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SOME WEAKLY INNER AUTOMORPHISMS OF THE CUNTZ ALGEBRAS

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ABSTRACT. Let V be an $n \times n$ unitary matrix. Then V induces an automorphism O_V of O_∞ and O_n. It is shown in this paper that O_V is weakly inner. Let F_n be the UHF C^*-subalgebra of O_n and U be a unitary operator in the diagonal maximal abelian *-subalgebra of F_n. Then the automorphism λ_U of O_n defined by $\lambda_U(S_i) = U^* S_i$, for each generator S_i of O_n, is weakly inner. Any automorphism conjugate to either one mentioned above is also weakly inner.

1. INTRODUCTION

In 1977, J. Cuntz studied a class of separable simple infinite C^*-algebras [3], which are denoted by O_n's for $n = 2, 3, \ldots$. O_n is also a crossed product of a UHF-subalgebra F_n (of O_n) by a single endomorphism of F_n, scaling the trace of F_n. Since 1977 there has been a good amount of work on various properties of O_n. In this paper we concentrate on some automorphisms of O_n and O_∞ (see the definition in Section 2). Earlier works exhibited some outer automorphisms [1], [2], [6], [7]. A theorem proved by Kishimoto [10] in answering a longstanding question raised in [11] by C. Lance states that an automorphism of a separable simple unital C^*-algebra is inner if and only if it is universally weakly inner (see the definition in Section 2). Here we try to determine which outer automorphisms are weakly inner (see the definition in Section 2) and succeed in describing a large class of weakly inner automorphisms, containing outer automorphisms studied in [2], [6], [7]. The notion of weak innerness can first be found in [9]. The results are contained in Sections 3 and 4.

2. NOTATION AND SOME LEMMAS

Let H be an infinite-dimensional Hilbert space. For $n = 2, 3, \ldots$, let $\{S_1, \ldots, S_n\}$ be n isometries on H such that

$$\sum_{i=1}^{n} S_i S_i^* = I. \tag{2.1}$$

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Let O_n be the C^*-algebra generated by $S_1, \ldots, S_n$. For $n = \infty$ let $\{S_i\}_{i=1}^n$ be a sequence of isometries on H such that

$$\sum_{i=1}^{k} S_i S_i^* \leq I \quad \text{for } k = 1, 2, \ldots. \tag{2.2}$$

Let O_∞ be the C^*-algebra generated by $\{S_i\}_{i=1}^\infty$. It turns out that the definition of O_n for $n = 1, 2, \ldots, \infty$ is space free and depends only on a set of isometries satisfying (2.1) or (2.2) (see 1.12 in [3]). These algebras are contained in every simple C^*-algebra which contains an infinite projection [4]. Following the notation by Cuntz in [3], for $k = 0, 1, \ldots$ we let W_k^n be the set of all k-tuples $(j_1, \ldots, j_k)$ with $j_i \in \{1, \ldots, n\}$, for $i = 1, \ldots, n$, and $W = \bigcup_{k=0}^\infty W_k^n$ with $W_0^n = \{0\}$. We write $S_0 = 1$, and for $\mu = (j_1, \ldots, j_k) \in W_k^n$ we set $S_\mu = S_{j_1} \cdots S_{j_k}$ and $l(\mu) = k$.

For a finite positive integer $n \geq 2$, O_n has a C^*-subalgebra, F_n, the closed linear span of terms of the form $S_\mu S_\nu^*$ with $l(\mu) = l(\nu)$, which is a UHF-algebra of type n^∞. Consider

$$F_n = \overline{\bigcup_{k=1}^{\infty} \mathbb{A}_k},$$

where

$$\mathbb{A}_k = \bigotimes_{i=1}^{k} M_{n_i}$$

and $n_i = n$ for $i = 1, \ldots, k$, and M_n is the algebra of $n \times n$ (complex) matrices. Let D_k be the subalgebra of all diagonal matrices of $\mathbb{A}_k$ and

$$D = \overline{\bigcup_{k=1}^{\infty} D_k}.$$

D is a maximal abelian subalgebra in both F_n and O_n, and D is also the closed linear span of terms of the form $S_\mu S_\mu^*$ for $\mu \in W_k^n$, for $k = 0, 1, \ldots$. Let P be the conditional expectation of O_n onto D. In the following definition we write S for S_1 and S^{-1} for S_1^*. Let

$$\mathbb{A} = \left\{ \sum_{i=-n}^{-1} S^i A_i + A_0 + \sum_{i=1}^{n} A_i S^i \,\middle|\, A_i \in \bigcup_{k=1}^{\infty} \mathbb{A}_k \right\}.$$

It is easy to see that $\mathbb{A}$ is a norm-dense *-algebra of O_n. Let α be any endomorphism of O_n such that $\alpha(1) = 1$. Then there is a unitary element $Z_\alpha \in O_n$ such that $\alpha(S_i) = Z_\alpha^* S_i$ $(i = 1, 2, \ldots, n)$. In fact, $Z_\alpha^* = \sum_{i=1}^n \alpha(S_i) S_i^*$. On the other hand, every unitary element $U \in O_n$ determines a unique isometric endomorphism λ_U of O_n such that $\lambda_U(S_i) = U^* S_i$ $(i = 1, \ldots, n)$ and $\lambda_U(1) = 1$. This was first observed by M. Takesaki and communicated to J. Cuntz. From Proposition 1.1 in [5] we know that Z_α satisfies a cocycle identity, λ is a twisted representation, and λ_U is an automorphism if and only if there is a unitary $V \in O_n$ such that $\lambda_U(V) = U^*$. Let Φ denote the endomorphism of O_n defined by $\Phi(x) = \sum_{i=1}^n S_i x S_i^*$. Let $V = (V_{ij})$ be a unitary operator on C_n, the n-dimensional complex Hilbert space. Then it is easy to check that $\{T_i = \sum_{k=1}^n V_{ki} S_k ; i = 1, 2, \ldots, n\}$ generate the same C^*-algebra O_n and V

induces on O_n an automorphism O_V by $O_V(S_i) = T_i$, for $i = 1, 2, \ldots, n$. In 1979, Archbold studied the case $n = 2$ and $V = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ and found O_V is outer yet weakly inner [1]. An automorphism β of a C^*-algebra A is said to be universally weakly inner if β^{**} on A^{**} is inner, and β is said to be weakly inner if there exists a faithful representation π of A such that, in identifying A and $\pi(A)$, β extends to an inner automorphism on the weak-operator closure $\overline{\pi(A)}^w$. Subsequently it was shown in [6], [7] that O_V is outer for all unitary V in C^n for $n = 2, 3, \ldots, \infty$. In 1981, Kishimoto [10] resolved a longstanding question with an affirmative answer, that is, an automorphism on a separable simple unital C^*-algebras is inner if and only if it is universally weakly inner. Thus the remaining question for O_V for $n = 3, \ldots, \infty$ and unitary V is whether O_V's are weakly inner. One consequence of the results in this paper is that O_V's are all weakly inner.

First we show a lemma which will lead us to a desired representation, π, of O_n in which a given automorphism has an inner extension to the weak-operator closure of $\pi(O_n)$.

2.3. **Lemma.** *Let A be a simple C^*-algebra and β be an automorphism of A. Suppose that φ is a β-invariant state of A, i.e., $(\varphi \circ \beta)(x) = \varphi(x)$ for all x in A. Then β, the automorphism of the image $\pi_\varphi(A)$ of A under the GNS representation π_φ induced by φ, can be extended to an automorphism $\hat{\beta}$ on the weak-operator closure of $\pi_\varphi(A)$. Furthermore $\hat{\beta}$ is spatial.*

Proof. Let H_φ be the GNS representation space of π_φ and f be the cyclic vector of $\pi_\varphi(A)$ with $\varphi(x) = \langle \pi_\varphi(x)f, f\rangle$ for all x in A. The operator U defined on H_φ by $U(\pi_\varphi(x)f) = \pi_\varphi(\beta(x))f (x \in A)$ is a unitary operator. Let $\hat{\beta}(x) = UxU^*$ for $x \in \overline{\pi_\varphi(A)}^w$. It is easy to see that $\hat{\beta}|_{\pi_\varphi(A)} = \beta$. Q.E.D.

2.4. *Remark.* Let α, β be two automorphism of a simple C^*-algebra A. If a pure state φ is β-invariant, then $\varphi_0\alpha$ is $\alpha^{-1}\beta\alpha$-invariant and pure. It is now clear that to establish weak innerness of a given automorphism β of A it is enough to find a pure state φ and an automorphism α such that φ is $\alpha^{-1}\beta\alpha$-invariant.

3. Some weakly inner automorphisms on O_n, n: finite

Let us first observe that $\Phi(\mathbb{A}_k) \subseteq \mathbb{A}_{k+1}$ for all $k = 1, 2, \ldots,$ and $\Phi(x)S_i = S_ix$ for all x in O_n, $i = 1, \ldots, n$.

3.1. **Lemma.** *Let U be a unitary operator in D and $U = \lim_{l\to\infty} U_l$, $U_l \in D_l \simeq \bigotimes_{i=1}^l D_i$, where D_i is the diagonal subalgebra of M_{n_i}. Then*

(a)

$$\lambda_U|_{\mathbb{A}_k} = Ad(U^*\Phi(U^*)\cdots\Phi^{k-1}(U^*)),$$

(b)

$$\lambda_U(x) = \lim_{l\to\infty} Ad(U_l^*\Phi(U_l^*)\cdots\Phi^{k-1}(U_l^*))(x) = \lim_{l\to\infty} \lambda_{U_l}(x) \textit{ for } x \in \mathbb{A}_k.$$

Proof. (b) follows obviously from (a).

We prove (a) by induction of k. Because of $\lambda_U(x) = U^*xU$ for x in $\mathbb{A}_1$, it is obvious that (a) holds for $k = 1$. Suppose that (a) holds for $k - 1$. We show (a) holds for k. Let an arbitrary element in $\mathbb{A}_k$ be the form $x \otimes y$, where

x is in $\mathbb{A}_{k-1}$ and y is an $n \times n$ matrix, i.e., $y = (\beta_{ij})$. Note that

$$x \otimes y = \sum_{i,j=1}^{n} \beta_{ij} S_i x S_j^*$$

and

$$\begin{aligned}
\lambda_U(x \otimes y) &= \sum_{i,j=1}^{n} \beta_{ij} U^* S_i \lambda_U(x) S_j^* U \\
&= \sum_{i,j=1}^{n} \beta_{ij} U^* S_i Ad(U^* \Phi(U^*) \cdots \Phi^{k-2}(U^*))(x) S_j^* U \\
&= U^* \left\{ \sum_{i,j=1}^{n} \beta_{ij} S_i (U^* \Phi(U^*) \cdots \Phi^{k-2}(U^*)) x (\Phi^{k-2}(U) \cdots \Phi(U) U) S_j^* \right\} U \\
&= U^* \left\{ \sum_{i,j=1}^{n} \beta_{ij} \Phi(U^* \Phi(U^*) \cdots \Phi^{k-2}(U^*)) S_i x S_j^* \Phi(\Phi^{k-2}(U) \cdots \Phi(U) \right\} U \\
&\quad (\text{for } \Phi(x) S_i = S_i x) \\
&= \sum_{i,j=1}^{n} \beta_{ij} Ad(U^* \Phi(U^*) \cdots \Phi^{k-1}(U^*))(S_i x S_j^*) \\
&= Ad(U^* \Phi(U^*) \cdots \Phi^{k-1}(U^*)) \left(\sum_{i,j=1}^{n} \beta_{ij} S_i x S_j^* \right).
\end{aligned}$$

Let x be in $\mathbb{A}_k$ and $x = (\alpha_{ij})$, $i, j = 1, \ldots, n^k$.
Then

$$P(x) = \begin{bmatrix} \alpha_{11} & 0 & \cdots & 0 \\ 0 & \alpha_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{n^k n^k} \end{bmatrix} = \text{diag}(x).$$

3.2. **Proposition.** *Let $\theta = \lambda_U$ for some unitary operator U in D. Then $P \circ \theta = P$.*

Proof. Let A be in $\mathbb{A}_k$ and V be a unitary operator in D_k. We observe that

(1) $\lambda_V(A) \in \bigcup_{j=1}^{\infty} \mathbb{A}_j$.

(2) For $l > 0$, $\lambda_V((S_1^*)^l A) = (S_1^* V)^l \lambda_V(A) \notin \mathbb{A} = \bigcup_{j=1}^{\infty} \mathbb{A}_j$, due to Lemma 3.1.

For $x \in \mathbb{A}$ with

$$x = \sum_{-N}^{-1} S^i A_i + A_0 + \sum_{1}^{N} A_i S^i, \qquad A_i \in \bigcup_{j=1}^{\infty} \mathbb{A}_j,$$

and

$$U = \lim_{l \to \infty} U_l, \qquad U_l \in D_i,$$

we have

$$\begin{aligned} P \circ \lambda_U(x) &= \lim_{l\to\infty} P \circ (\lambda_{U_l}(x)) \\ &= \lim_{l\to\infty} P \circ (\lambda_{U_l}(A_0)) \\ &= \lim_{l\to\infty} \operatorname{diag} Ad(U_l^*\Phi(U_l^*)\cdots\Phi^{k-1}(U_l^*))(A_0). \end{aligned}$$

Since $U_l^*\Phi(U_l^*), \ldots, \Phi^{k-1}(U_l^*)$ is a diagonal matrix in A_t for some large $t \geq k$, realizing $A_0 \in \mathbb{A}_t$ we see that the diagonal entries in A_0 are fixed under $Ad(U_l^*\Phi(U_l^*), \ldots, \Phi^{k-1}(U_l^*))$. Hence

$$P \circ \lambda_U(x) = \lim_{l\to\infty} \operatorname{diag}(A_0) = P(x).$$

Since A is norm-dense in O_n, we have $P \circ \lambda_U = P$. Q.E.D.

3.3. **Theorem.** *λ_U, for some unitary operator U in D, is weakly inner on O_n.*

Proof. Let Δ denote the pure-state space of D. Thus, Δ consists of all infinite sequences, $w = (j_1, j_2, \ldots)$, with $j_i \in \{1, 2, \ldots, n\}$ for $i = 1, 2, \ldots$. For any $w \in \Delta$, we note that the state on O_n defined by $w \circ P$ is λ_U-invariant for any unitary operator $U \in D$, for $(w \circ P) \circ \lambda_U = w \circ (P \circ \lambda_U) = w \circ P$ by Proposition 3.2. Hence, it suffices to show $w \circ P$ is a pure state on O_n for some $w \in \Delta$ to establish that λ_U is weakly inner on O_n by Remark 2.4. For the rest of the proof, we let $w = (j_1, j_2, \ldots)$ be a fixed element in Δ and $\phi = w \circ P$. We show that ϕ is pure.

Let the GNS representation triple induced by ϕ be denoted by $\{\pi, K, \xi\}$. We denote $(j_1, j_2, \ldots, j_k)$ by μ_k for $k = 0, 1, \ldots$. For $\nu \in W_k^n$, $\nu = (i_1, \ldots, i_k)$, and $\eta \in W_l^n$, $\eta = (i_1, \ldots, i_l)$ with $l < k$, we let $(i_{l+1}, \ldots, i_k)$ be denoted by $\nu\backslash\eta$. We observe that the following two equivalence classes are the same: $[S_\nu S_{\mu_l}^*\xi]$, $[S_\eta S_{\mu_k}^*\xi]$ if $\nu\backslash\eta = \mu_l\backslash\mu_k$ or $\eta\backslash\nu = \mu_k\backslash\mu_l$, and hence it is denoted by (ν, l). We also note that the following family of vectors in K is orthonormal, $\{(\nu, l)\}$, for $\langle\pi(S_{\nu_1})\pi(S_{\mu_l}^*)\xi, \pi(S_{\nu_2})\pi(S_{\mu_k}^*)\xi\rangle = \phi(S_{\mu_k}S_{\nu_2}^*S_{\nu_1}S_{\mu_l}^*) = \delta_{(\nu_1, l), (\nu_2, k)}$, for any $\nu_1, \nu_2 \in W$. Since $\|\pi(S_\eta S_\nu^*)\xi\| = 0$ for any $\eta, \nu \in W$ with $\nu \neq \mu_k$, $k = 0, 1, \ldots$, it follows that the orthonormal family mentioned above is actually an orthonormal basis for K. For any operator $T \in K$, we denote T by $[T_{\nu l, \eta k}]$, where $T_{\nu l, \eta k} = \langle T\pi(S_\nu S_{\mu_l}^*)\xi, \pi(S_\eta S_{\mu_k}^*)\xi\rangle$. Let T be an arbitrary operator in $\pi(O_n)'$, and we may assume that $T(\xi) = \pi(x)(\xi)$ for some $x \in O_n$. Then $T_{\nu l, \nu l} = \langle T\pi(S_{\mu_l}S_{\mu_l}^*)\xi, \xi\rangle = \phi(S_{\mu_l}S_{\mu_l}^*x) = w(S_{\mu_l}S_{\mu_l}^*P(x)) = w(S_{\mu_l}S_{\mu_l}^*)w(P(x)) = w(P(x)) = \langle T\xi, \xi\rangle$, denoted by $T_{0,0}$. Finally, we show $(T^*T)_{\nu l, \eta k} = 0$ for all $T \in \pi(O_n)'$, when $(\nu, l) \neq (\eta, k)$. Here we may assume that $T(\xi) = \pi(x)(\xi)$ for some $x \in O_n$. In fact, $(T^*T)_{\nu l, \eta k} = \langle\pi(S_\nu S_{\mu_l}^*)\pi(x)\xi, \pi(S_\eta S_{\mu_k}^*)\pi(x)\xi\rangle = \phi(x^*S_{\mu_k}S_\eta^*S_\nu S_{\mu_l}^*x) = 0$, when $(\nu, l) \neq (\eta, k)$. Then, T^*T is a scalar multiple of the identity operator for all $T \in \pi(O_n)'$, and $\pi(O_n)'$ consists of scalars of the identity operator. Therefore, π is irreducible, and ϕ is pure. Q.E.D.

3.4. *Remark.* A different proof for some of the above ϕ's that are pure can be seen in [12] (see Proposition 3.3 in [12]). A stronger result of pure-state extension of w is also stated in [5] (see Proposition 3.1 in [5]). However, there is no proof provided for Proposition 3.1 in [5].

3.5. **Corollary.** *Let V be a unitary operator on C^n. Then O_V is weakly inner.*

Proof. If V is diagonal, then $O_V = \lambda_{V^*}$ and O_V is weakly inner by Theorem 3.3, for $V \in D$ and $\varphi \circ P$ is a O_V-invariant pure state for any pure state φ on D. In general, there is a unitary operator W on C^n such that WVW^* is diagonal. After Remark 2.4 and an observation that $O_{WVW^*} = O_W O_V O_{W^*} = O_W O_V (O_W)^{-1}$ is weakly inner with an O_{WVW^*}-invariant pure state, ϕ, as in Theorem 3.3, we see that O_V, having an O_V-invariant pure state, $\phi \circ O_W$, is weakly inner. Q.E.D.

3.6. *Remark.* It follows from Remark 2.4 that any automorphism of O_n conjugate to λ_U in Theorem 3.3 is weakly inner. A result similar to Corollary 3.5 can also be found in [1], in which the approach is quite different.

4. FOCK REPRESENTATION OF O_∞

For any unitary operator V on C^n, $n = 2, 3, 4, \dots$, we can define an *-automorphism O_V on O_∞ by $O_V(S_i) = \sum_{k=1}^n V_{ki} S_k$, $i = 1, 2, \dots, n$. Likewise, any permutation V (may be infinite) of $\{2, 3, 4, \dots\}$ induces an *-automorphism O_V in the obvious way by permuting the generators according to V. In this section we show that the O_V of O_∞ mentioned above is weakly inner. Let H be an infinite-dimensional separable Hilbert space. Consider the FOCK space, $F(H)$, which is defined to be $\bigoplus_{r=0}^\infty [\otimes^r H]$, where $\bigotimes^r H$ is the Hilbert space tensor product of r copies of H and $\bigotimes^0 H$ is a one-dimensional Hilbert space C spanned by a unit vector Ω, the vacuum. Define a linear map $O_F : H \to B(F(H))$ by

$$O_F(f)(f_1 \otimes f_2 \otimes \cdots \otimes f_r) = f \otimes f_1 \otimes \cdots \otimes f_r,$$
$$O_F(f)(\Omega) = f.$$

Let $\{h_1, h_2, \dots\}$ be an orthonormal basis of H and P_0 be the orthogonal projection of H onto $\bigotimes^0 H$. Then we have

$$O_F(f)^* O_F(g) = \langle g, f \rangle I \quad \text{for all } f, g \in H,$$
$$\sum_{i=1}^\infty O_F(h_i) O_F(h_i)^* + P_0 = I \tag{4.1}$$

where the infinite sum is taken in the strong-operator topology.

Comparing (4.1) with (2.2) we conclude that O_F induces a faithful representation π of O_∞ by $\pi(S_i) = O_F(h_i)$, $i = 1, 2, \dots$, which is called the FOCK representation of O_∞.

4.2. **Lemma.** (1) *$\{\Omega\} \cup \{h_i | i = 1, 2, \dots\} \cup \{h_i \otimes h_j | i, j = 1, 2, \dots,\} \cup \cdots$ form an orthonormal basis for $F(H)$.*

(2) $P_0 \in \pi(O_\infty)''$.

(3) *For any element x in the above orthonormal basis there exists a rank-one partial isometry T_x in $\pi(O_\infty)''$ such that T_x maps $\bigotimes^0 H$ onto the subspace spanned by x.*

Proof. (1) is obvious.

(2) follows from the second condition in (4.1).

(3) Suppose $x = h_{i_1} \otimes h_{i_2} \otimes \cdots \otimes h_{i_k}$; then

$$T_x = O_F(h_{i_1}) O_F(h_{i_2}) \cdots O_F(h_{i_k}) P_0. \quad \text{Q.E.D.}$$

4.3. **Lemma.** (1) $\pi(O_\infty)'' = B(F(H))$.

(2) *The vector state of* $\pi(O_\infty)$, $w_\Omega(A) = \langle A\Omega, \Omega\rangle$, *is* β*-invariant for* $\beta = O_V$ *where* V *is a unitary operator in* C^n *for some* n *or a permutation of* $\{1, 2, 3, \ldots\}$.

Proof. (1) follows from (3) in Lemma 4.2, for $\pi(O_\infty)' = \{\text{scalars} \cdot I\}$.

(2) Any word consisting of $\pi(S_i)$, $\pi(S_i)^*$, $i = 1, 2, \ldots$, can be reduced to αI for some scalar α or $\pi(S_{j_1}) \cdots \pi(S_{j_l})\pi(S_{i_1})^* \cdots \pi(S_{i_m})^*$ with $l \geq 1$ or $m \geq 1$. The null space, N, of the bounded linear functional w_Ω is a closed subspace spanned by words in their reduced form $\pi(S_{j_1}) \cdots \pi(S_{j_l})\pi(S_{i_1})^* \cdots \pi(S_{i_m})^*$ with $l \geq 1$ or $m \geq 1$. It is easy to see that N is invariant under β described in (2) of Lemma 4.3. Thus w_Ω and $w_\Omega \circ \beta$ have the same null space and $w_\Omega(I) = w_\Omega \circ \beta(I) = 1$. Hence $w_\Omega = w_\Omega \circ \beta$. Q.E.D.

4.4. **Theorem.** *Every automorphism* O_V *of* O_∞, *with* V *being a unitary operator on* C^n *for some* n *or a permutation of* $\{1, 2, 3, \ldots\}$, *is weakly inner.*

Proof. From (1), (2) of Lemma 4.3 we see that π is an irreducible representation of O_∞ and $w_\Omega \circ \pi$ is an O_V-invariant pure state of O_∞. Hence by Remark 2.4, Theorem 4.4 is proved. Q.E.D.

4.5. *Concluding remark.* The similar construction of FOCK representation for O_n, n: finite, does not yield the same result. The FOCK representation for O_∞ can also be found in [8]. A result like Theorem 4.4 is also recorded in [1] with a different proof. It also follows from Remark 2.4 that any automorphism of O_∞ conjugate to O_V in Theorem 4.4 is weakly inner.

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