ENTIRE PROPAGATOR

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(Communicated by Palle E. T. Jorgensen)

ABSTRACT. We construct an entire solution for an evolutionary Cauchy problem when the initial data is an entire vector of an appropriate operator in a Banach space. We give a sufficient condition for an entire domination. Our results are applied to some partial differential equations including the Dirac system.

1. INTRODUCTION

In this paper we construct an analytic solution in the whole complex plane for the following initial value evolution problem:

$$(EP) \quad \begin{cases} du/dz = A(z)u(z) & (z \in \mathbb{C}), \\ u(w) = \phi & (w \in \mathbb{C}). \end{cases}$$

Let A be a linear operator on a Banach space X , and $D(A)$ its domain. We designate by $\mathcal{E}(A)$ the space of entire vectors defined by

$$\mathcal{E}(A) \equiv \left\{ \phi \in \bigcap_{n=0}^{+\infty} \mathcal{D}(A^n) \mid \sum_{k=0}^{+\infty} \|A^k \phi\| \frac{t^k}{k!} < +\infty \quad \forall t \geq 0 \right\},$$

endowed with a Fréchet space topology defined by the calibration $\{p_l\}_{l \in \mathbb{N}}$ where

$$p_l(\phi) = \sum_{k \geq 0} \|A^k \phi\| \frac{l^k}{k!} \quad \text{for } \phi \in \mathcal{E}(A).$$

In [1], one of the authors showed that if $-A(z)$ is independent of z and generates a strongly continuous holomorphic semigroup, then

$$(1) \quad \mathcal{E}(A) = Z(A),$$

where $Z(A)$, is the solution space for A which is the set of all $x \in X$ such that (EP) has a mild solution. Furthermore, if one endows $Z(A)$ with the family of seminorms

$$\|\phi\|_n = \sup_{t \in [0, n]} \|u(t)\|$$

then $Z(A)$ becomes a Fréchet space (see [dL2]), and the identity (1) holds algebraically and topologically. Our aim in this paper is to investigate a similar

Received by the editors July 20, 1992.

1991 *Mathematics Subject Classification.* Primary 47D06.

result in the case where $A(z)$ depends analytically on z . To this end we construct an entire propagator under some domination conditions on the family $\{A(z)\}_{z \in \mathbb{C}}$. Contrary to the analytic domination conditions given by Nelson [5], or those of Goodman and Jorgensen [2], which both have a local character, our domination conditions are more restrictive but ensure sufficient conditions for some linear operator B to get $\mathcal{E}(A) \subseteq \mathcal{E}(A + B)$.

We achieve this paper by applying our results to some evolutionary Cauchy problems of partial differential equations, in particular an important application to the Dirac operator is given.

2. ENTIRE PROPAGATOR

All definitions and results concerning analytic functions of several complex variables valued on a locally convex space which are used in the sequel can be found in [3].

Definition 1. A two-parameter family $\{S(z, w)\}_{(z, w) \in \mathbb{C}^2}$ of bounded linear operators on a Fréchet space Y is called an *entire propagator* on Y if the following two conditions are satisfied:

- (i) $S(z, z) = I$, $S(z, \zeta)S(\zeta, w) = S(z, \zeta)$ ($\forall z, \zeta, w \in \mathbb{C}$);
- (ii) $(z, w) \mapsto S(z, w)$ is a strongly holomorphic function on $\mathbb{C}^2$.

Let A be a linear closed operator on a Banach space X , such that $\mathcal{E}(A) \neq \{0\}$. We assume that the family $\{A(z)\}_{z \in \mathbb{C}}$ satisfies the following assumptions:

- (H1) $\mathcal{E}(A) \subseteq \bigcap_{z \in \mathbb{C}} \mathcal{D}(A(z))$;
- (H2) For each $\phi \in \mathcal{E}(A)$, $z \mapsto A(z)\phi$ is an $\mathcal{E}(A)$ -valued entire function;
- (H3) There exist two functions a and b , bounded on the compact subsets of $\mathbb{C}$ such that

$$\|A^n A(z)\phi\| \leq a(z)\|A^{n+1}\phi\| + b(z)\|A^n\phi\|$$

for all $n \in \mathbb{N}$, $\phi \in \mathcal{E}(A)$ and $z \in \mathbb{C}$.

The existence theorem can be announced as follows:

Theorem 1. Suppose that the assumptions (H1)–(H3) hold. Then there exists an entire propagator $S(z, w)$ on $\mathcal{E}(A)$ such that

- (i) $(z, w) \mapsto S(z, w)$ is holomorphic on $\mathbb{C}^2$ for the topology of uniform convergence on the bounded subsets of $\mathcal{E}(A)$;
- (ii) $\frac{\partial}{\partial z} S(z, w) = A(z)S(z, w)$; and
- (iii) $\frac{\partial}{\partial w} S(z, w) = -S(z, w)A(w)$.

Before starting the proof of this theorem, we give some technical preliminaries.

Lemma 1. Under the same assumptions,

- (a) $A \in \mathcal{L}(\mathcal{E}(A))$ and for $R > 0$, $\phi \in \mathcal{E}(A)$ and $k \in \mathbb{N}$, we have

$$p_l(A^k \phi) \leq \frac{k!}{R^k} p_{l+k}(\phi) \quad \forall l \geq 0, k \geq R;$$

- (b) $p_t(A^n A(z)\phi) \leq a(z)p_t(A^{n+1}\phi) + b(z)p_t(A^n\phi)$ for all $t \geq 0$, $\phi \in \mathcal{E}(A)$, $z \in \mathbb{C}$ and $n \in \mathbb{N}$;
- (c) $A(z) \in \mathcal{L}(\mathcal{E}(A)) \quad \forall z \in \mathbb{C}$;

(d) if $z \mapsto v(z)$ is an $\mathcal{E}(A)$ -valued entire function, then $z \mapsto A(z)v(z)$ is also an $\mathcal{E}(A)$ -valued entire function; and

(e) if $(z, w) \mapsto v(z, w)$ is an $\mathcal{E}(A)$ -valued holomorphic function on $\mathbb{C}^2$, then $(z, w) \mapsto \int_w^0 A(\zeta)v(\zeta, w) d\zeta$ is also an $\mathcal{E}(A)$ -valued holomorphic function on $\mathbb{C}^2$.

Proof. (a) This assumption is a direct consequence of the fact that the series

$$\sum_{n \geq 0} \|A^{k+n}\phi\| \frac{z^n}{n!}$$

are convergent and define an entire function when $\phi \in \mathcal{E}(A)$.

(b) (H3) implies that, for any $t \geq 0$, $\phi \in \mathcal{E}(A)$, $z \in \mathbb{C}$, and $n \in \mathbb{N}$,

$$\sum_{k=0}^N \|A^k A^n A(z)\phi\| \frac{t^k}{k!} \leq a(z) \sum_{k=0}^N \|A^{k+n+1}\phi\| \frac{t^k}{k!} + b(z) \sum_{k=0}^N \|A^{k+n}\phi\| \frac{t^k}{k!}.$$

Letting $N \rightarrow \infty$ we obtain the desired conclusion.

(c) The continuity of $A(z)$ is a consequence of (b) with $n = 0$, of the continuity of A on $\mathcal{E}(A)$ and the fact that for all t, t' such that $t \geq t'$, $p_t(\phi) \geq p_{t'}(\phi)$.

(d) For any $t \geq 0$ and $z, \zeta \in \mathbb{C}$

$$p_t \left(\frac{A(z)v(z) - A(\zeta)v(\zeta)}{z - \zeta} - A'(\zeta)v(\zeta) - A(\zeta)v'(\zeta) \right) \leq A + B + C$$

where

$$\begin{aligned} A &= p_t \left(\frac{A(z)v(z) - A(z)v(\zeta)}{z - \zeta} - A(z)v'(\zeta) \right), \\ B &= p_t \left(\frac{A(z)v(\zeta) - A(\zeta)v(\zeta)}{z - \zeta} - A'(\zeta)v(\zeta) \right), \\ C &= p_t(A(z)v'(\zeta) - A(\zeta)v'(\zeta)). \end{aligned}$$

According to (b)

$$A \leq a(z)p_t \left(A \frac{v(z) - v(\zeta)}{z - \zeta} - Av'(\zeta) \right) + b(z)p_t \left(\frac{v(z) - v(\zeta)}{z - \zeta} - v'(\zeta) \right).$$

Since $a(\cdot)$ and $b(\cdot)$ are bounded on the compact subsets of $\mathbb{C}$, the holomorphy of $z \mapsto v(z)$ on $\mathcal{E}(A)$ and (a) imply that $A \mapsto 0$ as $z \mapsto \zeta$. Cancellation of B and C is a consequence of the strong holomorphy of $A(z)$.

(e) According to (d), for any $w \in \mathbb{C}$, $\zeta \mapsto A(\zeta)v(\zeta, w)$ is holomorphic on $\mathbb{C}$. Since $A(z) \in \mathcal{L}(\mathcal{E}(A))$ for any $\psi^* \in \mathcal{E}(A)^*$, Hartogs Theorem implies that

$$(\zeta, w) \mapsto \langle v(\zeta, w), A(\zeta)^* \psi^* \rangle$$

is an entire function on $\mathbb{C}^2$. Finally the equivalence between weak and strong holomorphy implies the assertion. $\square$

Proof of Theorem 1. We define inductively

$$S_n(z, w)\phi = \int_w^z A(\zeta)S_{n-1}(\zeta, w)\phi d\zeta \quad (n \in \mathbb{N}^*)$$

with $S_0(z, w)\phi \equiv \emptyset$. The integral is taken on any contour running from w to z . Then we prove by induction that $(z, w) \mapsto S_n(z, w)\phi$ defines an $\mathcal{E}(A)$ -valued holomorphic function and that, for all $k \in \mathbb{N}$, $t \geq 0$, and $z, w \in \mathbb{C}$, there exists $r > 0$ such that

$$(2) \quad p_t(A^k S_n(z, w)\phi) \leq \frac{(\alpha(r)|z - w|)^n}{n!} \left[\sum_{l=0}^n C_n^l p_t(A^{k+l}\phi) \right].$$

The number r is such that the contour going from w to z is included in the closed ball $B(0, r)$ and $\alpha(r) = \sup(\sup_{z \in B(0, r)} a(z), \sup_{z \in B(0, r)} b(z))$. In fact by Lemma 1(b) and by induction we have

$$\begin{aligned} p_t(A^k S_{n+1}(z, w)\phi) &\leq \int_0^{z-w} [a(\zeta + w)p_t(A^{k+1}S_n(\zeta + w, w)\phi) \\ &\quad + b(\zeta + w)p_t(A^k S_n(\zeta + w, w)\phi)] d|\zeta| \\ &\leq \alpha(r)^{n+1} \int_0^{z-w} \frac{|\zeta|^n}{n!} \left[\sum_{l=0}^n C_n^l p_t(A^{k+l+1}\phi) + \sum_{l=0}^n C_n^l p_t(A^{k+l}\phi) \right] d|\zeta|, \end{aligned}$$

which proves (2). So assertion (a) of the Lemma 1 together with (2) imply

$$\begin{aligned} (3) \quad p_t(S_n(z, w)\phi) &\leq \frac{(\alpha(r)|z - w|)^n}{n!} \left[\sum_{l=0}^n C_n^l \frac{l!}{R^l} p_{t+R}(\phi) \right] \\ &\leq \frac{(\alpha(r)|z - w|)^n}{R^n} e^R p_{t+R}(\phi). \end{aligned}$$

Thus the series

$$(4) \quad S(z, w) = \sum_{n \geq 0} S_n(z, w)\phi$$

converges uniformly on the compact subsets of $\mathbb{C}^2$, and $(z, w) \mapsto S(z, w)\phi$ is analytic on $\mathbb{C}^2$ for all $\phi \in \mathcal{E}(A)$. From relation (3) we get

$$p_t(S(z, w)\phi) \leq \frac{Re^R}{R - \alpha(r)|z - w|} p_{t+R}(\phi).$$

Consequently, the linear operators $S(z, w)$ are bounded on $\mathcal{E}(A)$ and the families $\{S(z, w)\phi : z \in K\}$ and $\{S(z, w)\phi : w \in K\}$, where K is compact, are equicontinuous. Thus, by virtue of the strong holomorphy of $z \mapsto S(z, w)$ on $\mathbb{C}$, the partial derivatives $\frac{\partial}{\partial z} S(z, w)$ and $\frac{\partial}{\partial w} S(z, w)$ exist for the uniform topology on the bounded sets of $\mathcal{E}(A)$. Since $\mathcal{E}(A)$ is a Fréchet space, $\mathcal{L}(\mathcal{E}(A))$ equipped with the uniform topology is a complete locally convex space and Hartogs Theorem implies that $S(z, w)$ is analytic for the uniform topology of $\mathcal{L}(\mathcal{E}(A))$. It is easy to verify from (4) that

$$S(z, w)\phi = \phi + \int_w^z A(\zeta)S(\zeta, w)\phi d\zeta.$$

Hence $S(z, z) = I$. Property (ii) of Definition 1 together with (ii) of Theorem 1 come from this equality. To prove (iii), we define recursively

$T_0(z, w)\phi \equiv \phi$ and

$$T_n(z, w)\phi = \int_w^z T_{n-1}(z, \zeta)A(\zeta)\phi d\zeta.$$

As before we can prove by induction that

$$p_t(T_n(z, w)\phi) \leq \frac{(\alpha(r)|z-w|)^n}{n!} \left[\sum_{l=0}^n C_n^l p_t(A^l \phi) \right] \quad \forall \phi \in \mathcal{E}(A)$$

and obtain the convergence of the series

$$T(z, w) = \sum_{n \geq 0} T_n(z, w)$$

which is holomorphic on $\mathbb{C}^2$ for the uniform topology of $\mathcal{L}(\mathcal{E}(A))$. This gives

$$T(z, w)\phi = \phi + \int_w^z T(z, \zeta)A(\zeta)\phi d\zeta$$

for all $\phi \in \mathcal{E}(A)$. So, we have $T(w, w) = I$ and $\frac{\partial}{\partial w} T(z, w) = -T(z, w) \times A(w)$.

Let z and w be complex numbers. The holomorphy of the function $\zeta \rightarrow T(z, \zeta)S(\zeta, w)$ implies that

$$\frac{\partial}{\partial \zeta} T(z, \zeta)S(\zeta, w) = -T(z, \zeta)A(\zeta)S(\zeta, w) + T(z, \zeta)A(\zeta)S(\zeta, w) = 0,$$

which shows that $T(z, \zeta)S(\zeta, w)$ is independent of ζ . By taking successively $\zeta = z$ and $\zeta = w$, we have $T(z, w) = S(z, w)$.

The uniqueness of the entire propagator will be a consequence of the following theorem.

Theorem 2. Suppose that the assumptions (H1)-(H3) hold. Then for all $w \in \mathbb{C}$ and all $\phi \in \mathcal{E}(A)$, (EP) has a unique solution u given by $u(z) = S(z, w)\phi$ where $S(z, w)$ is the propagator given by Theorem 1.

Proof. From (ii) in Theorem 1, it follows that u defined by $u(z) = S(z, w)\phi$ is a solution of (EP). To prove the uniqueness, let v be another solution of (EP). Since $\{S(z, w)\}$ is analytic in $\mathbb{C} \times \mathbb{C}$ for the uniform topology of $\mathcal{L}(\mathcal{E}(A))$, the function $\zeta \rightarrow S(z, \zeta)v(\zeta)$ is entire and

$$\frac{\partial}{\partial \zeta} S(z, \zeta)v(\zeta) = -S(z, \zeta)A(\zeta)v(\zeta) + S(z, \zeta)A(\zeta)v(\zeta) = 0.$$

Consequently, $S(z, \zeta)v(\zeta)$ is constant in ζ . Letting $\zeta = z$ and $\zeta = w$ we find $S(z, w)\phi = v(z)$. This uniqueness result implies the uniqueness of the propagator $S(z, w)$ in the sense of Definition 1.

Corollary 1. Suppose that the assumptions (H1)-(H3) hold. Then the Cauchy problem for (EP) is well posed in $\mathcal{E}(A)$. More precisely, for any initial data in $\mathcal{E}(A)$, for any continuous seminorm p_t , and any compact subset K of $\mathbb{C}^2$, there exists $M \in \mathbb{R}^+$ and a continuous seminorm $p_{t'}$ such that

$$p_t(u(z, w)) \leq M p_{t'}(\phi) \quad \forall (z, w) \in K.$$

Proof. The proof is a direct consequence of Theorem 1 and of the local equicontinuity of the entire propagator on the compact subsets of $\mathbb{C}^2$. $\square$

Remark 1. If there exists a complex number λ , $|\lambda| \leq 1$, such that $AA(z) = \lambda A(z)A$ and

$$(H3)' \quad \|A(z)\phi\| \leq a(z)\|A\phi\| + b(z)\|\phi\|$$

for all $z \in \mathbb{C}$ and all $\phi \in \mathcal{E}(A)$ with the assumptions on a and b as in (H3), then the assumptions (H1)–(H3) are satisfied. In particular, this is the case when A and $A(z)$ commute.

Remark 2. It is not hard to see that we can improve assertion (H3) by replacing it with: there exist m functions $\{a_j : j = 1, \dots, m\}$, bounded on compact subsets of $\mathbb{C}$, such that, for all $n \in \mathbb{N}$, $\phi \in \mathcal{E}(A)$, and $z \in \mathbb{C}$,

$$\|A^n A(z)\phi\| \leq \sum_{j=1}^m a_j(z) \|A^{n+j}\phi\|,$$

and get the same result as Theorem 1. Suppose now that $A(z)$ is independent of z and $A(z) = A + B$; then Theorem 1 gives a theorem of perturbation for entire vectors. The following corollary makes this notion more precise.

Corollary 2. Let A be a closed linear operator on X such that $\mathcal{E}(A) \neq \{0\}$. Suppose B is a linear operator for which there exist two constants $a \geq 0$ and $b \geq 0$ such that

$$\|A^n B\phi\| \leq a\|A^{n+1}\phi\| + b\|A^n\phi\| \quad \forall n \in \mathbb{N}, \phi \in \mathcal{E}(A).$$

Then $e^{z(A+B)}\phi$ exists on $\mathcal{E}(A)$ and $\mathcal{E}(A) \subseteq \mathcal{E}(A+B)$.

Proof. Take in Theorem 1, $w = 0$ and $A(z) = A + B$. This implies that the series

$$\sum_{n \geq 0} (A+B)^n \phi \frac{z^n}{n!}$$

is convergent in $\mathcal{E}(A)$. $\square$

3. EXAMPLES

1. A parabolic partial differential equation. Let Ω be a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^n$. In this section, we consider on $L^p(\Omega)$ ($1 < p < \infty$) the following linear evolution problem:

$$(P1) \quad \begin{cases} \frac{\partial u(z, x)}{\partial z} = A(z, D)u(z, x) & (x \in \Omega, z \in \mathbb{C}), \\ u(z, x) = 0 & (x \in \partial\Omega, z \in \mathbb{C}), \\ u(0, x) = \phi(x) & (x \in \Omega), \end{cases}$$

with $A(z, D) = \sum_{|\alpha| \leq 2m} a_\alpha(z) D^\alpha$.

We define $|\alpha| = \sum_{j=1}^n \alpha_j$, $x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ and $D^\alpha = D_1^{\alpha_1} \cdots D_n^{\alpha_n}$ where $D_j = \partial/\partial x_j$.

For all α such that $|\alpha| \leq 2m$, we suppose the function $z \rightarrow a_\alpha(z)$ is entire. The operator $A(z)$ is defined by $\mathcal{D}(A(z)) = W^{2m,p}(\Omega) \cap W_0^{m,p}(\Omega)$ and $A(z)f = A(z, D)f$ for all $f \in \mathcal{D}(A(z))$. Let A be a strongly elliptic operator of order $2m$ defined on $L^p(\Omega)$ by $\mathcal{D}(A) = W^{2m,p}(\Omega) \cap W_0^{m,p}(\Omega)$. For such operators, we have the following estimate (see, e.g., [6]).

There exists a constant $C \geq 0$ such that

$$\|u\|_{2m,p} \leq C(\|Au\|_p + \|u\|_p) \quad \forall u \in W^{2m,p}(\Omega) \cap W_0^{m,p}(\Omega).$$

Since A generates a holomorphic strongly continuous semigroup and $\mathcal{E}(A)$ is dense in $L^p(\Omega)$ (see [4]),

$$\|A(z)\phi\|_p \leq \sum_{|\alpha| \leq 2m} |a_\alpha(z)| \|D^\alpha \phi\|_p \leq a(z) [\|A\phi\|_p + \|\phi\|_p]$$

and Remark 1 imply that (P1) admits a unique solution $u \in \mathcal{H}(\mathbb{C}, \mathcal{E}(A))$.

2. An integro-differential equation. Let Δ be the Laplacian operator on $L^2(\mathbb{R}^m)$ and $A(z)$, the operator defined for all $z \in \mathbb{C}$ on $L^2(\mathbb{R}^m)$ by $A(z)u = -\Delta u + K(z) * u$ and $\mathcal{D}(A(z)) = \mathcal{D}(-\Delta)$. We assume that $z \mapsto K(z)$ is a $L^1(\mathbb{R}^m)$ -valued entire function.

The space $\mathcal{E}(-\Delta)$ is dense (see [4]), and we consider in this space the abstract Cauchy problem

$$(P2) \quad \begin{cases} du(z)/dz = A(z)u(z), \\ u(0) = \phi. \end{cases}$$

Let $\|\cdot\|_2$ and $\|\cdot\|_1$ be the respective norms of $L^2(\mathbb{R}^m)$ and of $L^1(\mathbb{R}^m)$. If $\phi \in \mathcal{E}(-\Delta)$, then

$$\|K(z) * \phi\|_2 \leq \|K(z)\|_1 \|\phi\|_2.$$

By virtue of the commutation of Δ and $A(z)$, we are in the framework of Remark 1 and

$$p_t \left(\left[\left(\frac{K(z) - K(z')}{z - z'} - K'(z') \right) * \phi \right] \right) \leq \left\| \frac{K(z) - K(z')}{z - z'} - K'(z') \right\|_1 p_t(\phi)$$

for all $t \geq 0$ and $n \in \mathbb{N}$; thus, Theorem 1 can be applied. Hence (P2) admits a unique solution $u \in \mathcal{H}(\mathbb{C}, \mathcal{E}(-\Delta))$.

3. Dirac operator. The Dirac operator for a free electron is defined by $\mathbb{D}_\hbar = \mathbb{A} + \frac{c^2 \mu}{i\hbar} \mathbb{J}$, where

$$\mathbb{A} = c \begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix}, \quad \text{with } A = \begin{pmatrix} \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_1} - i \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_1} + i \frac{\partial}{\partial x_2} & -\frac{\partial}{\partial x_3} \end{pmatrix},$$

and

$$\mathbb{J} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \text{with } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here μ is the mass of the electron, c is the speed of light, and $\hbar$ is Planck's constant. iA is selfadjoint on the Hilbert space $H = L^2[\mathbb{R}^3, \mathcal{L}(\mathbb{C}^4)]$, so that $\mathcal{E}(A)$ is dense in H . Physically, one is interested on the behavior of $u_\hbar(t, x)$, the solution of $\frac{d}{dt}u = \mathbb{D}_\hbar u$, as $\hbar \rightarrow 0$. Instead of this, we replace $\frac{c^2 \mu}{i\hbar}$ by z and look to the holomorphy of $z \mapsto u(z)$, where $u(z)$ is the solution of $\frac{d}{dz}u = \mathbb{D}_z u = (\mathbb{A} + z\mathbb{J})u$, on a neighborhood of imaginary axes. In spite of the fact that $\mathbb{A}$ and $\mathbb{J}$ are not commutative, however, we are in the scope of the Remark 1 and we have $\|\mathbb{D}_z u\| \leq \|\mathbb{A}u\| + |z|\|u\|$. Thus, Theorem 1 is applicable and $u(z) \in \mathcal{H}(\mathbb{C}, \mathcal{E}(A))$ whenever $u(0) \in \mathcal{E}(A)$.

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