 4 to express the discrete series characters on elliptic Cartan subgroups as invariant integrals of their matrix coefficients. It is interesting that the integral formula from Theorem 2 has support on all Cartan subgroups, whereas the invariant integrals vanish on the Cartan subgroups which are not elliptic. Related work also appears in Clozel [1].

2. The integral formula for the character of a discrete series representation

We use the notational conventions of [2, 3, 8]. In particular, for $\mathbf{X}$ an $\mathbb{F}$-group, we write X to denote its corresponding group of $\mathbb{F}$-points.

Let Z denote the split component of G. Let (P, A) $(P = MN)$ denote a minimal p-pair of G and K an A-special maximal compact subgroup of G. In the following K_1 denotes an arbitrary open subgroup of K. Fix Haar measures dg on G and $d\dot{g}$ on G/Z such that $\int_{K_1} dg = \int_{K_1/Z\cap K_1} d\dot{g} = 1$.

Let π be an admissible representation of G acting in a complex vector space V. Let $C_c^\infty(G)$ denote the convolution algebra of compactly supported locally constant functions on G. For any $f \in C_c^\infty(G)$

$$\pi(f) = \int_G f(g)\,\pi(g)\,dg$$

is an operator of finite rank acting in V. The mapping

$$f \mapsto \Theta_\pi(f) = \operatorname{tr}(\pi(f)) \qquad (f \in C_c^\infty(G))$$

is the (distributional) character of π. Harish-Chandra has proved in [3] that if V is a module of finite type under $C_c^\infty(G)$, then there is a locally constant function $\Theta_\pi(g)$ defined for all $g \in G'$ such that

$$\Theta_\pi(f) = \int_{G'} f(g)\,\Theta_\pi(g)\,dg$$

for all $f \in C_c^\infty(G)$ with support in G'. His proof depends upon the following assertion, proved in [3] only for the case $K_1 = K$:

Theorem 1 (Harish-Chandra). *Assume that π is an admissible representation of G in a complex vector space V and that V is a left $C_c^\infty(G)$-module of finite type under π. Then*

$$g \mapsto \int_{K_1} \pi(kgk^{-1})\,dk \qquad (g \in G')$$

is a locally constant function with range in the space of finite rank operators on V.

As we shall need Theorem 1 for arbitrary K_1, we shall indicate the modification in Harish-Chandra's argument needed to prove the more general version in §4.

For the remainder of §2 assume Theorem 1, as stated. Let π now be an irreducible, admissible discrete series representation which is unitary on the pre-Hilbert space V. Write $\langle u, v\rangle$ for the inner product of $u, v \in V$. Let $\mathscr{A}(\pi)$ denote the vector space spanned by functions of the form

$$x \mapsto \langle \pi(x)u, v\rangle \qquad (x \in G;\ u, v \in V).$$

With respect to the fixed Haar measure $d\dot{x}$ on G/Z the formal degree $d(\pi)$ of π is defined such that

$$d(\pi)^{-1}\langle u_1, u_2\rangle \operatorname{conj}\langle v_1, v_2\rangle = \int_{G/Z} \langle \pi(x)u_1, v_1\rangle \operatorname{conj}\langle \pi(x)u_2, v_2\rangle\, d\dot{x}.$$

Theorem 2. *Let $\theta(x) \in \mathscr{A}(\pi)$ and let $g \in G'$. Then*

$$\theta(1)\Theta_\pi(g) = d(\pi) \int_{G/Z} \int_{K_1} \theta(xkgk^{-1}x^{-1})\, dk\, d\dot{x}.$$

Remark. Since $\int_{G/Z\times K_1} |\theta(xkgk^{-1}x^{-1})|\, d\dot{x} \times dk$ does not exist, in general, it is not possible to use the right invariance of the Haar measure $d\dot{x}$ to absorb the integration over K_1 into the integration over G/Z. Indeed, without the integration over K_1, the integrand would be constant on cosets of the centralizer of g; the centralizer of g not being compact (when g is not an elliptic element), the integral would diverge trivially.

Proof. Since the operator

$$T_g = \int_{K_1} \pi(kgk^{-1})\, dk$$

has finite rank, there exists an open normal subgroup $\widetilde{K} \subset K_1$ such that

$$\pi(k_1)\, T_g\, \pi(k_2) = T_g$$

for all $k_1, k_2 \in \widetilde{K}$. Let V_2 denote the subspace consisting of all $\widetilde{K}$-fixed vectors in V. By the choice of $\widetilde{K}$ we may assume $\dim(V_2) > 0$. Choose an orthonormal basis $\{e_i\}$ for V such that $e_1, \ldots, e_N$ is an orthonormal basis for V_2. Without loss of generality we assume that $\theta(g) = \langle \pi(g)u, v\rangle$ for some $u, v \in V$. Then

$$\begin{aligned}\int_{K_1} \theta(xkgk^{-1}x^{-1})\, dk &= \int_{K_1} \langle \pi(kgk^{-1})\pi(x^{-1})u,\ \pi(x^{-1})v\rangle dk \\ &= \sum_{i,j=1}^{N} \langle T_g e_i, e_j\rangle\, \langle \pi(x^{-1})u, e_i\rangle \operatorname{conj}\langle \pi(x^{-1})v, e_j\rangle \\ &= \sum_{i,j=1}^{N} \langle T_g e_i, e_j\rangle\, \langle \pi(x)e_j, v\rangle \operatorname{conj}\langle \pi(x)e_i, u\rangle.\end{aligned}$$

Since, for the discrete series representation π,

$$\int_{G/Z} \langle \pi(x)e_j, v\rangle \operatorname{conj}\langle \pi(x)e_i, u\rangle\, d\dot{x} = d(\pi)^{-1}\langle e_j, e_i\rangle\, \langle u, v\rangle = d(\pi)^{-1}\delta_{ij}\, \theta(1)$$

and since we can interchange $\int_{G/Z}$ and the finite summation $\sum_{i,j=1}^{N}$, we obtain

$$d(\pi)\int_{G/Z}\int_{K_1}\theta(xkgk^{-1}x^{-1})\,dk\,d\dot{x} = d(\pi)\sum_{i,j=1}^{N}\langle T_g e_i, e_j\rangle d(\pi)^{-1}\delta_{ij}\,\theta(1)$$
$$= \operatorname{tr}(T_g)\theta(1).$$

Finally, if $f \in C_c^\infty(G')$, then

$$\Theta_\pi(f) = \operatorname{tr}\left(\int_{G'} f(g)\,\pi(g)\,dg\right) = \operatorname{tr}\left(\int_{G'}\int_{K_1} f(k^{-1}gk)\,dk\;\pi(g)\,dg\right)$$
$$= \operatorname{tr}\left(\int_{G'} f(g)\int_{K_1}\pi(kgk^{-1})\,dk\,dg\right) = \int_{G'} f(g)\operatorname{tr}(T_g)\,dg.$$

This concludes the proof of Theorem 2.

3. On the invariant integrals of Schwartz functions

In this section we use [3] to construct a proof of the convergence of invariant integrals for Schwartz functions. Let

$$\varphi\colon G \to \mathrm{GL}_n(\mathbb{F})$$

be an irreducible faithful rational representation of G on $V = \mathbb{F}^n$, defined over $\mathbb{F}$. For $T \in \mathscr{M}_n(\mathbb{F})$ (the space of n by n matrices over $\mathbb{F}$) define

$$\|T\| = \max_{i,j}|T_{ij}|,$$

and for $x \in G$ define

$$\|x\| = \inf_{z\in Z}\max(\|\varphi(xz)\|,\ \|\varphi(xz)^{-1}\|).$$

Define the relations $\prec$ and $\asymp$ as in [8, p. 149]. In [3, p. 101] Harish-Chandra defines

$$\beta(g) = \sup_x\,(f_{\alpha_1,g}(x)) \qquad (g \in G',\ \ K_1 = K).$$

Let Γ be an elliptic Cartan subgroup of G, let $\Gamma' = \Gamma \cap G'$ denote the set of regular elements in Γ, and let Ξ denote the spherical function used in Harish-Chandra's definition of the Schwartz space for G ([2, §14] or [8, §4.2]).

Lemma 3. *Let ω be a compact subset of Γ'. Then there are positive constants c and r such that, for any $g \in \omega$,*

$$\int_K \Xi(m^{-1}k^{-1}gkm)\,dk \le c\beta(g)\,\Xi(m)^2 \tag{1}$$

and

$$\|m^{-1}k^{-1}gkm\| \ge c^{-1}\|m\|^r \tag{2}$$

for all $m \in M$.

Proof. A stronger version of (1) is proved in [3, p. 101] (where $\omega \subset \Gamma'$ only has to be precompact in Γ and Γ need not be elliptic). (Harish-Chandra ostensibly proves the assertion of (1) only for $m \in M^+$, but one can use the K-invariance of Ξ to obtain his assertion for any $m \in M$.) We prove only (2), using an idea from [5]. The assertion (2) factors through G/Z, so for the rest of the proof we assume that $Z = \{1\}$. Set

$$\Omega = \{k^{-1}gk : \ k \in K \text{ and } g \in \omega \cup \omega^{-1}\},$$

a compact set of elliptic regular points of G. Let Λ be the set of weights of φ for A, the split component of M, and let E_λ $(\lambda \in \Lambda)$ be the corresponding projection onto the λ-eigenspace V_λ. Without loss of generality we assume that for $\gamma \in \Omega$ and $m \in M$,

$$\begin{aligned}\|\varphi(m^{-1}\gamma m)\| &= \max\{\|E_\lambda\, \varphi(m^{-1}\gamma m)\, E_\nu\| : \ \lambda, \nu \in \Lambda\} \\ &\asymp \max\{q^{\langle \nu-\lambda,\ H(m)\rangle}\, \|E_\lambda\, \varphi(\gamma)\, E_\nu\| : \ \lambda, \nu \in \Lambda\}.\end{aligned}$$

For $m \in M$ with $m \notin {}^0M$ [8, p. 8] we have the flag in V,

$$F_\lambda = \bigoplus\{V_\nu : \ \langle \nu, H(m)\rangle \leq \langle \lambda, H(m)\rangle\} \qquad (\nu, \lambda \in \Lambda).$$

Let $P_m = M_m N_m$ be the proper parabolic subgroup in G which stabilizes this flag, so $\varphi(M_m)$ consists of block diagonal matrices and $\varphi(N_m)$ of block upper triangular matrices with respect to F_λ. Thus

$$P_m = \{x \in G : \ E_\lambda\, \varphi(x)\, E_\nu = 0 \text{ if } \langle \nu, H(m)\rangle \geq \langle \lambda, H(m)\rangle\}.$$

Define

$$l(m, \gamma) = \max\{\langle \nu - \lambda, H(m)\rangle : \ \|E_\lambda\, \varphi(\gamma)\, E_\nu\| \neq 0\}$$

for $m \in M$ with $m \notin {}^0M$ and $\gamma \in \Omega$ (cf. [5, p. 306]). Clearly, if $l(m, \gamma) \leq 0$, then $\gamma \in P_m$ (cf. [5, p. 305]); conjugating by N_m, we may assume that $\gamma \in M_m$. It follows that

$$\Gamma = \operatorname{cent}(\gamma)^0 \supset A_m = \text{ split center of } M_m,$$

which is impossible since Γ is elliptic. Set

$$L(m, \gamma) = \max\{\|E_\lambda\, \varphi(\gamma)\, E_\nu\| : \ \langle \nu - \lambda, H(m)\rangle = l(m, \gamma)\}.$$

Then

$$\|\varphi(m^{-1}\gamma m)\| \geq L(m, \gamma)\, q^{l(m, \gamma)} \qquad (\gamma \in \Omega,\ m \in M).$$

Next note that L and l are strictly positive (for $m \notin {}^0M$) locally constant functions. As γ varies over the compact set Ω, the set of values assumed by $L(m, \gamma)$ is a finite set of positive numbers; let L be the smallest of these. Since the $\lambda - \nu$ are integral linear combinations of roots, it is clear that l extends to the real Lie algebra,

$$l : \ \mathfrak{a}_{\mathbb{R}} \times \Omega \to \mathbb{R}_+$$

as a continuous function which is convex and positively homogeneous of degree one on $\mathfrak{a}_{\mathbb{R}}$ and locally constant on Ω.

On the other hand, if $H(a)$ lies in a closed positive Weyl chamber $\mathfrak{a}^+$ and λ is the highest weight for φ relative to $\mathfrak{a}^+$, then $\|a\| = q^{\langle \lambda, H(a)\rangle}$ relative to a basis of eigenvectors for A, since the other weights are of the form $\lambda - \sum m_\alpha \alpha$ with integers $m_\alpha \geq 0$ and $\alpha \geq 0$ on A^+. Thus

$$a \mapsto \sigma(a) = \max\{\langle \lambda, H(a)\rangle : \ \lambda \text{ is an extreme weight of } \varphi\}$$

extends to a continuous, convex function on $\mathfrak{a}_{\mathbb{R}}$ which is positively homogeneous of degree one and strictly positive away from zero (strictly positive because φ is faithful). Let $\mathscr{S}$ be the unit sphere in $\mathfrak{a}_{\mathbb{R}}$. Then

$$r = \inf\{l(H, \gamma)/\sigma(H) : (H, \gamma) \in \mathscr{S} \times \Omega\}$$

is positive, and

$$\|\varphi(m^{-1}\gamma m)\| \geq L\, q^{l(m, \gamma)} \geq L\, q^{r\sigma(H(m))} \succ L\|m\|^r$$

for $\gamma \in \Omega$. But also $\gamma^{-1} \in \Omega$, so $\|m^{-1}\gamma m\| \succ \|m\|^r$. This completes the proof of the lemma.

Recall that the Schwartz space $\mathscr{C}(G)$ is the space of functions f on G such that $f \in C(G//K_0)$, the space of K_0-bi-invariant functions, for some compact open subgroup $K_0 \subset G$, and that

$$|f|_N = \sup_{x \in G} |f(x)|\, \Xi(x)^{-1}\, (1 + \log\|x\|\,)^N$$

is finite for each $N \in \mathbb{N}$. If Γ is a Cartan subgroup of G, A_Γ is its split component, and $f \in \mathscr{C}(G)$, then

$$F_f(g) = |D(g)|^{1/2} \int_{G/A_\Gamma} f(xgx^{-1})\, dx^* \qquad (g \in \Gamma')$$

is called the invariant integral. Here $D(g)$ is the lowest coefficient in the characteristic polynomial of $\mathrm{Ad}(g) - 1$ and dx^* is the invariant measure on G/A_Γ.

Theorem 4. *There exists an integer N with the following property. For any compact set $\omega \subset \Gamma'$ there is a constant $C > 0$ such that for all $f \in \mathscr{C}(G)$*

$$|F_f(g)| \leq C|f|_N$$

for all $g \in \omega$. Moreover, F_f is locally constant on Γ' (for every $f \in \mathscr{C}(G)$), and for a fixed compact open subgroup $K_0 \subset G$, the space of restrictions

$$\{F_f|\omega : f \in \mathscr{C}(G//K_0)\}$$

is a finite-dimensional vector space.

Proof. If Γ is not elliptic, then choose a parabolic subgroup $P = MN$ so that $A = A_\Gamma$ is the split component of M, and $\Gamma \subset M$ is elliptic. Then we have the continuous map

$$f \mapsto f^P : \mathscr{C}(G) \to \mathscr{C}(M), \qquad |f^P|_{M, n} \leq C_n |f|_{G, n+d_A}$$

(for some integer d_A and all integers n [8, p. 176]) which satisfies

$$F_f^{G/\Gamma}(g) = F_{\overline{f}^P}^{M/\Gamma}(g) \qquad (g \in \Gamma')$$

(where $\overline{f}(x) = \int_K f(kxk^{-1})\, dk$) [4, p. 58].

This reduces the proof to the case of an elliptic Cartan subgroup Γ. We use the Cartan integration formula [8, p. 149]. For $g \in \omega$ (and letting μ denote our normalized Haar measure)

$$
\begin{aligned}
&|D(g)|^{-1/2}|F_f^{\Gamma}(g)| \\
&\quad = \left| \int_{K\times M^+ \times K} f(l^{-1}m^{-1}k^{-1}gkml)\ \mu(KmK)\ dk\,dm\,dl \right| \\
&\quad \prec \int_{M^+\times K} \Xi(m^{-1}k^{-1}gkm)(1+\log\|m^{-1}k^{-1}gkm\|)^{-n}\mu(KmK)dkdm \\
&\quad \prec \int_{M^+} (1+r\log\|m\|)^{-n}\ \delta_{P_0}(m)\ \int_K \Xi(m^{-1}k^{-1}gkm)\,dk\,dm \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Lemma 3 and [8, Lemma 4.1.1])} \\
&\quad \prec \beta(g)\int_{M^+} (1+r\log\|m\|)^{-n}\ \delta_{P_0}(m)\ \Xi(m)^2\,dm \quad \text{(Lemma 3)} \\
&\quad \prec \beta(g) \sum_{M^+/{}^0M} (1+r\log\|m\|)^{-n+2r_0} \quad \text{[8, p. 154]} \\
&\quad \prec \beta(g) \quad \text{[8, p. 150].}
\end{aligned}
$$

This implies the first sentence of the theorem. The rest comes directly from [3, Theorem 3] and the fact that $C_c(G//K_0)$ is dense in $\mathscr{C}(G//K_0)$.

As a corollary to the last result, note that if Γ is elliptic (so $A_\Gamma = Z$), the integral in Theorem 2 is absolutely convergent, since θ is a matrix entry of a discrete series representation, so lies in $\mathscr{C}(G)$. Thus we can reverse the order of integration and absorb the integration over K into the Haar invariance of $d\dot{x}$. Moreover, since θ is a cusp form, we obtain

Corollary 5. *Under the assumptions of Theorem 2, if $g \in G'$ is regular elliptic, then*

$$d(\pi) \int_{G/Z} \theta(xgx^{-1})\ d\dot{x} = \theta(1)\,\Theta_\pi(g),$$

and if g is regular but not elliptic and Γ is the centralizer of g, then

$$\int_{G/A_\Gamma} \theta(xgx^{-1})\ d\dot{x} = 0$$

(*cf.* [1, *p.* 9]).

4. The proof of Theorem 1

We use notation like [3, p. 98], and where notation or terminology is not explained we have used Harish-Chandra's. (To read this proof it will be necessary to have Harish-Chandra's article close at hand.)

For simplicity assume that K_1 is a normal subgroup of K. There is no loss of generality in this assumption inasmuch as every open subgroup of G contains an open normal subgroup of K. Moreover, if Theorem 1 is true for an open subgroup of K_1, then it is obviously true for K_1, too. Let $K_1, \ldots, K_n$ be the cosets of K_1 in K, and for $1 \le i \le n$ define

$$T_{g,i} = \int_{K_i} \pi(kgk^{-1})\,dk, \quad T_g = T_{g,1} \qquad (g \in G').$$

For every $k \in K$ there exists $i = i(k)$ such that

$$T_g \pi(k) = \pi(k) T_{g,i} \qquad (g \in G', \ k \in K).$$

Let P_0 be an open compact subgroup of P chosen as at the top of [3, p. 99]. Let $\alpha_i \in C_c^\infty(G \times P)$ be the characteristic function of $K_i \times P_0$ for each i. Harish-Chandra's "submersion principle" applied to the submersion of $G \times P \to G$ defined by

$$(x, p) \mapsto xgx^{-1}p \quad (\text{fixed } g \in G')$$

implies the existence of functions $f_{\alpha_i, g} \in C_c^\infty(G)$ such that

$$\int_{K_i \times P_0} F(kgk^{-1}p)\, dk\, d_l p = \int_{G \times P} \alpha_i(x, p) F(xgx^{-1}p)\, dx\, d_l p$$
$$= \int_G f_{\alpha_i, g}(y) F(y)\, dy$$

for any locally integrable function F on G ($d_l p$ denotes a left Haar measure on P). Let V_0 denote a finite-dimensional subspace of V such that $\pi(C_c^\infty(G))V_0 = V$ and such that V_0 is the space of all K_0-fixed vectors for some open normal subgroup K_0 of K. Then for $F = \pi$ and on the vector space spanning set for V

$$\{\pi(km)v \mid k \in K, \ m \in M^+, \ v \in V_0\}$$

we obtain the relation

$$T_g \pi(km)v = \pi(k) T_{g,i} \int_{P_0} \pi(p)\, d_l p\, \pi(m)v = \pi(k)\pi(f_{\alpha_i, g})\pi(m)v,$$

which implies that T_g is an operator of finite rank. In fact, choosing an open normal subgroup $\widetilde{K} \subset K$ such that $f_{\alpha_i, g} \in C_c^\infty(G//\widetilde{K})$ for each $i = 1, \dots, n$ we have

$$\pi(E_{\widetilde{K}}) T_g \pi(km)v = \pi(k)\pi(E_{\widetilde{K}} f_{\alpha_i, g})\pi(m)v = T_g \pi(km)v,$$

where $E_{\widetilde{K}}$ is the identity element in the convolution algebra $C_c^\infty(G//\widetilde{K})$. Assuming that $\widetilde{K} \subset K_1$ and using the fact that T_g commutes with $\pi(K_1)$, we conclude that, since

$$\pi(E_{\widetilde{K}}) T_g = T_g \pi(E_{\widetilde{K}}) = T_g,$$

T_g is of finite rank. (Harish-Chandra also shows that $(g, x) \mapsto f_{\alpha_i, g}(x)$ lies in $C^\infty(G' \times G)$ and is compactly supported in x. This implies that $T_{g,i}$ is locally constant in g.)

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