CLOSURES OF WEAKENED ANALYTIC GROUPS

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ABSTRACT. Let $(G, \mathcal{G})$ be a topological group with dense subgroup L , and suppose that L is an analytic group in a topology τ that is stronger than the topology that L inherits from $\mathcal{G}$. It is known that L contains a τ -closed abelian subgroup H that completely determines the topology of L . We now prove that the $\mathcal{G}$ -closure $\overline{H}$ of H similarly determines the topology of G . $(G, \mathcal{G})$ always has a left-completion in the category of topological groups, and the properties of $\overline{H}$ determine whether $(G, \mathcal{G})$ is locally compact, analytic, metrizable, left-complete, or finite dimensional. We discuss the relationship between these results and recent work of Goto.

1. INTRODUCTION

Analytic groups with the “wrong” topology—that is, with a topology weaker than the usual one—arise in several contexts, including the study of immersions of analytic groups, and have been investigated by Goto [4] and by the author [9, 10]. In this paper we extend our previous work by examining topological groups G in which a “weakened analytic” group L is dense and showing that the topology of such a group is completely determined by the closure of a certain abelian subgroup of L . This subgroup then determines whether G is locally compact, metrizable, analytic, complete, or finite dimensional. Moreover, G always has a left-completion in the category of topological groups. These theorems, which resemble but extend beyond recent results of Goto [4], are stated precisely in §3. The first step in the proof is the construction in §4 of a specific completion of L , which is then shown to be a completion of G as well. In §5 we analyze the topology of G by studying how it “sits” in this completion. Section 6 discusses in detail the relationship between Goto’s work and ours, and in §7 we close with a conjecture that affects the scope of our results.

2. NOTATION AND DEFINITIONS

Topologies for abstract groups are assumed to be Hausdorff and to make the group operations continuous. If $(G, \mathcal{G})$ is a topological group and A is a subgroup of G , then $\mathcal{G}_A$ denotes the topology that A inherits from G . If (L, τ) is an analytic (i.e., connected Lie) group and $\mathcal{U}$ is a topology for L

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that is weaker than τ , then $(L, \mathcal{U})$ is a *weakened analytic* (WA) group. When different topologies for the same abstract group are under simultaneous consideration, topological statements about that group will mention the particular topology involved.

The topological group $(G, \mathcal{G})$ is *complete* if it is complete in its left uniform structure $\mathcal{L}(\mathcal{G})$; in that case, of course, G is also complete in its right and two-sided uniform structures. If G , with its left uniform structure, has a completion in the category of topological groups, we say simply that $(G, \mathcal{G})$ *has a completion*.

Suppose the abstract group G contains subgroups A and B such that $G = AB$ and A is normal in G . We now describe a technique that generalizes slightly a notion introduced in [10, §3] and enables us to combine topologies for A and B into a topology for G . Let $I(b)$ denote conjugation by an element b of B , and let $\mathcal{A}$ and $\mathcal{B}$ be topologies for A and B , respectively, that make the evaluation map $f: A \times B \rightarrow A$, defined by $f(a, b) = bab^{-1}$, $(\mathcal{A} \times \mathcal{B}, \mathcal{A})$ -continuous. Then $\mathcal{A} \times \mathcal{B}$ is a group topology for the semidirect product $A \rtimes B$, and we can define a surjective homomorphism $\alpha: A \rtimes B \rightarrow G$ by $\alpha(a, b) = ab$. If $A \cap B$ is $\mathcal{A}$ -closed and inherits a weaker topology from $\mathcal{B}$ than from $\mathcal{A}$, then the unique topology for G that makes α open and continuous is a (Hausdorff) group topology for G , which is called the *standard extension* of $\mathcal{A}$ and $\mathcal{B}$ and is denoted by $\mathcal{E}(\mathcal{A}, \mathcal{B})$.

3. MAIN RESULTS

To facilitate the statement of our principal theorems, we first summarize for easy reference the properties of (WA) groups that will be used in this paper.

Theorem 3.1 [9, Theorem 3.1]. *Let $(L, \mathcal{U})$ be a (WA) group, with τ the analytic topology for L and $\mathfrak{l}$ its Lie algebra. Then L contains a τ -closed abelian subgroup H , which depends on (L, τ) but not on $\mathcal{U}$, with the following properties:*

(i) *The function $I: H \rightarrow \text{Aut}(L)$, which assigns to each $x \in H$ the inner automorphism of L by x , is $\mathcal{U}_H$ -continuous if $\text{Aut}(L)$ is given the generalized compact-open topology.*

(ii) $\mathcal{U} = \mathcal{E}(\tau, \mathcal{U}_H)$.

(iii) *If (L, τ) is (CA)—that is, if the adjoint image of L is closed in the general linear group $\text{GL}(\mathfrak{l})$ —then H is the center $Z(L)$ of L .*

(iv) *If (L, τ) is not (CA), then $H = V \times Z(L)$, where V is a vector group.*

H is called a *decisive* subgroup of (L, τ) . The existence of decisive subgroups is proved in [10] and their structure is further investigated in [9].

Throughout the remainder of this paper, $(G, \mathcal{G})$ will denote a topological group in which a (WA) group $(L, \mathcal{G}_L)$ is dense. Let τ be the analytic topology for L and H the abelian decisive subgroup of Theorem 3.1. Our principal result, detailed in the following two theorems, is that the topology $\mathcal{G}$ of G is completely determined by the $\mathcal{G}$ -closure $\overline{H}$ of H in G .

Theorem 3.2. *Let the notation be as in the preceding paragraph. Then*

(i) $(G, \mathcal{G})$ *has a completion in the category of topological groups.*

(ii) L *is normal in G , $G = L\overline{H}$, $L \cap \overline{H} = H$, and $\mathcal{G} = \mathcal{E}(\tau, \mathcal{G}_{\overline{H}})$.*

(iii) $(G, \mathcal{G})$ *is topologically isomorphic with $(L \rtimes \overline{H})/\{(h, h^{-1}): h \in H\}$,*

where the latter has the quotient topology from $\tau \times \mathcal{G}_{\overline{H}}$ and $\overline{H}$ acts on L by inner automorphism.

(iv) If (L, τ) is (CA), then $\overline{H}$ is the center of G .

Theorem 3.3. (i) $(G, \mathcal{G})$ is locally compact, metrizable, complete, or analytic $\Leftrightarrow (\overline{H}, \mathcal{G}_{\overline{H}})$ is, respectively, locally compact, metrizable, complete, or Lie.

(ii) If $(G, \mathcal{G})$ is metrizable, then $(G, \mathcal{G})$ has finite topological dimension $\Leftrightarrow (\overline{H}, \mathcal{G}_{\overline{H}})$ does.

(iii) L is the $\mathcal{G}$ -arc component of e in $G \Leftrightarrow H$ contains the $\mathcal{G}$ -arc component of e in $\overline{H}$.

Before proving these theorems, we supplement our introduction with the following remarks about their motivation and context. First we observe that, in the case where $G = L$, both theorems tell us a good deal about the topology of L . Second, we point out that Theorem 3.3(iii) can be useful in studying the locally arcwise-connected group associated with $(G, \mathcal{G})$ or some larger group. Third, in response to a suggestion from the referee, we note that, in the analysis of the topology of $\overline{H}$, the closure $\overline{Z(L)}$ of $Z(L)$ may offer a greater challenge than the closure $\overline{V}$ of V . Since the adjoint representation maps V injectively into a torus, the choices for the topology $\mathcal{G}_{\overline{V}}$ are limited by the fact that it must be weaker than the usual topology but strong enough to make the mapping of $\overline{V}$ into the torus $\overline{\text{Ad}(V)}$ continuous. On the other hand, the fact that the adjoint representation is trivial on $\overline{Z(L)}$ implies that it can have any (Hausdorff) topology that gives $Z(L)$ a topology weaker than the usual one. Since it is possible, however, for $\overline{H}$ properly to contain $\overline{V \cdot Z(L)}$, the analysis of $\overline{H}$ does not reduce simply to the study of the topologies inherited by $\overline{V}$ and $\overline{Z(L)}$. An example where this complication occurs may be found in §6. Finally, we note that Theorem 3.2(iv) duplicates [4, 4.1(i)] and that portions of Theorems 3.2(ii) and 3.3(i) are similar but not identical to theorems in [4]. This relationship will be more fully explored in §6.

4. COMPLETING A (WA) GROUP

In this section we use the abelian decisive subgroup H to construct a completion of the (WA) group $(L, \mathcal{G}_L)$ and also to prove an important lemma about the completion's topology. We begin by noting that $(H, \mathcal{G}_H)$, as an abelian group, does have a completion $(K, \mathcal{K})$ in the category of topological groups and that $I: H \rightarrow \text{Aut}(L)$, which by Theorem 3.1 is $\mathcal{G}_H$ -continuous, can therefore be extended to a $\mathcal{K}$ -continuous homomorphism $\overline{I}: K \rightarrow \text{Aut}(L)$. Thus the semidirect product $(L \otimes K, \tau \times \mathcal{K})$ is a topological group, and the fact that $\mathcal{K}_H = \mathcal{G}_H$ is weaker than τ_H implies that $B = \{(h, h^{-1}): h \in H\}$ is a closed subgroup. Since H is abelian, it follows that K is abelian and each element of $\overline{I}(K)$ leaves H pointwise fixed. These facts make it easy to verify that B is normal (in fact, central) in $L \otimes K$. Let $M = (L \otimes K)/B$, with $\mathcal{M}$ the quotient topology and $\pi: L \otimes K \rightarrow M$ the natural projection. The goal of this section is to establish the following theorem.

Theorem 4.1. $(M, \mathcal{M})$ is a completion of $(L, \mathcal{G}_L)$, and the injection $i: (L, \mathcal{G}_L) \rightarrow (M, \mathcal{M})$ is given by $i(x) = \pi(x, e)$ for $x \in L$.

The proof begins with the following lemma, which will also be useful in §5.

Lemma 4.2. *L contains a subset C with the following properties: (i) C is τ -homeomorphic with a cube in Euclidean space, (ii) the restriction of π to $(C \times K, \tau_C \times \mathcal{K})$ is a homeomorphism onto $\pi(C \times K)$, and (iii) $\pi(C \times K)$ is a complete neighborhood of e .*

Proof. As a closed subgroup of the analytic group (L, τ) , H has a local cross-section in L ; that is, L contains a subset C that is τ -homeomorphic with a cube and such that the natural mapping of (L, τ) onto the set of left cosets L/H is a homeomorphism of C onto a neighborhood of e in L/H . (See, e.g., [2, Proposition 1, p. 110].) Therefore CH is a τ -neighborhood of H and the mapping $\varphi: C \times H \rightarrow L$, defined by $\varphi(c, h) = ch$, is, with the relative topologies from τ , a homeomorphism of $C \times H$ onto CH . It follows easily that π is injective on $C \times K$, and the equality $\pi(C \times K) = \pi((C \times K)B) = \pi(CH \times K)$ implies that $\pi(C \times K)$ is an $\mathcal{M}$ -neighborhood of e .

To show that π is an open mapping of $(C \times K, \tau_C \times \mathcal{K})$ onto $\pi(C \times K)$, let $(c, k) \in C \times K$ and let $\{x_\alpha: \alpha \in A\}$ be a net in $\pi(C \times K)$ that converges to $\pi(c, k)$. Then there exist a subnet $\{x_{\alpha(j)}: j \in J\}$ of $\{x_\alpha\}$, a net $\{(c_j, k_j): j \in J\}$ in $C \times K$, and a net $\{(h_j, h_j^{-1}): j \in J\}$ in B such that $\pi(c_j, k_j) = x_{\alpha(j)}$ and $(c_j, k_j)(h_j, h_j^{-1}) = (c_j h_j, k_j h_j^{-1}) \rightarrow (c, k)$ in $\tau \times \mathcal{K}$. Since C is a local cross-section to H , $c_j \rightarrow c$ in τ , and $h_j \rightarrow e$ in τ and thus also in $\mathcal{G}_H = \mathcal{K}_H$. Therefore $k_j \rightarrow k$ in $\mathcal{K}$ and π maps $C \times K$ homeomorphically onto $\pi(C \times K)$. In particular, $\pi(e \times K)$ is topologically isomorphic to the complete group $(K, \mathcal{K})$ and, therefore, is complete. To finish the proof of the lemma we note that $\pi(C \times K)$ must, as the product of the compact set $\pi(C \times e)$ and the complete set $\pi(e \times K)$, be complete. $\square$

We may now prove Theorem 4.1. By Theorem 3.1(ii) and the definition of "standard extension," the mapping $\gamma: L \otimes H \rightarrow L$, defined by $\gamma(x, h) = xh$, is a $(\tau \times \mathcal{G}_H, \mathcal{G}_L)$ -continuous and open homomorphism with kernel B , and thus i is a homeomorphism of $(L, \mathcal{G}_L)$ onto the dense subgroup $(L \otimes H)/B$ of $(M, \mathcal{M})$. By [1, Proposition 4, p. 245] and Lemma 4.2, $(M, \mathcal{M})$ is complete and the theorem is proved. $\square$

5. PROOF OF MAIN THEOREMS

We begin with Theorem 3.2. Adopting the notation of Theorem 4.1, we deduce from Theorem 4.1 and the remark on [8, p. 187] that $(M, \mathcal{M})$ is a completion of $(G, \mathcal{G})$. Thus part (i) is proved, and, as we now show, (ii) follows easily from the identification of G with a dense subgroup of $(M, \mathcal{M})$. Since $i(L) = \pi(L \times H)$, G must have the form $\pi(L \times F)$, where F is a subgroup of K that contains H . The normality of L in G is then clear and the homeomorphism of $e \times F$ with $\pi(e \times F)$ that follows from Lemma 4.2 allows us to regard F as a subgroup of G , with $\mathcal{G}_F = \mathcal{K}_F$. Identifying L with $i(L)$, one can easily verify that $G = LF$ and $F \cap L = H$, and the fact that H is $\mathcal{K}$ -dense in K implies that $F = \overline{H}$. If we define a mapping $\beta: L \otimes \overline{H} \rightarrow G$ by $\beta(x, f) = xf$, then β is a surjective, $(\tau \times \mathcal{G}_{\overline{H}}, \mathcal{G})$ -continuous homomorphism with kernel $\{(h, h^{-1}): h \in H\}$, whence β is open and $\mathcal{G} = \mathcal{G}(\tau, \mathcal{G}_{\overline{H}})$. This completes the proof of (ii), from which (iii) immediately follows. We conclude the proof by observing that (iv) is an easy consequence of Theorem 3.1(iii) and the equality $G = L\overline{H}$. $\square$

Turning our attention to Theorem 3.3, we note that the $(\Rightarrow)$ portion of each part is trivial. To prove the other half of the theorem, we first apply Lemma 4.2 to obtain a subset C of L that is τ -homeomorphic with a cube and such that the homomorphism β in the proof of Theorem 3.2 maps $(C \times \bar{H}, \tau \times \mathcal{G}_{\bar{H}})$ homeomorphically onto the $\mathcal{G}$ -neighborhood $C\bar{H}$ of e . Since C is τ -compact, this homeomorphism implies that $(G, \mathcal{G})$ is locally compact, metrizable, or analytic under the conditions given in (i). To establish the remainder of (i), we simply note that $C\bar{H}$ must be $\mathcal{L}(\mathcal{G})$ -complete if $\bar{H}$ is and invoke [1, Proposition 4, p. 245]. Since $\bar{H}$ is $\mathcal{G}$ -separable by Theorem 3.1(iii), (iv), the hypothesis of metrizability in (ii) allows us to use [5, Theorem III.4, p. 33] to finish the proof of the second part of Theorem 3.3. Finally, to prove the $(\Leftarrow)$ part of (iii), let $\lambda: [0, 1] \rightarrow G$ be a $\mathcal{G}$ -arc with initial point e . The homeomorphism of $C \times \bar{H}$ with $C\bar{H}$ provides a lifting $\bar{\lambda}$ of λ to a $\tau \times \mathcal{G}_{\bar{H}}$ -arc in $L \times \bar{H}$. By hypothesis, $\bar{\lambda}$ lies in $L \times H$ and thus, $\beta(\bar{\lambda}) = \lambda$ lies in L . $\square$

6. EXAMPLES AND COMPARISON WITH RELATED RESULTS

According to Theorems 3.2 and 3.3, the topology of $(G, \mathcal{G})$ is completely determined by its restriction to $\bar{H}$. As we noted in §3, it is tempting to try to reduce the subsequent investigation of $\mathcal{G}_{\bar{H}}$ to the separate analysis of the topologies of $\bar{V}$ and $\overline{Z(L)}$. Such a reduction is not always possible, however, for $\mathcal{G}_H$ might not be a “product topology” (i.e., might not make the projections of H onto V and $Z(L)$ continuous), and in that case $\mathcal{G}_{\bar{H}}$ need not be determined by its restrictions to $\bar{V}$ and $\overline{Z(L)}$. In fact, $\bar{H} = \overline{V \times Z(L)}$ might not, even as a set, equal the product $\bar{V} \cdot \overline{Z(L)}$, as the following example shows. We begin by choosing inductively a sequence $\{n_j\}$ of integers such that, for all positive integers j , $n_{j+1} \geq n_j!$ and the distance between $\exp(2\pi i \sqrt{2}n_j)$ and 1 is at most $1/j$. Using [9, Proposition 4.4], we can construct a norm μ for $\mathbf{R}^2$ that is weaker than the usual norm and such that $\mu(n_j, n_j) \leq 1/j$ and $0 \times \mathbf{R}$ inherits the usual topology from μ . By symmetry and the definition of μ , μ also induces the usual topology on $\mathbf{R} \times 0$, and it follows that both $\mathbf{R} \times 0$ and $0 \times \mathbf{R}$ are complete subsets of $(\mathbf{R}^2, \mu)$. If we associate with each $(a, b) \in \mathbf{R}^2$ the homeomorphism of $\mathbf{C}^2$ given by

$$(z_1, z_2) \mapsto (z_1 \exp(2\pi i \sqrt{2}a), z_2 \exp(2\pi ia)),$$

then $(\mathbf{R}^2, \mu)$ acts continuously on $\mathbf{C}^2$ with the usual topology $\mathcal{G}$ and $(\mathbf{C}^2 \otimes \mathbf{R}^2, \mathcal{G} \times \mu)$ is a (WA) group that we will denote by $(L, \mathcal{U})$. It is easy to verify that $H = \mathbf{R}^2$, $V = \mathbf{R} \times 0$, and $Z(L) = 0 \times \mathbf{R}$, so that both V and $Z(L)$ are complete in the μ -topology. If $(G, \mathcal{G})$ is the completion of $(L, \mathcal{U})$ and $\bar{H}$ is the $\mathcal{G}$ -closure of H , then [6, Corollary 6, p. 99] implies that H is a proper subset of $\bar{H}$. On the other hand, since $\bar{V} = V$ and $\overline{Z(L)} = Z(L)$, $\bar{H}$ must properly contain $\bar{V} \cdot \overline{Z(L)}$ and thus $\mathcal{G}_{\bar{H}}$ is not the standard extension of $\mathcal{G}_{\bar{V}}$ and $\mathcal{G}_{\overline{Z(L)}}$ to $\bar{H}$.

Our second example deals with the relationship between our results and those obtained by Goto in [4]. Readers familiar with that paper may have noted that our decisive subgroup H plays a role similar to that of his gm -torus, which we will denote here by J . In particular, if $\overline{v(J)}$ is the $\mathcal{G}$ -closure of the vector part of J , then Goto has shown that $G = L\overline{v(J)}$ and that $(G, \mathcal{G})$ is locally

compact or analytic if and only if $\overline{v(J)}$ is [4, Theorems 3, 5, 6]. Thus H can be replaced by $v(J)$ in some portions of Theorems 3.2 and 3.3, and it is natural to ask whether that substitution is valid in other cases, as well.

Before answering this question, we recall how J and H are defined. Let $\mathfrak{l}$ be the Lie algebra of (L, τ) , and let $\overline{\text{Ad}(L)}$ be the closure of the adjoint image of L in the general linear group $\text{GL}(\mathfrak{l})$. If T is a maximal torus in $\overline{\text{Ad}(L)}$, then $J = \text{Ad}^{-1}(T)$ is a τ -closed, connected, abelian subgroup of L and, therefore, is the product of a vector group $v(J)$ and a torus [4, 7.2]. To define H , we let N be maximal among the connected subgroups of the analytic group $\text{Ad}(L)$ that contain the commutator subgroup of $\text{Ad}(L)$ and are closed in $\overline{\text{Ad}(L)}$. We then choose a torus S that is complementary in T to the torus $N \cap T$ and let $H = \text{Ad}^{-1}(S)$. Then $H = \mathbf{R}^p \times T^q \times \mathbf{Z}^r \times D$, where $\mathbf{R}$, T^q , $\mathbf{Z}$, and D denote, respectively, the real numbers, the q -dimensional toroid, the integers, and a finite group, and where p , q , and r are nonnegative integers. (Further details about the definition of H can be found in [9, §3].) Thus H is always a subgroup of J and $v(J)$ contains the noncompact but not the compact part of H .

We may now compare the utility of H , J , $v(J)$, and $\overline{v(J)}$ in describing the topologies of G and L . As the example in [10, §8] shows, H can be a proper subgroup of J and $\mathcal{G}_L$ does not necessarily equal $\mathcal{G}(\tau, \mathcal{G}_{v(J)})$, so that J is “too big” and $v(J)$ is “too small” to determine the topology of L . It is, however, possible to use the decisiveness of H and the relationship between H and $v(J)$ to show that $\mathcal{G} = \mathcal{G}(\tau, \mathcal{G}_{\overline{v(J)}})$, whence we obtain a topological isomorphism between $(G, \mathcal{G})$ and $(L \otimes \overline{v(J)}) / \{(x, x^{-1}) : x \in L \cap \overline{v(J)}\}$, where the latter has the quotient topology from $\tau \times \mathcal{G}_{\overline{v(J)}}$. As the following example will demonstrate, this representation of G has the disadvantage that $v(J)$ can be a proper subgroup of $L \cap \overline{v(J)}$, so that one must examine the way in which $\overline{v(J)}$ is “glued on” to L . Theorem 3.2(iii) shows that this difficulty does not arise if one uses $\overline{H}$ instead of $\overline{v(J)}$.

To illustrate the possibility that $\overline{v(J)}$ contains points of L that are not in $v(J)$, we let $L = \mathbf{R} \times T^1$ and $G = L$. Since L is abelian, it follows that $\text{Ad}(L)$ is trivial, $J = H = L$, and $v(J) = \mathbf{R} \times 1$. By 4.3 and 4.5 in [9], there is a topology $\mathcal{U}$ for L , weaker than the usual topology, in which the sequence $\{(n!, (-1)^n)\}$ converges to the identity. Therefore $\{((2n+1)!, 1)\}$ $\mathcal{U}$ -converges to $(0, -1)$, $\overline{v(J)}$ includes $\mathbf{R} \times \{\pm 1\}$, and $L \cap \overline{v(J)}$ properly contains $v(J)$. (The appropriate choice of $\mathcal{U}$ will ensure that $\overline{v(J)}$ in fact equals $\mathbf{R} \times \{\pm 1\}$.)

7. A CONJECTURE

Our principal results can be viewed as structure theorems for groups in which a (WA) group is dense, and we now ask whether any “unexpected” groups fall into this category. If $(A, \mathcal{A})$ is a separable, metrizable group of finite topological dimension and L is the arc-component of the identity in A , then [3, Corollary 7.3] implies that $(L, \mathcal{A}_L)$ is a (WA) group. Therefore Theorems 3.2 and 3.3 apply to the $\mathcal{A}$ -closure $\overline{L}$ of L , and we wish to ask under what circumstances $\overline{L}$ must equal A . In response to this question, A. Gleason has

conjectured that $\overline{L} = A$ if $(A, \mathcal{A})$ is connected and complete. The hypothesis of connectedness is obviously necessary, and the connected group without nontrivial arcs in [7] shows that completeness is necessary as well.

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