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WEAK CONVERGENCE AND WEAK COMPACTNESS IN ABSTRACT M SPACES

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ABSTRACT. This paper presents some properties of bounded linear functionals on σ complete abstract M spaces, from which some criteria for weak convergence and weak compactness in such spaces are obtained.

1. ABSTRACT M SPACES AND ABSTRACT L SPACES

Definition 1 [1, 2]. Let X be a Banach lattice.

(1) X is called an abstract M space $(X \in AM)$ if $x \wedge y = 0$ implies
$$\|x+y\| = \max\{\|x\|, \|y\|\}.$$

(2) X is called an abstract L space $(X \in AL)$ if $x \wedge y = 0$ implies
$$\|x+y\| = \|x\| + \|y\|.$$

For a Banach lattice X and $x \in X$, $f, g \in X^*$, as in [1], we define
$$(f \vee g)(x) = \sup\{f(u) + g(x-u) : 0 \le u \le x\} \qquad (x \ge 0),$$
$$(f \wedge g)(x) = \inf\{f(u) + g(x-u) : 0 \le u \le x\} \qquad (x \ge 0).$$
Then by Theorem 118.1 and 118.5 in [3], we have

Lemma 2. *Let X be a Banach lattice. Then*

(1) *$X \in AM$ implies $X^* \in AL$ and $X \in AL$ implies $X^* \in AM$;*

(2) *$X \in AM$ iff for any $x, y \in X$, $x, y \ge 0$ implies*
$$\|x \vee y\| = \max\{\|x\|, \|y\|\};$$

(3) *$X \in AL$ iff for any $x, y \in X$, $x, y \ge 0$ implies*
$$\|x+y\| = \|x\| + \|y\|.$$

Let X be a lattice and $x, u, v \in X$ and $u \ge 0$, $v \ge 0$. By Theorem 11.8 and 11.10 in [2], if $x = u - v$, then $u = x^+ + u \wedge v$ and $v = x^- + u \wedge v$, where $x^+ = x \vee 0$ and $x^- = (-x) \vee 0$. Especially, if $u \wedge v = 0$, then $u = x^+$ and $v = x^-$. If $X \in AL$, then $\|x\| = \|x^+\| + \|x^-\|$ and by Lemma 2, $\|u\| = \|x^+\| + \|u \wedge v\|$, $\|v\| = \|x^-\| + \|u \wedge v\|$. Hence, we have

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Lemma 3. *Let $X \in AL$ and $x \in X$. Then the above decomposition $x = x^+ - x^-$ is unique in the sense that if $x = u - v$, $u \geq 0$, $v \geq 0$, and $\|u\| + \|v\| = \|x\|$, then $u = x^+$ and $v = x^-$.*

For a subset E of a Banach lattice X and $x \in X$, we write

$$E^\perp = \{x \in X : x \perp y \text{ for all } y \in E\}, \qquad x^\perp = \{x\}^\perp,$$

where $x \perp y$ means $|x| \wedge |y| = 0$. If $x \in X = E + E^\perp$, then x can be uniquely decomposed into $x = u + v$, where $u \in E$ and $v \in E^\perp$. In this case, we write $x|_E = u$ and $f|_E(x) = f(u)$ for $f \in X^*$.

Definition 4. Let X be a Banach lattice. Then

(a) X is said to be σ complete, if for every order bounded sequence $\{x_n\}$ in X, $\bigvee_{n\geq 1}(x_n)$ exists in X.

(b) X is said to be bounded σ complete, provided that any norm bounded and order monotone sequence in X is order convergent.

Clearly, bounded σ complete Banach lattices are σ complete. The inverse does not hold; for instance, c_0 is σ complete but not bounded σ complete. Moreover, according to [1], the space $C(K)$ of all continuous functions on a compact Hausdorff topological space K is σ complete if and only if K is basically disconnected, i.e., the closure of every open F_σ subset of K is an open set.

For more detail about Banach lattices, also see [4] and [5].

2. Bounded linear functionals on abstract M spaces

For a Banach space X, we always denote by $B(X)$ and $S(X)$ the unit ball and the unit sphere of X respectively.

Theorem 5. *Let $X \in AM$ be σ complete and $f \in X^*$. Then for any $\varepsilon > 0$, there exists a subspace E of X such that $X = E + E^\perp$ and $\|f^+|_{E^\perp}\| < \varepsilon$, $\|f^-|_E\| < \varepsilon$.*

Proof. Pick $x \in S(X)$ satisfying $f(x) > \|f\| - \varepsilon$, and set $E = (x^-)^\perp$. Then $x^+ \in E$, $x^- \in E^\perp$, and by [1] $X = E + E^\perp$. Moreover, by Lemma 2,

$$\begin{aligned}
&\|f^+|_E\| + \|f^+|_{E^\perp}\| + \|f^-|_E\| + \|f^-|_{E^\perp}\| \\
&\quad = \|f^+\| + \|f^-\| = \|f\| < f(x) + \varepsilon \\
&\quad = f^+|_E(x) + f^+|_{E^\perp}(x) - f^-|_E(x) - f^-|_{E^\perp}(x) + \varepsilon.
\end{aligned}$$

Since $f^+|_{E^\perp}(x) \leq 0$ and $f^-|_E(x) \geq 0$, we find

$$\begin{aligned}
&\|f^+|_{E^\perp}\| + \|f^-|_E\| \\
&\quad = \|f^+\| - \|f^+|_E\| + \|f^-\| - \|f^-|_{E^\perp}\| \\
&\quad \leq \|f^+\| - f^+|_E(x) + \|f^-\| - f^-|_{E^\perp}(x) \\
&\quad < f^+|_{E^\perp}(x) - f^-|_E(x) + \varepsilon \leq \varepsilon. \quad \square
\end{aligned}$$

Theorem 6. *If a Banach lattice X is bounded σ complete and $B(X)$ is order closed, then every positive $f \in X^*$ (i.e., $f \geq 0$) is norm attainable, i.e., there exists $x \in S(X)$ satisfying $f(x) = \|f\|$.*

Proof. Pick $x_n(\geq 0) \in S(X)$ such that $f(x_n) \to \|f\|$. Since X is bounded σ complete and $B(X)$ is order closed, $y = \bigvee_n(x_n)$ exists in X and $\|y\| = 1$. Hence, $y \geq x_n \geq 0$ and $f \geq 0$ implies $\|f\| \geq f(y) \geq f(x_n) \to \|f\|$. $\square$

Remark. If $X \in AM$ is not bounded σ complete, then the conclusion of Theorem 6 may be false. For instance, if $X = c_0$ and $f = (c_n) \in l_1$ with infinitely many $c_n \neq 0$, then f is not norm attainable.

If $B(X)$ is not order closed, then the statement of the theorem is not necessarily true. For example, take $X = l_\infty$ and define

$$|||x||| = \sup_{n \geq 1} \left\{ |x_n|, k \cdot \limsup_{i \to \infty} |x_i| \right\}, \quad x = (x_n) \in l_\infty,$$

where $k > 1$ is a constant. Then the norm $|||\cdot|||$ satisfies $\|x\|_\infty \leq |||x||| \leq k\|x\|_\infty$ for all $x \in l_\infty$ and $\|x\|_\infty = |||x|||$ for all $x \in c_0$. But for any bounded linear functional $f = (c_n) \in l_1$ on l_∞ with infinitely many $c_n \neq 0$, f cannot attain its norm on $B(l_\infty, |||\cdot|||)$.

Theorem 7. *Let $X \in AM$ be bounded σ complete and $B(X)$ order closed. Then $f \in X^*$ is norm attainable iff there exists a subspace E of X such that $f^+ = f|_E$, $f^- = -f|_{E^\perp}$.*

Proof. Sufficiency. By Theorem 6, there exist $x, y (\geq 0) \in S(X)$ such that $f^+(x) = \|f^+\|$ and $f^-(y) = \|f^-\|$. Since $f^+ = f|_E$ and $f^- = -f|_{E^\perp}$, we may assume $x \in E$ and $y \in E^\perp$ (otherwise we replace x, y by $x|_E$, $y|_{E^\perp}$ respectively). Let $u = x - y$. Then $\|u\| = \|x - y\| = \max\{\|x\|, \|y\|\} = 1$ and hence, Lemma 2 implies

$$\begin{aligned} \|f\| &= \|f^+\| + \|f^-\| = f^+(x) + f^-(y) \\ &= f|_E(x) + f|_{E^\perp}(-y) = f(u). \end{aligned}$$

Necessity. Choose $x \in S(X)$ satisfying $f(x) = \|f\|$, and define $E = (x^-)^\perp$. Then $X = E + E^\perp$ and $x^+ \in E$, $x^- \in E^\perp$. Observe that $\|f\| = \|f|_E\| + \|f|_{E^\perp}\|$; to prove $f^+ = f|_E$ and $f^- = -f|_{E^\perp}$, it suffices to show $f|_E \geq 0$ and $-f|_{E^\perp} \geq 0$ thanks to Lemma 3. Indeed, if $f|_E(y) < 0$ for some $y (\geq 0) \in S(X)$, then we may assume $y \in E$. Therefore, $z = -x^- - y$ satisfies $\|z\| = \max\{\|x^-\|, \|y\|\} = 1$ and thus,

$$\begin{aligned} \|f^-\| &\geq f^-(-z) = f(z) - f^+(z) \geq f(z) \\ &= f|_{E^\perp}(-x^-) - f|_E(y) > f|_{E^\perp}(-x^-) = -f|_{E^\perp}(x). \end{aligned}$$

Since $\|f^+\| \geq f(x|_E) = f|_E(x)$, this leads to a contradiction that

$$\|f\| = \|f^+\| + \|f^-\| > f|_E(x) - f|_{E^\perp}(x) = f(x) = \|f\|.$$

Similarly, we can verify $-f|_{E^\perp} \geq 0$. $\square$

Definition 8 [6]. Let X be a Banach space. $x \in S(X)$ is called an extreme point of $B(X)$ if $x = \lambda y + (1 - \lambda) z$, $y, z \in B(X)$ and $\lambda \in (0, 1)$, imply $y = z$. In this case, we write $x \in \operatorname{ext} B(X)$.

Since by the Rainwater Theorem [6], $x_n \to 0$ weakly in a Banach space X iff $\{x_n\}$ is bounded, and $f(x_n) \to 0$ for every $f \in \operatorname{ext} B(X^*)$, we are encouraged to investigate the extreme points of the unit ball of a dual space.

Theorem 9. *Let $X \in AM$ be σ complete and $f \in S(X^*)$. Then $f \in \operatorname{ext} B(X^*)$ iff $f(x)f(y) = 0$ for all $x, y \in X$ satisfying $x \wedge y = 0$.*

Proof. Sufficiency. First we show $\|f^+\|\,\|f^-\| = 0$. In fact, for any $\varepsilon > 0$, by Theorem 5, there exist two orthogonal subspaces E, F of X such that

$X = E + F$ and $\|f^-|_E\| < \varepsilon$, $\|f^+|_F\| < \varepsilon$. Choose $x \in S(X)$ such that $f(x) > \|f\| - \varepsilon$, and let $x = u+v$, where $u \in E$ and $v \in F$. Then $f(u)f(v) = 0$ since $u \wedge v = 0$. If $f(v) = 0$, then

$$\begin{aligned}\|f\| - \varepsilon < f(x) &= f^+|_E(u) - f^-|_E(u) \\ &\leq \|f^+|_E\| + \|f^-|_E\| < \|f^+\| + \varepsilon.\end{aligned}$$

Letting $\varepsilon \to 0$, we find $\|f^-\| = \|f\| - \|f^+\| = 0$. Similarly, if $f(u) = 0$. Then $\|f^+\| = 0$. Hence, without loss of generality, we may assume $f = f^+$.

Let $g, h \in S(X^*)$ satisfy $2f = g + h$. Then $2f = (g^+ + h^+) - (g^- + h^-)$ and by Lemma 2,

$$\begin{aligned}\|2f\| &\leq \|g^+\| + \|h^+\| + \|g^-\| + \|h^-\| \\ &= \|g\| + \|h\| = 2 = \|2f\|.\end{aligned}$$

It follows from Lemma 3 that $g^+ + h^+ = 2f$ and $g^- = h^- = 0$.

Now we show $g = h = f$, i.e., $f \in \operatorname{ext} B(X^*)$. This follows if we prove that $g(y) = h(y) = 0$ whenever $f(y) = 0$ (by [7, §1.5, Theorem 1], this means $f = ag = bh$, but $f, g, h \in S(X^*)$ and $2f = g + h$, so $a = b = 1$). First we assume $y \geq 0$; then from $g(y) \geq 0$, $h(y) \geq 0$, and $g(y) + h(y) = 2f(y) = 0$ we have $g(y) = h(y) = 0$. For the general case, since $f(y) = 0$ and by the condition given in the theorem, $f(y^+)f(y^-) = 0$, we have $f(y^+) = f(y^-) = 0$. Hence, $g(y) = h(y) = 0$ follows from the first case.

Necessity. If there exist $x, y \in X$ satisfying $x \wedge y = 0$ but $f(x) > 0$ and $f(y) > 0$, then we set $E = y^\perp$, and then by [1] $X = E + E^\perp$. Let $g = f|_E$ and $h = f|_{E^\perp}$. Then $\|g\| > 0$, $\|h\| > 0$ since $x \in E$, $y \in E^\perp$. Therefore, from

$$f = \|g\| \frac{g}{\|g\|} + \|h\| \frac{h}{\|h\|}$$

and $\|g\| + \|h\| = \|f\| = 1$ according to Lemma 2, we see $f \in \operatorname{ext} B(X^*)$. □

3. Weak convergence and weak compactness in abstract M spaces

We begin with a lemma.

Lemma 10. *Let X be a σ complete lattice. Then for any $x_1, \dots, x_m \in X$, X can be decomposed into m many pairwise orthogonal subspaces. $X = E_1 + \cdots + E_m$ such that $(x_n - \bigwedge_{1 \leq m}(x_i))|_{E_n} = 0$, $1 \leq n \leq m$.*

Proof. Since for any $x, y, z \in X$, $(x - z) \wedge (y - z) = x \wedge y - z$, replacing z by $x \wedge y$, we obtain

$$(*) \qquad (x - x \wedge y) \perp (y - x \wedge y).$$

Set $\bigwedge_{1 \leq n \leq m}(x_n) = x'$ and $E_1 = (x_1 - x')^\perp$. Then by [1], $X = E_1 + E_1^\perp$. Moreover, replacing x, y by x_1, $\bigwedge_{2 \leq n \leq m}(x_n)$ in $(*)$ respectively, we see

$$(x_1 - x')|_{E_1} = 0, \qquad \left(\bigwedge\nolimits_{2 \leq n \leq m}(x_n) - x'\right)|_{E_1^\perp} = 0.$$

Let $E_2 = \{x \in E_1^\perp : x \perp (x_2 - x')|_{E_1^\perp}\}$. Then we also have $E_1^\perp = E_2 + E_2^\perp \cap E_1^\perp$. Again by $(*)$ (replace x, y by $x_2|_{E_1^\perp}$, $\bigwedge_{3 \leq n \leq m}(x_n)$ respectively there), we have

$$(x_2 - x')|_{E_2} = 0, \qquad \left(\bigwedge\nolimits_{3 \leq n \leq m}(x_n) - x'\right)|_{E_2^\perp} = 0.$$

And so on, we find pairwise orthogonal subspaces $E_1, \dots, E_{m-1}, E_m = E_{m-1}^\perp \cap E_{m-2}^\perp$ of X such that $X = E_1 + \cdots + E_m$ and $(x_n - x')|_{E_n} = 0$, $n \le m$. □

Theorem 11. *Let $X \in AM$ be σ complete. Then $x_n \to 0$ weakly in X iff $\{x_n\}$ is bounded and* $\lim_{m\to\infty} \|\bigwedge_{i\le m}(|x_{n_i}|)\| = 0$ *for all subsequences $\{x_{n_i}\}$ of $\{x_n\}$.*

Proof. Sufficiency. If $\{x_n\}$ does not converge to zero weakly, then by the Rainwater Theorem there exist some $f \in \operatorname{ext} B(X^*)$, $\varepsilon > 0$, and a subsequence of $\{x_n\}$, again denoted by $\{x_n\}$, such that $f(x_n) > \varepsilon$ for all $n \ge 1$. Since by the proof of Theorem 9, $f^+ = 0$ or $f^- = 0$ and $f(x_n) = f^+(x_n^+) + f^-(x_n^-) - f^-(x_n^+) - f^+(x_n^-)$, without loss of generality, we may assume $f \ge 0$ and $x_n \ge 0$ for all $n \ge 1$. Choose $m \ge 1$ such that $\|\bigwedge_{n\le m}(x_n)\| < \varepsilon$. Then by Lemma 10, X can be decomposed into the direct sum of pairwise orthogonal subspaces $E_1, \dots, E_m$ such that $x_n|_{E_n} = x'|_{E_n}$ for all $n \le m$, where $x' = \bigwedge_{n\le m}(x_n)$. By Theorem 9, there exists some $n \le m$ such that $f = f|_{E_n}$ which leads to a contradiction that

$$\varepsilon < f(x_n) = f(x_n|_{E_n}) = f(x'|_{E_n}) \le \|f\| \cdot \|x'\| < \varepsilon.$$

Necessity. Suppose that $x_n \to 0$ weakly in X. If the condition is not necessary, then there exist a constant $\varepsilon > 0$ and a subsequence of $\{x_n\}$, again denoted by $\{x_n\}$, satisfying $\|\bigwedge_{n\le m}(|x_n|)\| > 2\varepsilon$ for all $m \ge 1$. We first define $y_1^1 = x_1^+$ and $y_2^1 = x_1^-$. Suppose that $\{y_s^k : s \le 2^k, k \le m\}$ have already been defined. Then we set $y_{2s-1}^{m+1} = y_s^m \wedge x_{m+1}^+$ and $y_{2s}^{m+1} = y_s^m \wedge x_{m+1}^-$. By induction, we find $\{y_i^m\}$ satisfying $y_i^m \wedge y_j^m = 0$ for all $m \ge 1$ and all $i, j \le 2^m$ with $i \ne j$, and moreover, for any $k \le m$, we have either $x_k^+ \wedge y_s^m = 0$ or $x_k^- \wedge y_s^m = 0$ for each $s = 1, 2, \dots, 2^m$. Hence, if we pick $j \le 2^m$ such that $z_m = y_j^m$ satisfies $\|z_m\| = \max_{j\le 2^m} \|y_j^m\|$, then

$$\|z_m\| = \left\|\sum_{i\le 2^m} y_i^m\right\| = \left\|\bigwedge\nolimits_{n\le 2^m}(x_n)\right\| > 2\varepsilon.$$

Next, we select $f_m \in S(X^*)$ such that $f_m(z_m) = \|z_m\|$. Since $z_m \ge 0$ and $X^* \in AL$, we must have $f_m \ge 0$. In view of the Alaoglu Theorem [6], $\{f_m\}$ has a w^*-cluster $f \in B(X^*)$. It follows that for each fixed $n \ge 1$, we can find some $m \ge n$ such that $|f(x_n) - f_m(x_n)| < \varepsilon$. Let $F_m = z_m^\perp$ and $E_m = F_m^\perp$. Then $X = E_m + F_m$ by [1]. Note that $X^* \in AL$ implies $\|f_m\| = \|f_m|_{E_m}\| + \|f_m|_{F_m}\|$; from the fact

$$1 \ge \|f_m|_{E_m}\| \ge f_m\left(\frac{z_m}{\|z_m\|}\right) = 1$$

we see $\|f_m|_{F_m}\| = 0$. Since by the choice of z_m, $m \ge n$ implies that $x_n^+ \wedge z_m = 0$ or $x_n^- \wedge z_m = 0$, we may assume $x_n^+ \wedge z_m = 0$. Thus, $x_n^-|_{E_m} \ge z_m|_{E_m}$, and so

$$\begin{aligned} |f(x_n)| &\ge |f_m(x_n)| - |f(x_n) - f_m(x_n)| \\ &> |f_m(x_n)| - \varepsilon = |f_m|_{E_m}(x_n)| - \varepsilon \\ &\ge f_m|_{E_m}(z_m) - \varepsilon = f_m(z_m) - \varepsilon \\ &= \|z_m\| - \varepsilon > \varepsilon, \end{aligned}$$

which contradicts the hypothesis that $x_n \to 0$ weakly. □

Theorem 12. *Let X be a dual σ complete AM space. Then a bounded subset A of X is weakly compact iff*

$$\sup_{(x_n)\subset A} \lim_{m\to\infty} \inf_{x\in K} \left\| \bigwedge\nolimits_{n\le m} (|x_n - x|) \right\| = 0$$

where $K = K(x_n)$ is the set of sequentially w^*-clusters of $\{x_n\}$ and, as usual, we denote $\inf\{r : r \in E\} = +\infty$ for $E = \varnothing$.

Proof. Necessity. Let A be a weakly compact subset of X. Then for any sequence $\{x_n\}$ in A we can pick a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ weakly convergent to some point x in X and then obviously $x \in K = K(x_n)$. Therefore, it follows from Theorem 11

$$\begin{aligned} 0 &= \lim_{m\to\infty} \left\| \bigwedge\nolimits_{i\le m} (|x_{n_i} - x|) \right\| \\ &\ge \lim_{m\to\infty} \left\| \bigwedge\nolimits_{n\le m} (|x_n - x|) \right\| \\ &\ge \lim_{m\to\infty} \inf_{y\in K} \left\| \bigwedge\nolimits_{n\le m} (|x_n - y|) \right\| \ge 0. \end{aligned}$$

Sufficiency. For any sequence $\{x_n\}$ in A, by the given condition, $K = K(x_n) \ne \varnothing$, hence, $\{x_n\}$ contains a subsequence, again denoted by $\{x_n\}$, w^*-convergent to some point $x \in K$. Hence, for any subsequence $\{x_{n_i}\}$ of this subsequence, $K' = K(x_{n_i}) = \{x\}$ implies

$$\lim_{m\to\infty} \left\| \bigwedge\nolimits_{i\le m} (|x_{n_i} - x|) \right\| = \lim_{m\to\infty} \inf_{y\in K'} \left\| \bigwedge\nolimits_{i\le m} (|x_{n_i} - y) \right\| = 0.$$

By Theorem 11, $x_n \to x$ weakly. □

Remark 1. Replacing X in Theorem 11 or Theorem 12 by L_∞ or l_∞, we obtain criteria of weak convergence and weak compactness for those spaces. But for $X = l_\infty$, since w^*-convergence of a bounded sequence in X coincides with convergence in coordinates, which is also equivalent to weak convergence in $X = c_0$, we can prove, without any difficulties, the following corollary and from which one can easily deduce the relative results given in [8].

Corollary 13. *A bounded subset A of l_∞ or c_0 is weakly compact iff*

$$\sup_{(x_n)\subset A} \lim_{m\to\infty} \left\| \liminf_{k\to\infty} \min_{n\le m} (|x_n - x_k|) \right\| = 0.$$

Remark 2. By [1], if an AM space X has a strong unit e, i.e., $x \in B(X)$ if and only if $|x| \le e$, then X is order isometric to a $C(K)$ space for an appropriate compact Hausdorff space K. However, in this paper, the AM spaces are not assumed to have any units.

References

1. J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces.* II, Springer-Verlag, Berlin and New York, 1979.
2. W. A. J. Luxemburg and A. C. Zaanen, *Riesz spaces.* I, North-Holland, Amsterdam and London, 1971.
3. A. C. Zaanen, *Riesz spaces.* II, North-Holland, Amsterdam, New York, and Oxford, 1983.
4. L. V. Kantorovitch, B. Z. Vulih, and A. G. Pinsker, *Functional analysis in partially order spaces,* Gosudarstr, Izdat. Tehn. Teor. Lit., Moscow and Leningrad, 1950. (Russian)

5. K. Yosida, *Functional analysis*, Springer-Verlag, Berlin and New York, 1978.
6. J. Diestel, *Sequences and series in Banach spaces*, Springer-Verlag, Berlin and New York, 1984.
7. A. Wilansky, *Modern methods in topological vector spaces*, McGraw-Hill, New York, 1987.
8. N. Dunford and J. T. Schwartz, *Linear operators*, Interscience, New York, 1958.

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