PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 120, Number 2, February 1994

SOME GEOMETRIC PROPERTIES OF SPACES ASSOCIATED WITH MULTIPLE STABLE INTEGRALS

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(Communicated by Dale Alspach)

ABSTRACT. We investigate properties of vector lattices of multiply integrable functions with respect to a symmetric stable process.

1. INTRODUCTION

In this note we describe certain properties of spaces $\mathbb{S}_\alpha$ of multiparameter functions that are multiply integrable with respect to a symmetric α-stable Lévy process $X_\alpha(t)$, $0 < \alpha < 2$. Such spaces are special tensor products of L^α-spaces, and

$$\bigcap_{\beta>\alpha} L^\beta \hookrightarrow \mathbb{S}_\alpha \hookrightarrow L^\alpha,$$

where " $\hookrightarrow$ " denotes the continuous embedding, but a closed characterization is unknown at this time, for dimensions higher than two.

As is known, the distribution and the existence of the single stochastic integral $X_\alpha f = \int f\, dX_\alpha$ depends only on the integral $\int |f|^\alpha$. In [KaS] there was derived a recursive procedure for multiple integrability, but, except for $d = 1$ or $d = 2$, it did not provide any closed formula for the integrability. For the record, the double stable integrable $\iint f\, dX_\alpha\, dX_\alpha$ exists if and only if

$$(1) \qquad \iint |f(s,t)|^\alpha \left(1 + \ln_+ \frac{|f(s,t)|^\alpha}{\int |f(u,t)|^\alpha\, du \int |f(s,v)|^\alpha\, dv}\right) ds\, dt < \infty$$

(cf. [RW], and for its discrete counterpart, cf. [CRW]). The condition is clearly equivalent to the double integrability of $|f|^\alpha$ with respect to X_1. Hence, it is clear, at least in the case $d = 1$ or $d = 2$, that the spaces $\mathbb{S}_\alpha$ are obtainable from one another by a convexification (or concavification) procedure

$$\mathbb{S}_\alpha = \{f : f^\alpha \in \mathbb{S}_1\}.$$

In spite of the lack of concrete representation of integrable functions, the latter

Received by the editors May 10, 1992.
1991 *Mathematics Subject Classification*. Primary 46B20, 60H05; Secondary 46B30, 60E07.
Key words and phrases. Multiple stochastic integral, Banach lattice, F-lattice, stable distribution, L^p-space, p-convexity, p-concavity, Convexification, Rademacher type.

relation, obtained in a simple way in this note, allows us to examine properties of Banach or $-$F-lattices $\mathbb{S}_\alpha$, like reflexivity, "closeness" to L^α-spaces, Rademacher type, convexity type, etc.

2. Notation and known facts

For two quantities $A = A(\theta)$ and $B = B(\theta)$ depending on some parameter θ, we will write $A \asymp B$ ($A \preccurlyeq B$, respectively) if there is a constant $c > 0$ such that $c^{-1}A \le B \le cA$ (respectively, $A \le cB$), uniformly for θ. It will be clear from the context whether or not the constant c depends on additional parameters, like dimension or the stability index α.

Sequences of length d will be denoted by a boldface character, e.g., $\mathbf{u} = (u_1, \ldots, u_d)$. The relation $\mathbf{i} = (i_1, \ldots, i_d) \le n$ will mean $\max_{1\le k\le d} i_k \le n$. For an $x \in \mathbb{R}$ and $p \in \mathbb{R}$ we write $x^p \stackrel{\mathrm{df}}{=} |x|^p \operatorname{sgn}(x)$.

Throughout the paper, U and V, with or without subscripts, will denote random variables uniformly distributed on $[0, 1]$, while ε will be a Rademacher random variable, i.e., one taking the values ± 1 with probabilities $\frac{1}{2}$. Note that the random variable $\varepsilon U^{-1/\alpha}$ belongs to the domain of normal attraction of the symmetric α-stable law [F]. Let $[U_{ik}]_{k\in\mathbb{N},\ i=1,\ldots,d}$ and $[V_{ik}]_{k\in\mathbb{N},\ i=1,\ldots,d}$ be two independent random matrices whose entries consists of i.i.d. random variables. Let a matrix $[\varepsilon_{ik}]_{k\in\mathbb{N},\ i=1,\ldots,d}$ consist of independent Rademacher random variables and be also independent of both matrices of uniform random variables.

We will use the operator notation, $Xf = \int f\,dX$, for stochastic integrals. A symmetric α-stable Lévy process on $[0, 1]$ with values in $\mathbb{R}^d$ is denoted by $\mathbf{X} = \mathbf{X}_\alpha = (X^1_\alpha, \ldots, X^d_\alpha)$. Given a function $\mathbf{u} = \mathbf{u}(t) = (u_1(t), \ldots, u_d(t)) \in L^\alpha([0, 1]; \mathbb{R}^d)$, the distribution of the vector-valued coordinatewise integral $\mathbf{Xu} = (X^1 u_1, \ldots, X^d u_d)$ is given uniquely by a Lévy measure $\nu_\alpha(d\mathbf{x})$ or, separating the Lévy measure into the radial and angular components, by the corresponding spectral measure $\sigma_\alpha(d\mathbf{x})$ on the $(d-1)$-dimensional sphere S_d,

$$\mathsf{E}\exp\left\{ i\sum_{k=1}^{d} X_k u_k \right\} = \exp\left\{ -\int_0^1 \int_{\mathbb{R}^d} (1 - \cos(\mathbf{x}\cdot\mathbf{u}(t)))\nu_\alpha(d\mathbf{x})\,dt \right\}$$
$$= \exp\left\{ -\int_0^1 \int_{S_d} |\mathbf{x}\cdot\mathbf{u}(t)|^\alpha \sigma_\alpha(d\mathbf{x})\,dt \right\},$$

where $\mathbf{x}\cdot\mathbf{u}$ denotes the inner product in $\mathbb{R}^d$.

Let $\mathscr{D}$ be a linear space of functions on $[0, 1]^d$ which vanish on diagonal hyperplanes and are symmetric with respect to permutations of their arguments. Let $\mathbb{X}\colon \mathscr{D} \to L_0$ be a linear operator. Consider the conditions:

(1) $f\ (t_1, \ldots, t_d) = f_1(t_1)\cdots f_d(t_d) \in \mathscr{D}$, whenever $f_i \in L^\alpha$, and $\mathbb{X}f \stackrel{\mathrm{df}}{=} X^1 f_1 \cdots X^d f_d$.

(2) If $\{f_n\} \subset \mathscr{D}$, $f_n \to f$ in measure, and $\sup_n |f_n| \le g \in \mathscr{D}$, then $f \in \mathscr{D}$ and $\mathbb{X}f_n \xrightarrow{\mathsf{P}} \mathbb{X}f$.

The space of integrable functions $\mathbb{S}_\alpha$ is, by definition, a maximal space $\mathscr{D}$ with the above properties, while the operator $\mathbb{X}_\alpha = X^1_\alpha \cdots X^d_\alpha = \mathbb{X}\colon \mathbb{S}_\alpha \to L_0$ is termed the multiple stable integral [Kas]. This axiomatic definition is independent of a particular construction of a multiple stable integral.

Basic properties of the multiple stochastic integral and the space $\mathbb{S}_\alpha$, are gathered below. Recall that by an F-space we mean a linear space with a topology induced by a complete invariant metric (cf., e.g., [Ru, pp. 8, 9]). Denote

$$\|f\|_{\alpha,s} = (\mathsf{E}|\mathbb{X}_\alpha f|^s)^{1/s},$$

where s is any positive number less than α.

Theorem A. *The following properties hold:*

(i) *The functional* $\|\cdot\|_{\alpha,s}$ ($\|\cdot\|^s_{\alpha,s}$ *if* $s < 1$) *is a norm* (*s-norm, if* $s < 1$) *on* $\mathbb{S}_\alpha$, *turning* $\mathbb{S}_\alpha$ *into a Banach or* F*-space. Moreover, for a fixed* α, *the norms are equivalent for all* $s \in (0, \alpha)$.

(ii) *Decoupling principle. If* $\widehat{X}^1_\alpha, \dots, \widehat{X}^d_\alpha$ *are independent copies of the coordinate processes* $X^1_\alpha, \dots, X^d_\alpha$, *then*

$$\|\mathbb{X}_\alpha f\|_{\alpha,s} \asymp \|\widehat{X}^1_\alpha \cdots \widehat{X}^d_\alpha f\|_{\alpha,s}. \tag{2}$$

In particular, the class of integrable functions $\mathbb{S}_\alpha$ *does not depend on the Lévy or spectral measures.*

(iii) *The finite dimensional distributions of stochastic processes with a parameter space* $\mathbb{S}_\alpha$,

$$S^n_\alpha(f) = n^{-d/\alpha} \sum_{i \le n} \frac{f(V_{1i_1}, \dots, V_{di_d})}{U^{1/\alpha}_{1i_1} \cdots U^{1/\alpha}_{di_d}} \varepsilon_{1i_1} \cdots \varepsilon_{di_d},$$

converge weakly to the distribution of a multiple stable integral $\mathbb{X}_\alpha f$ *with independent coordinate processes.*

(iv) $\mathbb{S}_\alpha$ *is isomorphic to an* F*-space* $\{f\colon (S^n_\alpha(f))$ *is bounded in* $L^0\}$, *equipped with a complete metric generated by the modular*

$$\tilde{\rho}_{\alpha,s}(f) = \sup_n \|S^n_\alpha(f)\|_s.$$

(v) *The modular* $\tilde{\rho}_s$ *is equivalent to*

$$\begin{aligned}\rho_{\alpha,s}(f) &= \sup_n n^{-d/\alpha} \left\| \left(\sum_{\mathbf{i}} |f(V_{1i_1}, \dots, V_{1id})(U_{1i_1} \cdots U_{di_d})^{-1/\alpha}|^2 \right)^{1/2} \right\|_s \\ &= \sup_n n^{-d/\alpha} \left\| \left(\sum_{\mathbf{i}} | \, |f(V_{1i_1}, \dots, V_{di_d})|^\alpha (U_{1i_1} \cdots U_{di_d})^{-1} |^{2/\alpha} \right)^{\alpha/2} \right\|^{1/\alpha}_{s/\alpha}.\end{aligned}$$

(vi) *Under the natural ordering,* $\mathbb{S}_\alpha$ *becomes a vector lattice. The norm* (*or s-norm*) *induced by the modular* $\rho_{\alpha,s}$ *is monotone. Hence,* $|f| \le |g| \Rightarrow \|f\|_{\alpha,s} \preccurlyeq \|g\|_{\alpha,s}$.

Proof. (i) [S1, Theorem 3.5; KS1, Theorem 2.5 and Corollary 2.3].

(ii) [KS3, Proposition 5.1].

(iii) [S3, Proposition 3.1 and Corollary 3.2].

(iv) The connection between modulars and metrics is described, for example, in [M, pp. 2, 3]. The statement was proved in [S2, Theorem 6.6] in the case of an L^0-metric. Here, we can switch to a more convenient L_s-norm, for any $0 < s < \alpha$, due to hypercontractivity of the random variable $Z^{(\alpha)} = \varepsilon U^{-1/\alpha}$ (cf. [S1, Theorem 3.1; KS1, Theorem 2.5 and Corollary 2.3]).

(v) It suffices to apply the d-dimensional Khinchin inequality (cf., e.g., [KS2, inequality (2.4)].

(vi) Obviously. □

Lemma 1. *Let* $0 < q < p < r < \infty$. *Then, uniformly in all real sequences* (a_i),

$$\mathsf{E}\left(\sum_i (|a_i|U_i^{-1/p})^r\right)^{q/r} \asymp \left(\sum_i |a_i|^p\right)^{q/p} \asymp \mathsf{E}\sup_i (|a_i|U_i^{-1/p})^q.$$

Proof. Let $1 < p' < \infty$, and let (Y_i) be a sequence of independent random variables such that $\mathsf{P}(|Y| > t) = t^{-1/p'}$ for every i and every $t \geq 1$. Then for every $r' > p'$, by [LT, Lemma 1.f.8], we have

$$\mathsf{E}\left(\sum_i |a_i' Y_i|^{r'}\right)^{1/r'} \asymp \left(\sum_i |a_i'|^{p'}\right)^{1/p'} \asymp \mathsf{E}\sup_i |a_i' Y_i|.$$

The lemma follows by substituting

$$r' = r/q, \qquad p' = p/q, \qquad a_i' = |a_i|^q, \qquad Y_i = U_i^{-1/p'} = U^{-q/p}. \quad \square$$

Lemma 2. *Let* $r > p > q > 0$, *and let* $\mathbf{U} = [U_{ik}]_{i=1,\dots,d,\ k\in\mathbb{N}}$ *be a random matrix of mutually independent random variables uniformly distributed on* $[0, 1]$.

(i) *Let* $\{A(\mathbf{i})\colon \mathbf{i} \in \mathbb{N}^d\}$ *be a family of random variables independent of* $\mathbf{U}$. *Then, for every* $q \in (0, p)$.

$$\left\|\left(\sum_{\mathbf{i}} (|A(\mathbf{i})|(U_{1i_1}\cdots U_{di_d})^{-1/p})^r\right)^{1/r}\right\|_q \asymp \left\|\sup_{\mathbf{i}} |A(\mathbf{i})|(U_{1i_1}\cdots U_{di_d})^{-1/p}\right\|_q.$$

(ii) *Let* $0 < q < p < \infty$. *Then*

$$\left\|\sum_{\mathbf{i}} \delta(\mathbf{i})a(\mathbf{i})(U_{1i_1}\cdots U_{di_d})^{-1/p}\right\|_q \asymp \left\|\sum_{\mathbf{i}} a(\mathbf{i})(U_{1i_1}\cdots U_{di_d})^{-1/p}\right\|_q,$$

for all functions $a\colon \mathbb{N}^d \to \mathbb{R}_+$, $\delta\colon \mathbb{N}^d \to [-1, 1]$.

Proof. (i) The relation "$\succcurlyeq$" is obvious. To prove the relation "$\preccurlyeq$", we use Fubini's theorem which, in particular, allows us to consider nonrandom matrices $[A(\mathbf{i})]$. For simplicity, we prove the statement in the case $d = 2$. The general case follows in the similar way. Denote by E_1 and E_2 the expectations corresponding (in terms of Fubini's theorem and product measures or, equivalently in this context, by means of conditioning) to independent random sequences (U_{1i}) and (U_{2j}), respectively. Put

$$b_j = \left(\sum_i (|A(i, j)|U_{1i}^{-1/p})^r\right)^{1/r} \quad \text{and} \quad c_i = \sup_j |A(i, j)|U_{2j}^{-1/p}.$$

We have

$$\mathsf{E}\left(\sum_i \sum_j (|A(i,j)| U_{1i}^{-1/p} U_{2j}^{-1/p})^r\right)^{q/r} = \mathsf{E}_1\mathsf{E}_2\left(\sum_j (b_j U_{2j}^{-1/p})^r\right)^{q/r}$$
$$\preccurlyeq \mathsf{E}_1\mathsf{E}_2\left(\sup_j (b_j U_{2j}^{-1/p})^r\right)^{q/r} \preccurlyeq \mathsf{E}_2\mathsf{E}_1\left(\sum_i (c_i U_{1i}^{-1/p})^r\right)^{q/r}$$
$$\preccurlyeq \left\|\sup_{ij} |A(i,j)| U_{1i}^{-1/p} U_{2j}^{-1/p}\right\|^q .$$

(ii) The statement follows from the generalized Khinchin inequality and statement (i) of this lemma. □

3. Main results

Theorem 1. *Let* $0 < \alpha, \beta < 2$ *and* $f\colon [0,1]^d \to \mathbb{R}$ *be a symmetric function vanishing on diagonal hyperplanes.*

(i) f *is* $\mathbb{X}_\alpha$*-integrable if and only if* $|f|$ *is* $\mathbb{X}_\alpha$*-integrable.*

(ii) f *is* $\mathbb{X}_\alpha$*-integrable if and only if* f^α *is* $\mathbb{X}_1$*-integrable if and only if* $f^{\alpha/\beta}$ *is* $\mathbb{X}_\beta$*-integrable. Additionally, for any* s, $0 < s < \alpha$,

$$\mathsf{E}|\mathbb{X}_\alpha f|^s \asymp \mathsf{E}|\mathbb{X}_1 f^\alpha|^{s/\alpha} \asymp \mathsf{E}|\mathbb{X}_\beta f^{\alpha/\beta}|^{s\beta/\alpha}. \tag{3}$$

Proof. By the decoupling principle, we may deal with stable processes with independent coordinates. By Theorem A(iii), (iv), it suffices to consider the random operators $S_\alpha^n(f)$ instead of the integrals $\mathbb{X}_\alpha f$. Thus Lemma 2(ii) yields statement (i).

(ii) We infer from Lemma 2(i) that the modulars

$$\Lambda_{p,q,r}(A) = \left\|\left(\sum_{\mathbf{i}} |A(\mathbf{i})(U_{1i_1}\cdots U_{di_d})^{-1/p}|^r\right)^{1/r}\right\|_q ,$$

where $A\colon \mathbb{N}^d \to L_0$ and $r > p > q > 0$, are equivalent for all r. By hypercontractivity of $U^{-1/p}$ (cf. [S1, Theorem 3.1]), the modulars are also equivalent for all permissible q. Then, the substitutions $p = 1$, $r = 2/\alpha$, $A = f^\alpha$ prove the former equivalence in (3). The latter equivalence in (3) follows a fortiori. □

Theorem 1 enables us to reduce the investigation of the totality of spaces $\mathbb{S}_\alpha$ to that of a space marked by an arbitrarily selected index α. We will give a precise meaning of this statement.

Let $p > 0$. We call the norm $\|\cdot\|$ *p-convex* if

$$\left\|\left(\sum_i |f_i|^p\right)^{1/p}\right\| \preccurlyeq \left(\sum_i \|f_i\|^p\right)^{1/p}, \qquad \{f_i\} \subset \mathbb{S}_\alpha,$$

and *p-concave* if the relation "$\preccurlyeq$" in the above definition is replaced by the relation "$\succcurlyeq$". Observe that the function $(\sum_i |f_i|^p)^{1/p}$ is well defined in $\mathbb{S}_\alpha$, since $\mathbb{S}_\alpha$ is a solid vector lattice (i.e., $|f| \le g \in \mathbb{S}_\alpha \Rightarrow f \in \mathbb{S}_\alpha$). These notions are well known in the context of Banach lattices and indices $p \geq 1$ (cf. [Kri] or [LT, §1.d]). For an arbitrary F-lattice $\mathbb{L}$ of functions, define

$$\mathbb{L}^{(p)} = \{f\colon f^P \in \mathbb{L}\}.$$

If $\|\cdot\|$ denotes a homogeneous positive functional on $\mathbb{L}$, we also put

$$\|f\|_{(p)} = \|f^p\|^{1/p}.$$

It is easy to check that, if $\|\cdot\|$ is s-convex and r-concave, then $\|\cdot\|_{(p)}$ is ps-convex and pr-concave. The operation $\mathbb{L} \mapsto \mathbb{L}^{(p)}$ is usually called the *p-convexification* if $p > 1$, and *$1/p$-concavification* if $p < 1$ [LT, pp. 53–54].

Recall that a Banach space $\mathbb{L}$ is of Rademacher type r if

$$\mathsf{E}\left\|\sum_i x_i\varepsilon_i\right\| \preccurlyeq \left(\sum_i \|x_i\|^r\right)^{1/r}, \qquad \{x_i\} \subset \mathbb{L}.$$

By a suitable concavification, it is easy to formulate an analogous property in the context of spaces $\mathbb{S}_\beta$ and $\beta \leq 1$.

In the one-dimensional situation, the spaces $\mathbb{S}_1$ and L_1 are identical. For a higher dimension, the structure of the space is no longer explicit, even though $\mathbb{S}_1$, for $d = 2$, is clearly characterized by (1).

Theorem 2. *Let* $0 < \alpha < 2$.

(i) *For* $0 < \beta < 2$, $(\mathbb{S}_\beta)^{(\alpha/\beta)} = \mathbb{S}_\alpha$.

(ii) *Except for* $d = 1$, *there is no equivalent Banach lattice renorming of* $\mathbb{S}_1$.

(iii) $\mathbb{S}_\alpha$ *is r-concave for every* $r \geq \alpha$ *and r-convex for every* $0 < r < \alpha$. *In particular,* (a) *multiple* 1*-stable integrals satisfy the relation*

$$\left(\mathsf{E}\left(\sum Y_i\right)^s\right)^{1/s} \geq \sum (\mathsf{E}Y_i^s)^{1/s}\,; \tag{4}$$

and (b) $\mathbb{S}_\alpha$ *is r-uniformly smoothable for* $1 < r < \alpha$ *and, thus, it is a reflexive* (*even superreflexive*) *Banach lattice.*

(iv) $\mathbb{S}_\alpha$ *is not of Rademacher type* α *although it is of Rademacher type* r *for* $1 < r < \alpha$.

Proof. (i) follows immediately from Theorem 1(ii).

(ii) Let $\alpha > 1$. Assume the contrary. If there were an equivalent Banach lattice norm on $\mathbb{S}_1$, it would be 1-convex. It suffices to construct a two-dimensional counterexample based on the characterization (1).

Let $0 < c_i < 1$, and put $f_i = c_i \mathbb{1}_{[i,i+1)\times[i-1,i)}$. Then, by (1), $\mathbb{X}_1 \sum f_i$ converges in L_q for some $q < 1$ if and only if $\sum c_i \ln(1/c_i)$ converges. On the other hand, $\sum \|\mathbb{X}_1 f_i\|_q$ converges if and only if $\sum c_i$ converges. It is easy to find a suitable sequence (c_i), for instance, taking $c_i = (i \ln^{1+\varepsilon} i)^{-1}$, $0 < \varepsilon < 1$.

(iii) Let $\mathbb{X}_\alpha^{(d)}$ and $\mathbb{X}_\alpha^{(d-1)}$ be d- and $(d-1)$-dimensional multiple stable integrals, respectively. Then the first of the following relations is another byproduct of the decoupling principle (Proposition 2), and the last one uses s-convexity of L_α for $s < \alpha$:

$$\|\mathbb{X}_\alpha^{(d)} f\|_s \asymp \left(\mathsf{E}\left(\int_0^1 |\mathbb{X}_\alpha^{(d-1)} f(\cdot, t)|^\alpha\, dt\right)^{s/\alpha}\right)^{1/s}$$

$$= \left\|\left(\int_0^1 |\mathbb{X}_\alpha^{(d-1)} f(\cdot, t)|^\alpha\, dt\right)^{1/\alpha}\right\|_s \geq \left(\int_0^1 \|\mathbb{X}_\alpha^{(d-1)} f(\cdot, t)\|_s^\alpha\, dt\right)^{1/\alpha}.$$

To prove the r-concavity of $\mathbb{S}_\alpha$, $r \geq \alpha$, we use the induction with respect to d:

$$\left\|\mathbb{X}^{(d)}\left(\sum_i |f_i|^r\right)^{1/r}\right\|_s \succcurlyeq \left(\int_0^1 \left\|\mathbb{X}_\alpha^{(d-1)}\left(\sum_i |f_i(\cdot, t)|^r\right)^{1/r}\right\|_s^\alpha dt\right)^{1/\alpha}$$

$$\succcurlyeq \left(\int_0^1 \left(\sum_i \|\mathbb{X}_\alpha^{(d-1)} |f_i(\cdot, t)|\,\|_s^r\right)^{\alpha/r} dt\right)^{1/\alpha}$$

$$\succcurlyeq \left(\sum_i \left(\int_0^1 \|\mathbb{X}_\alpha^{(d-1)} |f_i(\cdot, t)|\,\|_s^\alpha\right)^{r/\alpha} dt\right)^{1/r}$$

$$\asymp \left(\sum_i \|\mathbb{X}^{(d)} f_i\|^r\right)^{1/r},$$

where, besides the inductive assumption, we have also used the r-concavity of L_α.

To prove the convexity, we proceed as follows. Let $r < q < \alpha$. By Theorem 1 and the triangle inequality in $L_{q/r}$, we have

$$\left\|\left(\sum_i |f_i|^r\right)^{1/r}\right\|_{\alpha,q} = \left\|\mathbb{X}_\alpha\left(\sum_i |f_i|^r\right)^{1/r}\right\|_q \asymp \left\|\mathbb{X}_{\alpha/r}\sum_i |f_i|^r\right\|_{q/r}^{1/r}$$

$$= \left\|\sum_i \mathbb{X}_{\alpha/r}|f_i|^r\right\|_{q/r}^{1/r} \leq \left(\sum_i \|\mathbb{X}_{\alpha/r}|f_i|^r\|_{q/r}\right)^{1/r}$$

$$\asymp \left(\sum_i \|\mathbb{X}_\alpha f_i\|_q^r\right)^{1/r} = \left(\sum_i \|f_i\|_{\alpha,q}^r\right)^{1/r}.$$

(iii)(a) Inequality (4) holds for $Y_i = \mathbb{X}_1 g_i$ since $\mathbb{S}_1$ is 1-concave. Note that this inequality shows that 1-stable processes behave, in a sense, like nonnegative random processes, even though there is no positive 1-stable process.

(iii)(b) Cf. [LT, p. 101].

(iv) This can be seen by modifying appropriately the example given in the proof of statement (ii) of this theorem. Indeed, in the aforementioned example, the functions f_k have disjoint supports; hence, $(\sum_k |f_k|^\alpha)^{1\alpha} = \sum_k f_k = |\sum_k \varepsilon_k f_k|$. Thus, the same argument applies. The second part of the statement is obvious, for a p-convex Banach lattice is of Rademacher type p. This completes the proof. □

The restriction to the parameter space $T = [0, 1]$ and the governing Lebesgue measure is not essential. One can easily extend the results to an arbitrary separable σ-finite measure space S. Also, the extension of main results (Theorems 1 and 2) to strictly stable processes is direct, through the appropriate symmetrization (cf., e.g., [KS3, Proposition 5.1 and p. 774]).

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