THE MAXIMAL IDEAL SPACE OF $\text{lip}_A(X, \alpha)$

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ABSTRACT. Let X be a compact subset of the complex plane $\mathbb{C}$, and let $0 < \alpha < 1$. We show that the maximal ideal space of $\text{lip}_A(X, \alpha)$ is X .

Let F be a closed nonempty set in $\mathbb{R}^n$, and take α with $0 < \alpha < 1$. Then $\text{Lip}(F, \alpha)$ is the algebra of bounded complex-valued functions f on F such that

$$p_\alpha(f) = \sup \left\{ \frac{|f(x) - f(y)|}{|x - y|^\alpha} : x, y \in F, x \neq y \right\}$$

is finite, and $\text{lip}(F, \alpha)$ is the subalgebra of functions f such that

$$\frac{|f(x) - f(y)|}{|x - y|^\alpha} \rightarrow 0 \quad \text{as } |x - y| \rightarrow 0.$$

For $f \in \text{Lip}(X, \alpha)$, set

$$\|f\|_\alpha = \|f\|_F + p_\alpha(f),$$

where $\|\cdot\|_F$ is the uniform norm on the closed set F . Then $(\text{Lip}(F, \alpha), \|\cdot\|_\alpha)$ is a Banach function algebra on F , and $\text{lip}(F, \alpha)$ is a closed subalgebra of $\text{Lip}(F, \alpha)$. These Lipschitz algebras were first studied by Sherbert [3].

Let X be a compact subset of the complex plane $\mathbb{C}$. We follow established custom in denoting by $C(X)$ the algebra of all continuous complex-valued functions on X and denoting by $A(X)$ the subalgebra of functions analytic on $\text{int}(X)$. We further define

$$\text{Lip}_A(X, \alpha) = \text{Lip}(X, \alpha) \cap A(X) \quad \text{and} \quad \text{lip}_A(X, \alpha) = \text{lip}(X, \alpha) \cap A(X),$$

so that $\text{Lip}_A(X, \alpha)$ and $\text{lip}_A(X, \alpha)$ are closed subalgebras of $\text{Lip}(X, \alpha)$. This paper concerns the maximal ideal space of the algebra $\text{lip}_A(X, \alpha)$. It is clear that when the interior of X is empty, $\text{lip}_A(X, \alpha)$ is the algebra $\text{lip}(X, \alpha)$ and its maximal ideal space is X [3]. So suppose that the interior of X is nonempty. We shall show that the maximal ideal space of $\text{lip}_A(X, \alpha)$ is X . The proof of this result depends on an extension theorem and some approximation lemmas.

An interesting extension theorem for the Lipschitz algebra $\text{Lip}(X, \alpha)$ is given in [4, Chapter VI, Theorem 3]. Here we modify the proof of that result to prove the following extension theorem for $\text{lip}(X, \alpha)$.

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Theorem 1. Let X be a compact nonempty set in $\mathbb{R}^n$ and $0 < \alpha < 1$. If $f \in \text{lip}(X, \alpha)$, then there exists an extension F of f defined on all $\mathbb{R}^n$, having compact support, such that $F \in \text{lip}(\mathbb{R}^n, \alpha)$.

Proof. Consider the function F defined by

$$\begin{cases} F(x) = f(x), & x \in X, \\ F(x) = \sum_k f(p_k) \varphi_k^*(x), & x \in X^c, \end{cases}$$

where $\{\varphi_k^*\}$ is the partition of unity described at the end of §1.3 in [4, Chapter VI] and the points $\{p_k\}$ are in X with the special property defined on page 172 in [4]. For each x we have $\varphi_k^*(x) = 0$ except for a finite number of indices k , and $\sum_k \varphi_k^*(x) \equiv 1$ if $x \in X^c$. Thus F is well defined and, moreover, it is bounded on $\mathbb{R}^n$.

It is sufficient to show that $|F(x) - F(y)|/|x - y|^\alpha \rightarrow 0$ as $|x - y| \rightarrow 0$ for all $x, y \in \mathbb{R}^n$, $x \neq y$. Let $\varepsilon > 0$. There exists $\delta_1 > 0$ such that $|f(x) - f(y)|/|x - y|^\alpha < \varepsilon$ whenever $x, y \in X$ with $|x - y| < \delta_1$. Take $\delta' \leq \frac{1}{9}\delta_1$ and $\delta \leq \varepsilon\delta'/(1 + \varepsilon)$. Let $x, y \in \mathbb{R}^n$ satisfy $0 < |x - y| < \delta$.

Suppose first that $y \in X$, $x \in X^c$. If $\varphi_k^*(x) \neq 0$, a simple calculation shows that $|y - p_k| \leq 9|x - y|$. So

$$\begin{aligned} \frac{|F(x) - F(y)|}{|x - y|^\alpha} &= \frac{|\sum_k f(p_k) \varphi_k^*(x) - f(y)|}{|x - y|^\alpha} \leq \frac{\sum_k |f(p_k) - f(y)| \varphi_k^*(x)}{|x - y|^\alpha} \\ &= \sum_{\varphi_k^*(x) \neq 0} \frac{|f(p_k) - f(y)| \varphi_k^*(x)}{|x - y|^\alpha} \\ &\leq 9^\alpha \sum_{\varphi_k^*(x) \neq 0} \frac{|f(p_k) - f(y)|}{|p_k - y|^\alpha} \varphi_k^*(x) < 9^\alpha \varepsilon. \end{aligned}$$

Suppose now that both y and x are in X^c . Let L be the line segment joining them; we consider the two cases: (i) the distance of L from X exceeds $\varepsilon^{-1}|x - y|$, (ii) the distance of L from X is not larger than $\varepsilon^{-1}|x - y|$. In the first case the quantitative estimate (14) in [4, p. 174] shows that we have

$$\begin{aligned} |F(x) - F(y)| &\leq |x - y| \sup_{x' \in L} |\nabla F(x')| \\ &\leq c|x - y| \sup_{x' \in L} (\text{dist}(x', X))^{\alpha-1}, \end{aligned}$$

where c is a constant independent of x, y . By assumption $\text{dist}(x', X) \geq \varepsilon^{-1}|x - y|$, $x' \in L$, whence $|F(x) - F(y)|/|x - y|^\alpha \leq c\varepsilon^{1-\alpha}$.

In the second case we can find points $x' \in L$ and $y' \in X$ so that $|x' - y'| \leq \varepsilon^{-1}|x - y|$. Therefore, $|y' - x| \leq (1 + \varepsilon^{-1})|x - y|$ and $|y' - y| \leq (1 + \varepsilon^{-1})|x - y|$, so we have

$$\begin{aligned} \frac{|F(x) - F(y)|}{|x - y|^\alpha} &\leq \left(\frac{1 + \varepsilon}{\varepsilon}\right)^\alpha \left\{ \frac{|F(x) - F(y')|}{|x - y'|^\alpha} + \frac{|F(y') - F(y)|}{|y' - y|^\alpha} \right\} \\ &< 2.9^\alpha \varepsilon \left(\frac{1 + \varepsilon}{\varepsilon}\right)^\alpha = 2.9^\alpha (1 + \varepsilon)^\alpha \varepsilon^{1-\alpha} \end{aligned}$$

by using the estimate above.

Finally if $x, y \in X$, we have $|F(x) - F(y)|/|x - y|^\alpha < \varepsilon$ by hypotheses.

Hence $|F(x) - F(y)|/|x - y|^\alpha \rightarrow 0$ as $|x - y| \rightarrow 0$.

If U is a bounded open subset of $\mathbb{R}^n$ containing X , then we can fix a C^∞ function g with the properties: $0 \leq g \leq 1$; $g(x) = 1$, $x \in X$; and $g(x) = 0$, $x \in \mathbb{R}^n \setminus U$. Hence the function gF is in $\text{lip}(\mathbb{R}^n, \alpha)$ with compact support.

Now we need a preliminary lemma that estimates an integral which we shall require. Although we present a brief proof for this lemma, it can be found in books such as O. D. Kellogg, *Potential theory*, 1929, or G. M. Leibowitz, *Lectures on complex function algebras*, 1970, of course, with some modifications.

Here I should thank the referee who suggested the first reference and made some comments which shorten some parts of the long calculations in this paper.

First we consider the following notation:

$$\Delta(z, r) = \{w: |z - w| < r\}, \\ \Delta_r = \Delta(0, r).$$

Lemma 1. *Let E be a Borel subset of $\mathbb{C}$, and let $0 < \beta < 2$. Then*

$$\iint_E \frac{d\xi d\eta}{|\zeta - z|^\beta} \leq 2\pi \frac{r^{2-\beta}}{2-\beta},$$

where $\pi r^2 = m(E)$ (m here is two-dimensional Lebesgue measure).

Proof. Let $D = \Delta(z, r)$. Since $(E \setminus D) \cup (E \cap D) = E$, $(D \setminus E) \cup (E \cap D) = D$, and $m(E) = m(D)$, we see that $m(E \setminus D) = m(D \setminus E)$; since $|\zeta - z| \geq r$ for $\zeta \in E \setminus D$, we conclude that

$$\iint_E \frac{d\xi d\eta}{|\zeta - z|^\beta} \leq \iint_D \frac{d\xi d\eta}{|\zeta - z|^\beta} = 2\pi \int_0^r \frac{\rho d\rho}{\rho^\beta} = 2\pi \frac{r^{2-\beta}}{2-\beta}.$$

Lemma 2. *Let $0 < \alpha < 1$. For every $z, w \in \mathbb{C}$,*

$$\iint_{\Delta_r} \frac{d\xi d\eta}{|\zeta - z| |\zeta - w|^{1-\alpha}} \leq 2\pi \frac{r^\alpha}{\alpha} \quad (\zeta = \xi + i\eta).$$

Proof. Let $p = 2 - \alpha$ and $q = p/(p - 1) = (2 - \alpha)/(1 - \alpha)$, so $1/p + 1/q = 1$. Applying Hölder's inequality and Lemma 1, we have

$$\begin{aligned} \iint_{\Delta_r} \frac{d\xi d\eta}{|\zeta - z| |\zeta - w|^{1-\alpha}} &\leq \left(\iint_{\Delta_r} \frac{d\xi d\eta}{|\zeta - z|^p} \right)^{1/p} \left(\iint_{\Delta_r} \frac{d\xi d\eta}{|\zeta - w|^{(1-\alpha)q}} \right)^{1/q} \\ &\leq 2\pi \left(\frac{r^{2-p}}{2-p} \right)^{1/p} \left(\frac{r^{2-p}}{2-p} \right)^{1/q} \\ &= 2\pi \frac{r^{2-p}}{2-p} = 2\pi \frac{r^\alpha}{\alpha}. \end{aligned}$$

Lemma 3. *Let α and r be positive numbers with $\alpha < 1$. Suppose X, Y are compact subsets of $\mathbb{C}$ such that Y contains X and the disc Δ_r . If $f \in \text{lip}(Y, \alpha)$, then the function*

$$h(z) = \frac{1}{\pi r^2} \iint_{\Delta_r} \frac{f(\zeta) - f(z)}{\zeta - z} d\xi d\eta \quad (z \in X, \zeta = \xi + i\eta)$$

is in $\text{lip}(X, \alpha)$. Moreover, if f is analytic on a subset of X , h is also analytic there.

Proof. Let $\varepsilon > 0$. Since $f \in \text{lip}(Y, \alpha)$, there exists $\delta_1 > 0$ such that for every $z, w \in Y$,

$$\frac{|f(z) - f(w)|}{|z - w|^\alpha} < \varepsilon \quad \text{whenever } 0 < |z - w| < \delta_1.$$

Choose $\delta < \min\{\frac{1}{2}\delta_1, \varepsilon^{1/(1-\alpha)}\}$. Let $z, w \in X$ with $0 < |z - w| < \delta < \delta_1/2$. Then we have

$$\begin{aligned} |h(z) - h(w)| &= \frac{1}{\pi r^2} \left| \iint_{\Delta_r} \left(\frac{f(\zeta) - f(z)}{\zeta - z} - \frac{f(\zeta) - f(w)}{\zeta - w} \right) d\xi d\eta \right| \\ &\leq \frac{|z - w|}{\pi r^2} \iint_{\Delta_r} \frac{|f(\zeta) - f(w)|}{|\zeta - z||\zeta - w|} d\xi d\eta + \frac{|f(z) - f(w)|}{\pi r^2} \iint_{\Delta_r} \frac{d\xi d\eta}{|\zeta - z|}; \end{aligned}$$

we use $|f(\zeta) - f(w)| \leq p_\alpha(f)|\zeta - w|^\alpha$ and Lemma 2 to estimate the first integral, and Lemma 1 to estimate the second, obtaining

$$|h(z) - h(w)| \leq 2p_\alpha(f)(r^{\alpha-2}/\alpha)|z - w| + (2/r)|f(z) - f(w)|,$$

from which it is obvious that $h \in \text{lip}(X, \alpha)$. Also as noted in [1, p. 30, Lemma 1.7], it is easy to see that h is analytic on subsets of X where f is analytic.

Theorem 2. Let X be a compact subset of $\mathbb{C}$, and let $0 < \alpha < 1$. If $f \in \text{lip}_A(X, \alpha)$ and $z_0 \in X$, then there is a sequence $\{h_n\}$ in $\text{lip}_A(X, \alpha)$ such that if $\tilde{h}_n(z) = (z - z_0)h_n(z)$, then

$$\|f - f(z_0) - \tilde{h}_n\|_\alpha = \|f - f(z_0) - \tilde{h}_n\|_X + p_\alpha(f - f(z_0) - \tilde{h}_n) \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Proof. For notational convenience we suppose $z_0 = 0 \in X$. It is sufficient to consider the case when $f(0) = 0$. Let Y be a compact plane set containing X and the discs $\Delta(z, 1)$ for all $z \in X$. We denote $\Delta(0, \frac{1}{n})$ by D_n , where n is a positive integer. By Theorem 1, f can be extended to a function in $\text{lip}(Y, \alpha)$. Denote the extension by f again, and then define for $n = 1, 2, 3, \dots$ functions h_n by

$$h_n(z) = \frac{n^2}{\pi} \iint_{D_n} \frac{f(\zeta) - f(z)}{\zeta - z} d\xi d\eta \quad (\zeta = \xi + i\eta).$$

By Lemma 3, $h_n \in \text{lip}_A(X, \alpha)$ for all $n \in \mathbb{N}$. We now show that

$$\|f - \tilde{h}_n\|_\alpha = \|f - zh_n\|_X + p_\alpha(f - zh_n) \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Letting $z, w \in X$, $z \neq w$, we have

$$\begin{aligned} zh_n(z) - f(z) &= \frac{n^2}{\pi} \iint_{D_n} z \frac{f(\zeta) - f(z)}{\zeta - z} d\xi d\eta - \frac{n^2}{\pi} \iint_{D_n} f(z) d\xi d\eta \\ &= \frac{n^2}{\pi} \iint_{D_n} \frac{zf(\zeta) - \zeta f(z)}{\zeta - z} d\xi d\eta \quad (\zeta = \xi + i\eta), \end{aligned}$$

thus

$$\begin{aligned}
& \frac{|zh_n(z) - f(z) - wh_n(w) + f(w)|}{|z - w|^\alpha} \\
&= \frac{1}{|z - w|^\alpha} \frac{n^2}{\pi} \left| \iint_{D_n} \left(\frac{zf(\zeta) - \zeta f(z)}{\zeta - z} - \frac{wf(\zeta) - \zeta f(w)}{\zeta - w} \right) d\xi d\eta \right| \\
&\leq \frac{n^2}{\pi} \iint_{D_n} \frac{|\zeta| |z - w|^{1-\alpha}}{|\zeta - z| |\zeta - w|^{1-\alpha}} \frac{|f(\zeta) - f(w)|}{|\zeta - w|^\alpha} d\xi d\eta \\
&\quad + \frac{n^2}{\pi} \frac{|f(z) - f(w)|}{|z - w|^\alpha} \iint_{D_n} \frac{|\zeta|}{|\zeta - z|} d\xi d\eta \\
&\leq \frac{n}{\pi} \iint_{D_n} \frac{|z - w|^{1-\alpha}}{|\zeta - z| |\zeta - w|^{1-\alpha}} \frac{|f(\zeta) - f(w)|}{|\zeta - w|^\alpha} d\xi d\eta \\
&\quad + \frac{n}{\pi} \frac{|f(z) - f(w)|}{|z - w|^\alpha} \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|} \\
&= I_1 + I_2.
\end{aligned}$$

We shall show that I_1 and I_2 tend to zero as $n \rightarrow \infty$. Given $\varepsilon > 0$, choose $k > 2$ such that $\alpha(2k)^\alpha > 2$ and such that $p_\alpha(f)/(k-1) < \varepsilon/4$, and choose $\delta > 0$ such that $|f(z) - f(w)| \leq (\varepsilon/4k)|z - w|^\alpha$ whenever $|z - w| < \delta$. We choose a positive integer N such that $(2k/N) < \delta$. Suppose $n > N$. For the integral I_2 we consider two cases:

(i) $|z| > \frac{k}{n}$ or $|w| > \frac{k}{n}$. Without loss of generality suppose $|z| > \frac{k}{n}$. Then

$$I_2 \leq \frac{n}{\pi} p_\alpha(f) \iint_{D_n} \frac{d\xi d\eta}{|z| - |\zeta|} \leq \frac{n}{\pi} p_\alpha(f) \frac{n}{k-1} \frac{\pi}{n^2} = \frac{p_\alpha(f)}{k-1} < \frac{\varepsilon}{4}.$$

(ii) $|z| \leq \frac{k}{n}$ and $|w| \leq \frac{k}{n}$. In this case $|z - w| < \delta$, so we have

$$I_2 \leq \frac{n}{\pi} \frac{\varepsilon}{4k} \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|} \leq \frac{\varepsilon}{2}$$

using Lemma 1. For the integral I_1 we consider four cases:

(i') $|z| \leq \frac{k}{n}$, $|w| \leq \frac{k}{n}$. In this case for every $\zeta \in D_n$, $|\zeta - w| < \delta$. So we have

$$I_1 \leq \frac{n}{\pi} \frac{\varepsilon}{4k} \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z| |\zeta - w|^{1-\alpha}} |z - w|^{1-\alpha}$$

which by Lemma 2 does not exceed

$$\frac{n}{\pi} \frac{\varepsilon}{4k} \frac{2\pi}{\alpha n^\alpha} \left(\frac{2k}{n} \right)^{1-\alpha} = (2k)^{1-\alpha} \frac{\varepsilon}{2\alpha k} = \frac{\varepsilon}{\alpha(2k)^\alpha} < \frac{\varepsilon}{2}.$$

(ii') $|z| > \frac{k}{n}$ and $|w| > \frac{k}{n}$. We have

$$\begin{aligned}
 I_1 &\leq \frac{n}{\pi} p_\alpha(f) \iint_{D_n} \frac{|\zeta - z|^{1-\alpha} + |\zeta - w|^{1-\alpha}}{|\zeta - z| |\zeta - w|^{1-\alpha}} d\xi d\eta \\
 &\leq \frac{n}{\pi} p_\alpha(f) \left\{ \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|^\alpha |\zeta - w|^{1-\alpha}} + \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|} \right\} \\
 &\leq \frac{n}{\pi} p_\alpha(f) \left\{ \iint_{D_n} \frac{d\xi d\eta}{(|z| - |\zeta|)^\alpha (|w| - |\zeta|)^{1-\alpha}} + \iint_{D_n} \frac{d\xi d\eta}{|z| - |\zeta|} \right\} \\
 &\leq \frac{n}{\pi} p_\alpha(f) \frac{2n}{k-1} \iint_{D_n} d\xi d\eta \\
 &= \frac{n}{\pi} p_\alpha(f) \frac{2n}{k-1} \cdot \frac{\pi}{n^2} = \frac{2p_\alpha(f)}{k-1} < \frac{\varepsilon}{2}.
 \end{aligned}$$

(iii') $|z| > \frac{k}{n}$, $|w| \leq \frac{k}{n}$. In this case for every $\zeta \in D_n$, $|\zeta - w| < \delta$. By Lemma 1

$$\begin{aligned}
 I_1 &\leq \frac{n}{\pi} \frac{\varepsilon}{4k} \left\{ \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|^\alpha |\zeta - w|^{1-\alpha}} + \iint_{D_n} \frac{d\xi d\eta}{|\zeta - z|} \right\} \\
 &\leq \frac{n}{\pi} \frac{\varepsilon}{4k} \left\{ \left(\frac{n}{k-1} \right)^\alpha \frac{2\pi}{(1+\alpha)n^{1+\alpha}} + \frac{n}{k-1} \frac{\pi}{n^2} \right\} \\
 &= \frac{\varepsilon}{4k} \left(\frac{2}{(1+\alpha)(k-1)^\alpha} + \frac{1}{k-1} \right) < \frac{3\varepsilon}{4k} < \frac{\varepsilon}{2}.
 \end{aligned}$$

(iv') $|z| \leq \frac{k}{n}$, $|w| > \frac{k}{n}$, as for (iii').

Hence also $I_1 < \frac{\varepsilon}{2}$ for every $z, w \in X$, $z \neq w$, when $n > N$.

Therefore, $\frac{|zh_n(z) - f(z) - wh_n(w) + f(w)|}{|z-w|^\alpha} < \varepsilon$ for all $z, w \in X$, $z \neq w$, by taking n large enough; that is,

$$\begin{aligned}
 p_\alpha(f - \tilde{h}_n) &= p_\alpha(f - zh_n) \\
 &= \sup_{\substack{z, w \in X \\ z \neq w}} \frac{|zh_n(z) - f(z) - wh_n(w) + f(w)|}{|z-w|^\alpha} \rightarrow 0 \quad \text{as } n \rightarrow \infty.
 \end{aligned}$$

We also know $f(0) = \tilde{h}_n(0) = 0$, so for every $z \in X$ we have

$$|f(z) - \tilde{h}_n(z)| \leq |z|^\alpha p_\alpha(f - \tilde{h}_n) \leq (\text{diam } X)^\alpha p_\alpha(f - \tilde{h}_n).$$

Thus

$$\|f - \tilde{h}_n\|_X \leq (\text{diam } X)^\alpha p_\alpha(f - \tilde{h}_n), \quad \text{so } \|f - \tilde{h}_n\|_X \rightarrow 0 \text{ as } n \rightarrow \infty.$$

This completes the proof.

Corollary 1. If $z_0 \in X$, then the ideal $(z - z_0)\text{lip}_A(X, \alpha)$ is dense in M_{z_0} , where z is the coordinate function on X , and M_{z_0} is the maximal ideal in $\text{lip}_A(X, \alpha)$ corresponding to the point z_0 .

Now for the characterization of the maximal ideal space of $\text{lip}_A(X, \alpha)$, we adopt the Arens method.

Corollary 2. *Let X be a compact subset of $\mathbb{C}$, and let $0 < \alpha < 1$. Then the maximal ideal space of $\text{lip}_A(X, \alpha)$ coincides with X .*

Proof. Let φ be a nonzero homomorphism of $\text{lip}_A(X, \alpha)$ into $\mathbb{C}$. If $z_0 = \varphi(z)$, where z is the coordinate function on X , then $\varphi((z - z_0)g) = 0$ for all $g \in \text{lip}_A(X, \alpha)$. It follows that $z_0 \in X$. Otherwise, the function g given by $g(z) = \frac{1}{z - z_0}$ lies in $\text{lip}_A(X, \alpha)$, and so $\varphi((z - z_0)g) = 1$ which is contrary to $z_0 = \varphi(z)$.

The above corollary shows that $\varphi = 0$ on M_{z_0} . Hence for every $f \in \text{lip}_A(X, \alpha)$, $\varphi(f) = f(z_0)$, that is, φ is the evaluation at the point $z_0 \in X$.

Remark. If $R(X) = A(X)$, where $R(X)$ is the uniform closure of rational functions with poles off X , then $\text{Lip}_A(X, \alpha)$ is uniformly dense in $A(X)$. For every $f \in \text{Lip}(X, \alpha)$, $\|f^n\|_\alpha \leq \|f\|_X^n + np_\alpha(f)\|f\|_X^{n-1}$, so that $\|\hat{f}\| = \|f\|_X$, where $\hat{f}$ is the Gelfand transform of f . Consequently by the main theorem in [2], $M_{\text{Lip}_A(X, \alpha)} \cong M_{A(X)} \cong X$.

Note that in general the maximal ideal space of $\text{Lip}_A(X, \alpha)$ is not yet characterized.

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