FUNCTIONS OF CONDITIONALLY NEGATIVE TYPE ON KAZHDAN GROUPS

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ABSTRACT. An explicit bound is given for functions of conditionally negative type on Kazhdan groups, in terms of a set of generators and the corresponding Kazhdan constants. This is used to estimate how far a set in an infinite measure space can be translated by the action of a Kazhdan group. Some estimates are given for Kazhdan constants.

1. INTRODUCTION

The groups G considered in this paper are always assumed to be locally compact with a countable base. If such a group has the discrete topology, it will be denoted by Γ . Property (T) was introduced by Kazhdan in 1966. His definition says that G has Property (T) if the trivial representation is isolated in the unitary dual of G . This concept has been the subject of considerable recent interest, and much information concerning it has been collected in [HaV], to which we refer for further details.

Among several characterizations of Property (T) for a group G we consider the following:

G is compactly generated and for any compact generating set K , there exists $\varepsilon > 0$, such that if $\pi: G \rightarrow \mathcal{U}(\mathcal{H})$ is a unitary representation of G on a Hilbert space $\mathcal{H}$, and $\zeta \in \mathcal{H}$ with $\|\zeta\| = 1$ is such that

$$\sup_{g \in K} \|\pi(g)\zeta - \zeta\| < \varepsilon,$$

then π has a nonzero invariant vector. The largest value of ε corresponding to a given G and K is the *Kazhdan constant* $\kappa(G, K)$ [HRV].

Another property equivalent to Property (T) is the absence of unbounded conditionally negative type functions [HaV, Chapter 5, Theorem 20]. This result was proved in [Gui, Del], and, independently, in [AkW].

We shall establish a bound on conditionally negative type functions on a Kazhdan group G in terms of the supremum on a compact generating set K and the Kazhdan constant $\kappa(G, K)$ (Theorem 2 and Corollary 3 below). This says roughly that a conditionally negative type function cannot grow too fast outside K .

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Conditionally negative type functions are more usually referred to as continuous negative definite functions in English. We use the European terminology, stressing that a conditionally negative type function is not simply the negative analogue of a positive type (continuous positive definite) function. We refer to [HaV, Chapter 5] for an introduction to these functions.

We show (Corollary 5) that for sufficiently large compact generating sets K the Kazhdan constants are greater than $1 - \varepsilon$ for any $\varepsilon > 0$. This leads to a further refinement of Theorem 2. Then we give an application to Kazhdan groups acting on infinite measure spaces to obtain a bound on how far sets can be translated by the group action (Theorem 7). A related result on translates of functions is given (Proposition 9). Finally we obtain an estimate for Kazhdan constants for paradoxical decompositions.

2. A BOUND FOR FUNCTIONS OF CONDITIONALLY NEGATIVE TYPE ON KAZHDAN GROUPS

Consider a real-valued function, ψ , of conditionally negative type on a group with Property (T). We know from [HaV, 5.20] that ψ is bounded. Here we find a bound for ψ as a function of the supremum of ψ on a compact generating set and the Kazhdan constant for that generating set.

First we need some terminology.

Let π be a unitary representation of a group G . Suppose K is a compact subset of G and $\varepsilon > 0$. A unit vector $\zeta \in \mathcal{H}_\pi$ is (ε, K) -invariant if

$$\sup_{g \in K} \|\pi(g)\zeta - \zeta\| < \varepsilon.$$

Thus G has Property (T) if and only if there is a number $\varepsilon > 0$ such that each unitary representation of G which has an (ε, K) -invariant unit vector also has a nonzero invariant vector.

We begin with a lemma, the proof of which is taken from the proof of [HaV, 1.16].

Lemma 1. *Let G be a group with Property (T), K a compact generating set for G , and δ a positive number. Let $\varepsilon = \kappa(G, K)$. If π is a representation of G on a Hilbert space $\mathcal{H}$, and $\zeta \in \mathcal{H}$ is an $(\varepsilon\delta, K)$ -invariant unit vector for π , then $\|\pi(g)\zeta - \zeta\| < 2\delta$ for every $g \in G$.*

Proof. Let $\pi: G \rightarrow \mathcal{U}(\mathcal{H})$ be a representation with $(\varepsilon\delta, K)$ -invariant unit vector ζ . If $\delta > 1$, then $\|\pi(g)\zeta - \zeta\| \leq 2\|\zeta\| < 2\delta$, since ζ is a unit vector. Suppose $\delta \leq 1$. Then, since ε is the Kazhdan constant for G and K , $\pi(G)$ has some nonzero invariant vector. Let $\mathcal{H}_0$ be the subspace of $\mathcal{H}$ consisting of vectors invariant by $\pi(G)$, and let $\mathcal{H}_1$ be the orthogonal complement of $\mathcal{H}_0$ in $\mathcal{H}$. Let $\zeta = \zeta_0 + \zeta_1$ be the decomposition of ζ into components in $\mathcal{H}_0$ and $\mathcal{H}_1$. If $\zeta_1 = 0$, then ζ is invariant and the lemma is true. Suppose $\zeta_1 \neq 0$. Then

$$\sup_{g \in K} \left\| \pi(g) \frac{\zeta_1}{\|\zeta_1\|} - \frac{\zeta_1}{\|\zeta_1\|} \right\| = \frac{1}{\|\zeta_1\|} \sup_{g \in K} \|\pi(g)\zeta - \zeta\| < \frac{\varepsilon\delta}{\|\zeta_1\|}.$$

Since $\mathcal{H}_1$ does not contain any nonzero invariant vectors, it cannot contain any (ε, K) -invariant unit vectors and so $\varepsilon\delta/\|\zeta_1\| > \varepsilon$. Hence $\|\zeta_1\| < \delta$. Then, for all $g \in G$,

$$\|\pi(g)\zeta - \zeta\| = \|\pi(g)\zeta_1 - \zeta_1\| \leq 2\|\zeta_1\| < 2\delta. \quad \square$$

Theorem 2. Let G be a group with Property (T), K a compact generating set for G , and $\psi: G \rightarrow \mathbf{R}$ a conditionally negative type function on G . Then for every $g \in G$

$$0 \leq \psi(g) \leq \left(\frac{2}{\kappa(G, K)} \right)^2 \sup_{h \in K} \psi(h).$$

Proof. It is well known that real-valued conditionally negative type functions take only nonnegative values.

Let $N = \sup\{\psi(g): g \in K\}$; this is finite since ψ is continuous and K is compact. Let $\varepsilon = \kappa(G, K)$.

By Schoenberg's Theorem [HaV, 5.16] for any $t > 0$ the function $\phi: G \rightarrow \mathbf{R}$ defined by $\phi(g) = e^{-t\psi(g)}$ is of positive type. Let $(\mathcal{H}, \pi, \zeta)$ be the cyclic representation of G induced by ϕ . So

$$e^{-t\psi(g)} = \phi(g) = \langle \pi(g)\zeta | \zeta \rangle \quad \forall g \in G.$$

Notice that $\mathcal{H}$ is a real Hilbert space since ϕ is real valued and that $\|\zeta\|^2 = \langle \pi(e)\zeta | \zeta \rangle = e^{-t\psi(e)} = 1$.

Suppose first that $N = 0$. Then, for every $g \in K$,

$$\langle \pi(g)\zeta | \zeta \rangle = e^{-t\psi(g)} = 1$$

and so ζ is invariant under the action of the generating set K . Hence, ζ is invariant under the action of all of G , and so for each $g \in G$

$$e^{-t\psi(g)} = \langle \pi(g)\zeta | \zeta \rangle = 1.$$

Hence, $\psi(g) = 0 = (\frac{2}{\varepsilon})^2 N$ for all $g \in G$.

Now suppose $N > 0$. Take δ such that $0 < \delta < 1/\sqrt{2}$ and t such that $0 < t < -\frac{1}{N} \ln(1 - (\varepsilon\delta)^2/2)$; noting that $(\varepsilon\delta)^2/2 < 1$ since $\varepsilon \leq 2$. Then

$$\begin{aligned} \sup_{g \in K} \|\pi(g)\zeta - \zeta\|^2 &= \sup_{g \in K} 2(1 - \langle \pi(g)\zeta | \zeta \rangle) = \sup_{g \in K} 2(1 - e^{-t\psi(g)}) \\ &= 2(1 - e^{-tN}) < (\varepsilon\delta)^2. \end{aligned}$$

It follows by Lemma 1 that

$$\|\pi(g)\zeta - \zeta\| < 2\delta \quad \forall g \in G.$$

Hence, for all $g \in G$

$$\psi(g) = -\frac{1}{t} \ln(1 - (1 - e^{-t\psi(g)})) = -\frac{1}{t} \ln\left(1 - \frac{1}{2}\|\pi(g)\zeta - \zeta\|^2\right) < -\frac{1}{t} \ln(1 - 2\delta^2).$$

Since this holds for every $t \in (0, -\frac{1}{N} \ln(1 - (\varepsilon\delta)^2/2))$, it follows that for all $g \in G$

$$\psi(g) \leq \frac{N \ln(1 - 2\delta^2)}{\ln(1 - (\varepsilon\delta)^2/2)}.$$

This in turn holds for every $\delta \in (0, 1/\sqrt{2})$, so for every $g \in G$

$$\psi(g) \leq N \lim_{\delta \rightarrow 0} \frac{\ln(1 - 2\delta^2)}{\ln(1 - (\varepsilon\delta)^2/2)} = \left(\frac{2}{\varepsilon}\right)^2 N. \quad \square$$

It is not hard to extend this result to complex-valued functions as follows.

Corollary 3. *Let G be a group with Property (T), K a compact generating set for G , and $\psi: G \rightarrow \mathbb{C}$ a conditionally negative type function on G . Then for every $g \in G$*

$$|\psi(g) - \psi(e)| \leq 2 \left(\frac{2}{\kappa(G, K)} \right)^4 \sup_{h \in K} |\psi(h) - \psi(e)|.$$

Proof. This follows from Chapter 5, Corollary 1.9 of [HaV], which says that the function $\psi_0: G \rightarrow \mathbb{R}$ defined by $\psi_0(g) = \Re([\psi(g) - \psi(e)]^{1/2})$ is of conditionally negative type (in the real-valued sense) and that $|\psi(g) - \psi(e)|^{1/2} \leq \sqrt{2}\psi_0(g)$ for all $g \in G$. $\square$

3. KAZHDAN CONSTANTS FOR LARGE GENERATING SETS

Let π be a unitary representation of Γ on a Hilbert space $\mathcal{H}$. Recall from [HRV, KaT] that

$$\kappa(\pi, K) = \inf_{\|\xi\|=1} \max_{k \in K} \|\xi - \pi(k)\xi\|.$$

Note that $\kappa(\pi, K) \leq 2$. Also $\kappa(\pi, K) = 0$ if and only if the trivial representation χ_1 is weakly contained in π [HRV, Lemma 4].

Assume that $K = K^{-1}$. Let K_n denote the set of words of length at most n in K . Note that as $n \rightarrow \infty$, $\kappa(\pi, K_n)$ increases, and hence converges to a limit $\kappa(\pi) \leq 2$.

Theorem 4. *If $\kappa(\pi, K) > 0$, then $\lim_{n \rightarrow \infty} \kappa(\pi, K_n) \geq 1$.*

Proof. The value of $\kappa(\pi, K)$ does not change if the identity element e is added to K , so we may assume that $e \in K$.

Fix n . Let $\varepsilon > 0$, and choose $\xi \in H_\pi$, $\|\xi\| = 1$, such that

$$\max_{g \in K_n} \|\xi - \pi(g)\xi\| < \kappa(\pi, K_n) + \varepsilon.$$

Let $h = \frac{1}{|K|} \sum_{k \in K} k \in C^*(\Gamma)$. Thus $h = h^*$ and $1 \in \text{Sp } h$ [HRV].

Also $h^n = \frac{1}{|K|^n} \sum k_1 k_2 \cdots k_n$, where the sum is over all $(k_1, k_2, \dots, k_n) \in K^n = K \times K \times \cdots \times K$. Now

$$\begin{aligned} 1 &\leq |1 - \langle \pi(h^n)\xi | \xi \rangle| + |\langle \pi(h^n)\xi | \xi \rangle| \\ &= \left| \frac{1}{|K|^n} \sum_{(k_1, k_2, \dots, k_n) \in K^n} (1 - \langle \pi(k_1 k_2 \cdots k_n)\xi | \xi \rangle) \right| + |\langle \pi(h^n)\xi | \xi \rangle| \\ &\leq \frac{1}{|K|^n} \sum |1 - \langle \pi(k_1 k_2 \cdots k_n)\xi | \xi \rangle| + \|\pi(h^n)\| \\ &\leq \frac{1}{|K|^n} \sum \|\xi - \pi(k_1 k_2 \cdots k_n)\xi\| + \|\pi(h^n)\| \\ &< \kappa(\pi, K_n) + \varepsilon + \|\pi(h^n)\|. \end{aligned}$$

Since $\varepsilon > 0$ is arbitrary, we have

$$\|\pi(h)\|^n \geq \|\pi(h^n)\| \geq 1 - \kappa(\pi, K_n).$$

Now suppose that $\lim_{n \rightarrow \infty} \kappa(\pi, K_n) = \kappa(\pi) < 1$. We show that this implies a contradiction.

For all n , $\|\pi(h)\|^n \geq 1 - \kappa(\pi) > 0$. Hence $\|\pi(h)\| \geq 1$, and so $\|\pi(h)\| = 1$. Since $e \in K$, we know that $-1 \notin \text{Sp } \pi(h)$. Therefore, $1 \in \text{Sp } \pi(h)$. We deduce from [HRV, Proposition I] that π weakly contains χ_1 , i.e., $\kappa(\pi, K) = 0$, contrary to assumption. $\square$

Recall that $\kappa(\Gamma, K) = \inf_{\pi} \kappa(\pi, K)$, where the infimum is over all unitary representations π of Γ on a separable Hilbert space which have no nonzero fixed vector.

Corollary 5. *Suppose that Γ has Kazhdan's Property (T). Then*

$$\sup\{\kappa(\Gamma, K_n) : n = 1, 2, \dots\} \geq 1.$$

Proof. Since Γ has Property (T), $\kappa(\Gamma, K) > 0$ [HaV]. Suppose that the supremum in Corollary 5 is strictly less than $1 - \varepsilon$, for some $\varepsilon > 0$. Then for each $n = 1, 2, \dots$, there is a unitary representation π_n with no nonzero fixed vector such that $\kappa(\pi_n, K_n) < 1 - \varepsilon$. Let $\pi = \bigoplus_{n \geq 1} \pi_n$. For each n ,

$$\kappa(\pi, K_n) \leq \kappa(\pi_n, K_n) < 1 - \varepsilon.$$

This contradicts the theorem and so proves the result. $\square$

Remark. If Γ is any infinite group, then the supremum in Corollary 5 is less than or equal to $\sqrt{2}$. Just consider the left regular representation λ of Γ , and let $\xi \in l^2(\Gamma)$ be the Dirac function at 1. Then $\|\xi - \lambda(g)\xi\| = \sqrt{2}$ for all $g \in \Gamma$.

From Corollary 5 we can obtain further information regarding the estimates in the previous section. For example,

Corollary 6. *Let Γ be a discrete Kazhdan group, and let $\varepsilon > 0$. Then there exists a sufficiently large generating set K for Γ such that, for every conditionally negative type function $\psi : \Gamma \rightarrow \mathbf{R}$,*

$$\sup_{g \in \Gamma} |\psi(g)| \leq (4 + \varepsilon) \sup_{g \in K} |\psi(g)|.$$

Proof. This follows from Corollary 5 and Theorem 2. $\square$

4. A DYNAMICAL CONSEQUENCE OF PROPERTY (T)

Theorem 7. *Let G be a locally compact group with compact generating set K . Suppose that there is a measure-preserving action of G on an infinite measure space $(X, \mathcal{B}, \mu)$ and there is a set $S \in \mathcal{B}$ such that $\mu(S\Delta gS) < \infty$ for all $g \in K$. Then $\mu(S\Delta gS) < \infty$ for all $g \in G$. Suppose, moreover, that G has Property (T) and that $\mu(S\Delta gS)$ is a continuous function of $g \in G$. Then $\sup_{g \in G} \mu(S\Delta gS) < \infty$. In fact,*

$$\sup_{g \in G} \mu(S\Delta gS) \leq \left(\frac{2}{\kappa(G, K)} \right)^2 \sup_{g \in K} \mu(S\Delta gS).$$

Proof. In order to prove the first assertion, observe that $\mu(S\Delta gS) = \mu(S\Delta g^{-1}S)$ for all $g \in G$. Now using the fact that for $g_1, g_2 \in S$, $S\Delta g_1 g_2 S \subset (S\Delta g_1 S) \cup (g_1 S\Delta g_1 g_2 S)$ we obtain by induction that $\mu(S\Delta gS) < \infty$ for all $g \in G$. In order to prove the second assertion, define a function $\eta : G \rightarrow L^2(X, \mu)$ by

$$\eta(g) = \chi_{gS} - \chi_S = \chi_{gS-S} - \chi_{S-gS}$$

where χ_A denotes the characteristic function of a set $A \in \mathcal{B}$. The function ψ defined by

$$\psi(g) = \|\eta(g)\|_2^2 = \mu(S\Delta gS)$$

satisfies $\psi(e) = 0$. To see that ψ is of conditionally negative definite type, note that

$$\psi(g^{-1}h) = \|\chi_{hS} - \chi_{gS}\|_2^2 = \|\eta(h) - \eta(g)\|_2^2.$$

Now if $g_1, g_2, \dots, g_n \in G$ and $\alpha_1, \alpha_2, \dots, \alpha_n$ are real numbers with $\sum \alpha_i = 0$, then, writing $\xi_i = \eta(g_i)$,

$$\begin{aligned} \sum \alpha_i \alpha_j \psi(g_i^{-1} g_j) &= \sum \alpha_i \alpha_j \|\xi_i - \xi_j\|_2^2 \\ &= -2 \sum \alpha_i \alpha_j \langle \xi_i, \xi_j \rangle = -2 \left\| \sum \alpha_i \xi_i \right\|_2^2 \leq 0. \end{aligned}$$

Since G has Property (T), it follows that ψ is bounded. This proves the result, and the stated inequality follows from Theorem 2. $\square$

Remarks. 1. The result can fail if G does not have Property (T). For example, let G be a free group. There exists a homomorphism from G onto $\mathbb{Z}$. Take $X = \mathbb{Z}$, μ = counting measure, and $S = \{1, 2, 3, \dots\}$, and let G act on X by translation. Then $\mu(S\Delta gS) < \infty$ for all $g \in G$ but $\sup_{g \in G} \mu(S\Delta gS) = \infty$.

2. Continuity of $g \mapsto \mu(S\Delta gS): G \rightarrow \mathbb{R} \cup \{\infty\}$ at the identity element e is sufficient to guarantee continuity on all of G . To see this, let $\varepsilon > 0$ and choose a neighbourhood U of e in G such that $\mu(S\Delta hS) < \varepsilon$ for all $h \in U$. Then for any $g \in G$, gU is a neighbourhood of g and, for all $h \in U$,

$$|\mu(S\Delta ghS) - \mu(S\Delta gS)| \leq \mu(gS\Delta ghS) = \mu(S\Delta hS) < \varepsilon.$$

Hence, $g \mapsto \mu(S\Delta gS)$ is continuous on G .

3. An alternative proof of the boundedness part of Theorem 7 can be given using the cohomological definition of Property (T) [HaV, Chapitre 4] and the cocycle η defined above. It follows that η is necessarily a coboundary and hence bounded.

Our next result is proved by the same argument as in the proof of Theorem 7.

Proposition 8. *Let G be a Kazhdan group with compact generating set K and Haar measure dh . For any continuous function $f: G \rightarrow \mathbb{C}$ we have*

$$\sup_{g \in G} \int_G |f(hg) - f(h)|^2 dh \leq \left(\frac{2}{\kappa(G, K)} \right)^2 \sup_{g \in K} \int_G |f(hg) - f(h)|^2 dh.$$

Proof. We may assume that the right-hand side of the inequality is finite. Define a function $\eta: G \rightarrow L^2(G)$ by $\eta(g)(h) = f(hg) - f(h)$. Now apply the argument as before to the conditionally negative definite function $\psi(g) = \|\eta(g)\|_2^2$. $\square$

Note that we have not assumed that $f \in L^2(G)$, as was the case for a similar inequality for a free group [Sak, Lemma 4.3.20].

5. AN ESTIMATE FOR KAZHDAN CONSTANTS FOR PARADOXICAL DECOMPOSITIONS

Suppose that a group Γ acts on a set X with no finitely additive G -invariant measure. For example, X could be a nonamenable group and $G = X$. Then by

Tarski's Theorem [Pat, 3.15], X is G -paradoxical. That is, there exists a partition $A_1, \dots, A_m, B_1, \dots, B_n$ of X and elements $x_1, \dots, x_m, y_1, \dots, y_n$ of Γ such that $\{x_1 A_1, \dots, x_m A_m\}$ is a partition of X and $\{y_1 B_1, \dots, y_n B_n\}$ is a partition of X . Let $K = \{x_1, \dots, x_m, y_1, \dots, y_n\}$, and consider the group $\langle K \rangle$ generated by K . Let π denote the quasi-regular representation of $\langle K \rangle$ on $l^2(X)$, that is,

$$\pi(g)f(x) = f(g^{-1}x), \quad g \in \langle K \rangle, f \in l^2(X), x \in X.$$

Proposition 9. $\kappa(\pi, K) \geq \frac{1}{2(m+n)}.$

Proof. For a subset S of X let E_S denote the orthogonal projection of $l^2(X)$ onto $l^2(S)$, that is,

$$E_S \xi(g) = \begin{cases} \xi(g) & \text{if } g \in S, \\ 0 & \text{otherwise.} \end{cases}$$

Direct calculation shows that $\pi(g)E_S\pi(g^{-1}) = E_{gS}$.

Let $v_i = \pi(x_i)$, $e_i = E_{A_i}$ ($1 \leq i \leq m$) and $w_j = \pi(y_j)$, $f_j = E_{B_j}$ ($1 \leq j \leq n$).

Then we can write the paradoxical decomposition in terms of bounded operators on $l^2(X)$ as follows:

$$1 = \sum e_i + \sum f_j = \sum v_i e_i v_i^* = \sum w_j f_j w_j^*.$$

Let $\varepsilon = \kappa(\pi, K)$, and let $\xi \in l^2(X)$ with $\|\xi\| = 1$. Then

$$\|v_i \xi - \xi\| \leq \varepsilon, \quad \|w_j \xi - \xi\| \leq \varepsilon.$$

Hence, also

$$\|v_i^* \xi - \xi\| \leq \varepsilon, \quad \|w_j^* \xi - \xi\| \leq \varepsilon$$

for $1 \leq i \leq m$, $1 \leq j \leq n$.

Now $\varepsilon \geq \|e_i(v_i^* \xi - \xi)\| \geq \|e_i v_i^* \xi\| - \|e_i \xi\|$. Therefore,

$$\|e_i v_i^* \xi\|^2 - \|e_i \xi\|^2 = \|e_i v_i^* \xi\| + \|e_i \xi\| \|e_i v_i^* \xi\| - \|e_i \xi\| \leq 2\|\xi\|\varepsilon \leq 2\varepsilon$$

and so $\|e_i v_i^* \xi\|^2 \leq \|e_i \xi\|^2 + 2\varepsilon$. Similarly $\|f_j w_j^* \xi\|^2 \leq \|f_j \xi\|^2 + 2\varepsilon$. It follows that

$$1 = \|\xi\|^2 = \sum_i \langle v_i e_i v_i^* \xi | \xi \rangle = \sum_i \|e_i v_i^* \xi\|^2 \leq \sum_i \|e_i \xi\|^2 + 2m\varepsilon$$

and similarly

$$1 \leq \sum_j \|f_j \xi\|^2 + 2n\varepsilon.$$

Now $1 = \sum e_i + \sum f_j$. Therefore,

$$1 = \sum_i \|e_i \xi\|^2 + \sum_j \|f_j \xi\|^2 \geq 1 - 2m\varepsilon + 1 - 2n\varepsilon,$$

that is, $2(m+n)\varepsilon \geq 1$, as required. $\square$

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