A VANISHING THEOREM FOR DONALDSON INVARIANTS

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ABSTRACT. Given a smooth simply connected 4-manifold M , we prove that if there is a smoothly embedded 2-torus T inside M , then the $SU(2)$ -Donaldson invariants of M vanish on collections of 2-homology classes, all of which are orthogonal to $[T]$ and at least two of which are multiples of $[T]$. From this we deduce obstructions to the representability of 2-homology classes of some algebraic surfaces by smoothly embedded tori, and we compute the group of self-diffeomorphisms of certain 4-manifolds with boundary.

1. INTRODUCTION

In this paper we prove the following vanishing theorem for Donaldson invariants, and we give some applications.

Theorem 1.1. *Let M be a smooth closed simply connected oriented 4-manifold with $b_2^+(M)$ odd and greater than one, and $T \hookrightarrow M$ a smoothly embedded torus with self-intersection zero. Let k be an integer, $k > \frac{3}{4}(1 + b_2^+(M))$. Then the Donaldson invariants of M vanish on any collection of 2-homology classes orthogonal to $[T]$ of which at least two are multiples of $[T]$.*

This theorem is consistent with results announced by O'Grady [OG2]:

O'Grady obtains Theorem 1.1 for k large enough when M is an algebraic surface and T an elliptic curve. The blow-up formula for the Donaldson polynomials [FM] together with Theorem 1.1 give a result (Corollary 3.2) already obtained by a different approach [L].

To motivate Theorem 1.2, we recall that the adjunction formula for a curve C inside a complex surface S is

$$(*) \quad C.C + C.k_S = 2 \text{ genus}(C) - 2,$$

where k_S is the canonical class of S . When k_S is numerically effective, e.g., if S is minimal irrational, this gives $C.C \leq 2 \text{ genus}(C) - 2$. Several results which extend this inequality to smoothly embedded surfaces $\Sigma \hookrightarrow S$ have been obtained ([KM], [L], [MMR], [R]). On the other hand a naive analogue of $(*)$ for smooth embeddings is bound to fail (there are, for instance, embedded

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spheres with self-intersection less than -2 inside a K3 surface). Nonetheless, Theorem 1.1 implies the following result, which, for the complex surfaces of the statement, is equivalent to $(*)$ for smoothly embedded surfaces of genus one:

Theorem 1.2. *Let S be a smooth simply connected complex surface with geometric genus greater than zero. Let k be an integer, $k > \frac{3}{4}(1 + b_2^+(S))$. Suppose that the Donaldson invariant $\gamma_k(S)$ is a polynomial in the intersection form q_S and the canonical class k_S , with k_S^l , for some $l > 1$, appearing non-trivially in its expression. Then any smoothly embedded torus $T \hookrightarrow S$ with $[T].[T] = 0$ satisfies $[T].k_S = 0$.*

There are several classes of surfaces which are known to satisfy the hypotheses of Theorem 1.2, e.g., minimal smooth elliptic surfaces with geometric genus greater than two [FM] or, putting together [OG1] and [Mo], smooth surfaces $S_n \subseteq \mathbb{P}^3$ of degree $n > 4$ divisible by 4.

Observe that if S is a surface satisfying the hypothesis of Theorem 1.2 and $T \hookrightarrow S$ is a smoothly embedded torus, then by Theorem 1.1 the Donaldson invariants of S vanish on collections of classes orthogonal to $[T]$ of which at least one is a multiple of $[T]$. So for such a surface a stronger version of Theorem 1.1 holds.

Theorem 1.2 can be applied to get information about the group of automorphisms of certain 4-manifolds with boundary considered in [G]. We compute the group of automorphisms of the intersection form induced by self-diffeomorphisms for such 4-manifolds (Theorem 3.5).

2. PROOF OF THE VANISHING THEOREM

In order to state the result we will use to prove the vanishing theorem, we need to review briefly the results of [MMR]. Let (Y, g_Y) be a closed, oriented, Riemannian 3-manifold, and (X, g) an oriented Riemannian 4-manifold with a cylindrical end orientation-preserving isometric to $[0, +\infty) \times Y$. The results we are going to describe hold for generic metrics g on X exponentially decaying to the metric g_Y along the end. We shall assume from now on to have chosen such a metric, and we shall omit it from the notation. Let $P \rightarrow X$ be a principal $SU(2)$ -bundle, $\mathcal{M}^0(X)$ and $\mathcal{M}(X)$ respectively the based and unbased L^2 -moduli spaces of ASD connections on P , and $\mathcal{M}_\delta^0(X)$ and $\mathcal{M}_\delta(X)$ the based and unbased subspaces of δ -exponentially decaying L^2 -connections ([Mr], [MMR], [T]).

Let $\mathcal{A}(Y)$ be the space of $SU(2)$ -connections on $\eta = P|_Y$, and $\chi(Y)$ the character variety, that is, the set of flat connections modulo gauge equivalence. Given $\Gamma \in \mathcal{A}(Y)$, denote by $\text{ad } \Gamma$ the induced connection on the adjoint bundle $\text{ad } \eta$. Recall that the set of flat connections on η can be identified with $\mathcal{R}(Y) = \text{Hom}(\pi_1(Y), SU(2))$ via the holonomy representation, and $\chi(Y)$ with $\mathcal{R}(Y)/SU(2)$, where $SU(2)$ acts on the right by conjugation.

Let $t: X \rightarrow \mathbb{R}$ be any smooth function extending the projection onto the first factor over the end of X . There is a continuous $SO(3)$ -equivariant map $\partial_X^0: \mathcal{M}^0(X) \rightarrow \mathcal{R}(Y)$ sending every element of $\mathcal{M}^0(X)$ down to the limit, for $t \rightarrow +\infty$, of its restriction to $\{t\} \times Y$, inducing a map $\partial_X: \mathcal{M}(X) \rightarrow \chi(Y)$.

Given $[A] \in \mathcal{M}(X)$, let $c_2([A]) = \frac{1}{8\pi^2} \int_X |F_A|^2 \text{vol}(g)$ be the square of the L^2 -norm of the curvature.

For $r \in [0, +\infty)$ and given a subset $V \subset \chi(Y)$ we shall use the notation $\mathcal{M}_r(X) = c_2^{-1}(r)$, $\mathcal{M}(X; V) = \partial_X^{-1}(V)$ and the corresponding based ones; similar notation will be used for the analogous subsets of the moduli spaces of δ -decaying connections.

For X equal to the complement of a smoothly embedded torus of square zero inside a smooth simply connected 4-manifold, the analysis of the Chern-Simons flow has been carried out in [GM] for $SO(3)$ -moduli spaces. Adapting the results of [GM] (in particular, Theorem VI.4.1) to the $SU(2)$ -setup immediately yields the following:

Theorem 2.1. *Let M be a smooth simply connected closed 4-manifold with $b_2^+(M)$ odd and greater than one, and $T \hookrightarrow M$ a smoothly embedded 2-torus with self-intersection zero. Let $X \subset M$ be the complement of a closed tubular neighborhood of T inside M . There exists a $\delta > 0$ such that, for generic metrics on X exponentially decaying to the product metric down the end, the L^2 -moduli space $\mathcal{M}(X)$ is smoothly stratified with respect to the filtration*

$$\mathcal{M}(X) \supset \mathcal{M}(X, \chi(Y)^{tr}) \supset \mathcal{M}_\delta(X, \chi(Y)^{tr}).$$

For any $[\omega] \in \mathcal{M}(X)$, $c_2([\omega]) \equiv 0 \pmod{\mathbb{Z}}$. Given a positive integer k , let $d = d(k) = 4k - \frac{3}{2}(1 + b_2^+(M))$. Then

- (1) $\dim\{\mathcal{M}_k(X) \setminus \mathcal{M}_k(X, \chi(Y)^{tr})\} = 2d(k) + 1$,
- (2) $\dim\{\mathcal{M}_k(X, \chi(Y)^{tr}) \setminus \mathcal{M}_{\delta,k}(X, \chi(Y)^{tr})\} = 2d(k) - 2$,
- (3) $\dim \mathcal{M}_{\delta,k}(X, \chi(Y)^{tr}) = 2d(k) - 6$.

Proof of Theorem 1.1. The proof is similar to those of XVI.0.9 in [MMR] and 7.1 in [L]. The condition $k > \frac{3}{4}(1 + b_2^+(M))$ assures that $\gamma_k(M)$ is defined. Write $M = X_1 \cup_{\partial X_1 = \partial X_2} X_2$, where X_1 is a tubular neighborhood of T . Let

$s > 0$, and let $\Sigma_1, \dots, \Sigma_d \hookrightarrow M$ be smoothly embedded surfaces in general position, with $\Sigma_1, \Sigma_2 \subset X_1$, $[\Sigma_1] = [\Sigma_2] = [T] \in H_2(M; \mathbb{Z})$, and $\Sigma_i \subset X_2$, $[\Sigma_i] = \alpha_{i-2} \in H_2(M; \mathbb{Z})$, $i = 3, \dots, d$. We choose on the X_i 's structures of smooth four-manifolds with cylindrical ends diffeomorphic to $[0, +\infty) \times \partial X_i$, $i = 1, 2$, and generic metrics g_1 and g_2 exponentially decaying to product metrics along the ends. Let $t_i: X_i \rightarrow \mathbb{R}$ be smooth functions extending the projections onto the first factor on the ends $[0, +\infty) \times \partial X_i$ inside X_i . Let $X_i^s = t_i^{-1}((-\infty, s])$ for any $s > 0$, and let $M^s = X_1^s \amalg X_2^s / t_1^{-1}(s-1, s] \sim t_2^{-1}(s-1, s]$. M^s can be provided with a smooth structure diffeomorphic to M , and one can build generic C^∞ metrics g_s on M^s interpolating g_1 and g_2 . Given the surfaces Σ_i there exist corresponding transverse codimension-two submanifolds $\Delta_{\Sigma_i}(s) \subset \mathcal{M}_k(M^s)$ so that, once an orientation for the moduli space is chosen, the value of the Donaldson invariant $\gamma_k(M^s)$ on the classes represented by the Σ_i 's equals the algebraic intersection number $\# \bigcap_{i \geq 1} \Delta_{\Sigma_i}(s)$ (this does not depend on s , because M^s is diffeomorphic to M for all s). But we claim

that for s sufficiently large $\bigcap_{i=1}^d \Delta_{\Sigma_i}(s) \cap \mathcal{M}_k(M^s)$ is empty. By contradiction, suppose $s_n \rightarrow +\infty$ and $[A_n] \in \bigcap \Delta_{\Sigma_i}(s_n) \cap \mathcal{M}_k(M^{s_n})$. Then, up to passing to a subsequence, by [MMR] there are points $x_1, \dots, x_r \in X_1$, $y_1, \dots, y_s \in X_2$

such that A_n restricted to $X_1^{s_n} \setminus \{x_i\}$ converges modulo gauge to an L^2 -ASD connection A_1 which extends to X_1 in the C^∞ topology, up to a gauge transformation, and analogously A_n restricted to $X_2^{s_n} \setminus \{y_j\}$ converges to A_2 on X_2 . Moreover, there are positive integers $\{n_i\}$ and $\{m_j\}$ attached to the points $\{x_i\}$ and $\{y_j\}$ such that if $[A_1] \in \mathcal{M}_e(X_1)$ and $[A_2] \in \mathcal{M}_f(X_2)$,

$$e + f + \sum_{i=1}^r n_i + \sum_{j=1}^s m_j \leq k = c_2(A_n).$$

Also, $[A_1]$ belongs to the intersection of analogous transverse codimension-two submanifolds $D_{\Sigma_1}, D_{\Sigma_2} \subseteq \mathcal{M}_e(X_1)$, while $[A_2]$ belongs to the intersection of submanifolds $D_{\Sigma_3}, \dots, D_{\Sigma_d} \subseteq \mathcal{M}_f(X_2)$ in general position, which are therefore also transverse to the stratification of Theorem 2.1. If $f = k$, we have $e = r = s = 0$, so that $A_1 \in \mathcal{M}_0(X_1) \cap D_{\Sigma_1} \cap D_{\Sigma_2}$. But since $\mathcal{M}_0 = \chi(S^1 \times S^1 \times D^2)$, $\dim \mathcal{M}_0(X_1) = 2$, and since D_{Σ_1} and D_{Σ_2} are in general position, this gives a contradiction. Hence we may assume $f < k$ and $e + r \geq 1$. For any given index $i \in \{3, \dots, d\}$, if $y_j \notin \Sigma_i$ for all $j = 1, \dots, s$, then $[A_2] \in D_{\Sigma_i}$, because D_{Σ_i} is closed and the sequence $\{A_n\}$ restricted to X_2 converges away from $\{y_j\}$. Hence for $i = 3, \dots, d$, $[A_2] \in D_{\Sigma_i}$ or $y_j \in \Sigma_i$ for some j , and we may choose the Σ_i 's so that there are no points y_j belonging to more than two surfaces. So by transversality of the D_{Σ_i} 's we have

$$(*) \quad 2d - 4 \leq 4s + \dim \mathcal{M}_f(X_2).$$

If $f = 0$, then it is easy to see that $\dim \mathcal{M}_f(X_2) = 0$, which gives $4s \geq 2d - 4$; on the other hand $k > \frac{3}{4}(1 + b_2^+(M))$ implies $2d > 4k$ and from $k \geq e + f + r + s$ we get $k \geq s + 1$, hence $4k \geq 4s + 4$, and we have the contradiction $4s > 4s$. So we may assume $f \neq 0$. Since $e + f + \sum n_i + \sum m_j \leq k$, $f \leq k - (e + r + s)$. Therefore by Theorem 2.1 $\dim \mathcal{M}_f(X_2) \leq 2d + 1 - 8(e + r + s)$, and by $(*)$ $2d - 4 \leq 4s + 2d + 1 - 8(e + r + s)$, i.e., $8(e + r) + 4s \leq 5$, which gives $e + r = 0$, contrary to our assumption. $\square$

3. APPLICATIONS

As a first example of application of Theorem 1.1 we have Theorem 3.1 and Corollary 3.2, which have already been obtained ([L], Theorems 7.1 and 7.3) in the particular cases $T.T = +1$ (which imply the general cases by the blow-up formula). Indeed, Theorem 3.1 is now a special case of a more general result of Kronheimer and Mrowka [KM].

Theorem 3.1. *Let M be a smooth closed simply connected oriented 4-manifold with $b_2^+(M) > 1$ odd, and suppose $T \hookrightarrow M$ is a smoothly embedded torus with positive self-intersection. Let k be an integer, $k > \frac{3}{4}(1 + b_2^+(M))$. Let $d = d(k) = 4k - \frac{3}{2}(1 + b_2^+(M))$. Then the Donaldson invariants $\gamma_k(M)$ vanish on any $d(k)$ -tuple of classes $\alpha_1, \dots, \alpha_{d(k)} \in H_2(M; \mathbb{Z})$ orthogonal to $[T]$.*

Proof. Let $n = T.T > 0$. The torus $T' = T \#^n \overline{\mathbb{CP}}^1 \subset \hat{M} = M \#^n \overline{\mathbb{CP}}^2$ has self-intersection zero, and by [FM] $\gamma_k(M \# \overline{\mathbb{CP}}^2) = \gamma_k(M) - 2 \frac{d!}{(d-4)!4!} e^4 \gamma_{k-1}(M) + \dots$, where e is the exceptional class. Hence, if e_n is the exceptional class of the n th copy of $\overline{\mathbb{CP}}^2$,

$$(*) \quad \gamma_{k+1}(\hat{M}) = \gamma_{k+1}(M \#^{n-1} \overline{\mathbb{CP}}^2) - 2 \frac{d!}{(d-4)!4!} e_n^4 \gamma_k(M \#^{n-1} \overline{\mathbb{CP}}^2) + \dots$$

Let $\alpha_1, \dots, \alpha_d \in H_2(M; \mathbb{Z}) \cap [T]^\perp$. The α_i 's, as classes in $H_2(\hat{M})$, are orthogonal to $[T']$. Hence by Theorem 1.1

$$\gamma_{k+1}(\hat{M})([T'], [T'], [T'], [T'], \alpha_1, \dots, \alpha_d) = 0.$$

On the other hand $(*)$ gives

$$\begin{aligned} \gamma_{k+1}(\hat{M})([T'], [T'], [T'], [T'], \alpha_1, \dots, \alpha_d) \\ = -2(e_n \cdot T')^4 \gamma_k(M)(\alpha_1, \dots, \alpha_d), \end{aligned}$$

Since $e_n \cdot T' \neq 0$, this concludes the proof. $\square$

Corollary 3.2. *Let M be a smooth closed simply connected oriented 4-manifold with $b_2^+(M) > 1$ odd, and suppose $T \hookrightarrow M$ is a smoothly embedded torus with positive self-intersection. Let k be an integer, $k > \frac{3}{4}(1 + b_2^+(M))$, and $I = (i_1, \dots, i_n)$ a multi-index with $|I| = \sum_j i_j < d = d(k)$. Then the generalized Donaldson invariants $\gamma_{k,I}(M)$ vanish on any $(d - |I|)$ -tuple of classes $\alpha_1, \dots, \alpha_{d-|I|} \in H_2(M; \mathbb{Z})$ orthogonal to $[T]$.*

Proof. This follows from Theorem 3.1 in exactly the same way Theorem 7.3 follows from Theorem 7.1 in [L] (use the definition of $\gamma_{k,I}(M)$ and Theorem 3.1 applied to M blown-up away from T). $\square$

Proof of Theorem 1.2. By hypothesis

$$\gamma_k(S) = \begin{cases} a_1 k_S q_S^n + a_3 k_S^3 q_S^{n-1} + \dots & \text{if } p_g \text{ is even,} \\ a_0 q_S^n + a_2 k_S^2 q_S^{n-1} + \dots & \text{if } p_g \text{ is odd,} \end{cases}$$

with $a_l \neq 0$ for some $l > 1$. Choose a d -tuple of classes $\alpha_1, \dots, \alpha_d$ of which l are equal to $[T]$ and the rest are all equal to a class α with $\alpha \cdot \alpha \neq 0$. By Theorem 1.1 $\gamma_k(\alpha_1, \dots, \alpha_d) = 0$, which implies

$$a_l (k_S \cdot T)^l q_S(\alpha)^{n-l} = 0. \quad \square$$

Remark. Theorem 1.2 can be strengthened to include the case of a torus of square zero $T \hookrightarrow \hat{S}$, where $\hat{S}$ is obtained by blowing-up a surface S satisfying the hypothesis of 1.2, and $[T]$ is orthogonal to the exceptional classes. In fact, choosing the classes $\alpha_1, \dots, \alpha_d$ in $H_2(S)$ as in the proof of Theorem 1.2, by [FM] $\gamma_k(\hat{S})(\alpha_1, \dots, \alpha_d) = \gamma_k(S)(\alpha_1, \dots, \alpha_d)$, and the argument works just as well.

We point out that Theorem 1.2 can be used to determine the minimal genus of smoothly embedded surfaces representing certain 2-homology classes. For example, let $S \rightarrow \mathbb{CP}^1$ be a smooth simply connected elliptic surface without multiple fibers with $p_g(S) = 3$. Then there is a section $s: \mathbb{CP}^1 \rightarrow S$, and if $C = s(\mathbb{CP}^1)$, $C \cdot C = -4$. Let $[f]$ denote the homology class of a smooth fiber of S . Construct a smoothly embedded surface $\Sigma \hookrightarrow S$ representing $[C] + 2[f]$, using a copy of C plus two smooth fibers by tubing at the intersection points. Then $\Sigma \cdot \Sigma = 0$, $\text{genus}(\Sigma) = 2$, and, since the Poincaré dual of k_S equals $2[f]$, $\Sigma \cdot k_S = 2$. As observed in the introduction, Theorem 1.2 applies to S , so $[\Sigma]$ cannot be represented by a smoothly embedded torus.

FIGURE 1

Now we give our last application. Let k, l, m be positive integers. Consider the 4-manifolds $M(k, l, m)$ obtained by attaching 2-handles to B^4 along the framed links of Figure 1.

Lemma 3.4. *For all $k, l, m > 0$ there is a smooth embedding $i: M(k, l, m) \hookrightarrow M(k, 1, 1) \#^{l+m-2} \mathbb{CP}^2$ such that $i_* H_2(M(k, l, m)) \subseteq H_2(M(k, 1, 1))$.*

Proof. Blow-up $l + m - 2$ -1 's in a link picture for $M(k, 1, 1)$ and slide them over the 0-framed handle to get a framed link containing Figure 1 plus algebraically unlinked -1 -framed unknots (see [K] for the calculus of links). $\square$

The intersection form of $M(k, l, m)$ is either isomorphic to $(1) \oplus (-1)$ or to $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, depending on k, l, m . It is easy to see that the group of automorphisms of either form is isomorphic to $\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$. Let $\mathcal{D}(k, l, m) \subseteq \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$ denote the image, under the natural map, of the group of self-diffeomorphisms of $M(k, l, m)$.

Theorem 3.5. *If $k > 3$, $\mathcal{D}(k, l, m) \cong \mathbb{Z}/2\mathbb{Z}$, generated by multiplication by -1 .*

Proof. Observe first of all that there are self-diffeomorphisms of $M(k, l, m)$ which are -1 on H_2 . One is visible in Figure 1, after rearranging the link in a symmetrical fashion, as a 180° rotation about a vertical line in the plane of the paper. We need to prove that if $k > 3$, there are no self-diffeomorphisms which are not ± 1 on H_2 . Let V_k be a smooth simply connected minimal elliptic surface without multiple fibers with geometric genus $p_g = k - 1$. $M(k, 1, 1)$ is nothing but the Gompf nucleus $N_k \subseteq V_k$ (see [G]). Hence by Lemma 3.4

$M(k, l, m)$ embeds inside $V_k \#^{l+m-2} \mathbb{CP}^2$ with the image of $H_2(M(k, l, m))$ orthogonal to the exceptional classes. Moreover, it follows from the proof of the lemma that the image of $H_2(M(k, l, m))$ contains the class $[f]$ of a smooth fiber of the elliptic fibration on V_k . In fact, $[f]$ comes from the class α given by the 2-handle attached to the 0-framed component in the link picture. Arguing by contradiction, if there are self-diffeomorphisms of $M(k, l, m)$ which are not ± 1 on H_2 , one can generate the entire automorphism group of the intersection form and, in particular, find a diffeomorphism sending the (non-trivial) class α to a class $\hat{\alpha}$ different from $\pm \alpha$, which implies $\hat{\alpha} \cdot \alpha \neq 0$. But α is clearly representable by a smoothly embedded torus, hence $\hat{\alpha}$ is so, and the same holds

for their images inside $H_2\left(V_k^{l+m-2}\overline{\mathbb{CP}^2}\right)$. Also, by what we observed before, $\hat{\alpha}$ is orthogonal to the exceptional classes. Moreover, by [FM] Theorem 1.2 applies to V_k if $k > 3$. Hence, since the Poincaré dual of the canonical class of $V_k^{l+m-2}\overline{\mathbb{CP}^2}$ equals a nonzero multiple of $[f]$ plus the Poincaré duals of the exceptional classes, the remark following Theorem 1.2 implies $\hat{\alpha}.\alpha = 0$, which gives a contradiction. $\square$

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