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THE LEAST CARDINAL FOR WHICH THE BAIRE CATEGORY THEOREM FAILS

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ABSTRACT. The least cardinal for which the Baire category theorem fails is equal to the least cardinal for which a Ramseyan theorem fails.

The Baire category theorem states that the real line is not the union of countably many meager (also known as first category) sets. Let $\mathrm{cov}(\mathcal{M})$ denote the least cardinal number such that there are that many first category subsets of the real line whose union is the entire real line. Then $\mathrm{cov}(\mathcal{M})$ is the least cardinal number for which the Baire category theorem fails.

This cardinal number, defined in terms of topological notions, appears in many different guises in combinatorial set theory. A long list of diverse guises of this cardinal number is already general knowledge for set theorists; Galvin gave a game-theoretic version (part of which is published in [4], and part of which is unpublished—however, see [8]), A. W. Miller gave a characterization in terms of sequences of positive integers [6], which was later given an elegant improvement by Bartoszynski [1]. It is also known to be the least cardinal number such that there is a set of real numbers of that cardinality which does not have Rothberger's property C''. A set X of real numbers has property C'' if, for every sequence $(\mathcal{U}_n\colon n = 1, 2, 3, \dots)$ of open covers of X, there is a sequence $(U_n\colon n = 1, 2, 3, \dots)$ such that, for each n, $U_n \in \mathcal{U}_n$ and $\{U_n\colon n = 1, 2, 3, \dots\}$ is a cover for X.

It seems that for the purposes of applications of set theory to other areas of mathematics, it would be useful to have as many non-trivial characterizations of this cardinal number as possible. In this paper we give a few more equivalent forms of this cardinal number. To explain some of our results, we need some terminology which is well-known in other contexts. Let κ be an infinite cardinal number which will be fixed for the duration of the paper.

A collection of subsets of κ is said to be a *cover* of κ if its union is equal to κ. We shall be interested in countable covers of κ. A cover of κ is said to be an ω-cover if it is countably infinite, κ itself is not a member of the cover, and if there is for every finite subset of κ an element of this cover which contains it. We shall let the symbol Ω denote the collection of ω-covers of κ.

Borrowing from Ramsey theory (see Section 8 of [3]), we shall use the symbol

$$\Omega \to (\Omega)^n_k$$

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to denote that: for all positive integers n and k, for every ω-cover $\mathcal{U}$ of κ, and for every function

$$f\colon [\mathcal{U}]^n \to \{1,\dots,k\}$$

there is an ω-cover $\mathcal{V} \subseteq \mathcal{U}$ of κ such that f restricted to $[\mathcal{V}]^n$ is constant.

Theorem 1. *The following statements are equivalent*:

1. $\kappa < \mathrm{cov}(\mathcal{M})$.
2. *for all positive integers k and n,* $\Omega \to (\Omega)^n_k$.

The paper is organized so that we prove Theorem 1 in the first section. A few remarks regarding extensions of what we discussed here constitute the second and final section of the paper.

1. The proof of Theorem 1

There is a natural duality between the theory of filters on ω and the theory of ω-covers of a set S. We make this explicit since it can be used to obtain some of our lemmas also directly from some of Canjar's results in [2].

Thus, let S be an infinite set and let $\mathcal{U}$ be an ω-cover of S. By our conventions $\mathcal{U}$ is a countable set and does not have S as an element. For a finite subset F of S, put

$$\mathcal{U}_F = \{U \in \mathcal{U} : F \subseteq U\}.$$

Then the family $\{\mathcal{U}_F : F \text{ is a subset of } S\}$ is a basis for a filter on the countable set $\mathcal{U}$. The filter so generated is a free filter, because there is no element of $\mathcal{U}$ which contains every finite subset of S.

Conversely, let $\{U_s : s \in S\}$ be a collection of infinite subsets of ω which is a basis of a free filter. Then define for each n in ω the set V_n to be $\{s \in S : n \in U_s\}$. Then the collection $\mathcal{U} = \{V_n : n < \omega\}$ is an ω-cover for S: for let F be a finite subset of S. Then $\bigcap_{s\in F} U_s$ is an infinite set since the collection generates a free filter. Pick an n in this intersection. Then F is a subset of V_n. Moreover, since the filter generated by $\{U_s : s \in S\}$ is a free filter, there is no n contained in each U_s. This translates to saying that S is not equal to any V_n.

Using this duality, Lemmas 2 and 3 can be obtained as direct consequences of Canjar's Lemma 7, as we shall indicate below. However, because of the simple and direct involvement of $\mathrm{cov}(\mathcal{M})$ in our proofs of these two lemmas, we kept them as part of our exposition.

Let us say that κ is a *Q-point cardinal number* if, for every ω-cover $\mathcal{U}$ of κ, and for every partition $(\mathcal{P}_n : n = 1,2,3,\dots)$ of $\mathcal{U}$ into disjoint finite subsets, there is a subset $\mathcal{V}$ of $\mathcal{U}$ such that $\mathcal{V}$ is an ω-cover of κ such that, for each n, $\mathcal{V} \cap \mathcal{P}$ has at most one element.

Lemma 2. *If κ is less than* $\mathrm{cov}(\mathcal{M})$*, then κ is a Q-point cardinal number.*

Proof. For let $\mathcal{U}$ be an ω-cover of X, and let $(\mathcal{P}_n : n = 1,2,3,\dots)$ be a partition of it into disjoint finite subsets. We may assume that each of these $\mathcal{P}_n$'s is nonempty. Endow each $\mathcal{P}_n$ with the discrete topology and endow the set $\prod_{n=1}^\infty \mathcal{P}_n$ with the product topology. For every nonempty finite subset F of κ, define

$$D_F = \left\{ f \in \prod_{n=1}^{\infty} \mathcal{P}_n : \text{for each } n, F \not\subseteq f(n) \right\}.$$

Then each D_F is a closed, nowhere dense subset of $\prod_{n=1}^{\infty} \mathcal{P}_n$. As κ is less than $\mathrm{cov}(\mathcal{M})$, we see that

$$\left(\prod_{n=1}^{\infty} \mathcal{P}_n\right) \setminus \bigcup \{D_F \colon F \in [\kappa]^{<\aleph_0} \text{ and } F \neq \varnothing\}$$

is nonempty. Let f be an element of this set. Then, for each n, $f(n)$ is an element of $\mathcal{P}_n$, and the set $\{f(n) \colon n = 1, 2, 3, \dots\}$ is an ω-cover of X. □

Here is how Lemma 2 could be obtained from Lemma 7 of Canjar's: Let $\kappa < \mathrm{cov}(\mathcal{M})$ be an infinite cardinal number, let $\mathcal{U} = (U_n : n < \omega)$ be an ω-cover of it, and let $(\mathcal{P}_n : n < \omega)$ be a partition of this ω-cover into disjoint finite sets. For each $\alpha < \kappa$ set $A_\alpha = \{n < \omega : \alpha \in U_n\}$. Then $\{A_\alpha \colon \alpha < \kappa\}$ is a family of κ subsets of ω which has the finite intersection property. Define a function $f \colon \omega \to \omega$ so that $f(n) = m$ if $U_n \in \mathcal{P}_m$. Then f is finite-to-one. By [2], Lemma 7, choose an infinite set $A \subset \omega$ such that $\{A\} \cup \{A_\alpha : \alpha < \kappa\}$ has the finite intersection property, and f is one-to-one on A. Put $\mathcal{V} = \{U_n : n \in A\}$. Then $\mathcal{V}$ is an ω-cover of κ, and it meets each $\mathcal{P}_n$ in at most one element.

Next, let us say that κ is a *P-point cardinal number* if, for each descending sequence

$$\mathcal{U}_1 \supseteq \mathcal{U}_2 \supseteq \cdots \supseteq \mathcal{U}_n \supseteq \cdots$$

of ω-covers of κ, there is an ω-cover $\mathcal{V}$ of κ such that for each n the set $\mathcal{V} \setminus \mathcal{U}_n$ is finite.

Lemma 3. *If κ is less than* $\mathrm{cov}(\mathcal{M})$*, then κ is a P-point cardinal number.*

Proof. $\mathcal{U}_1$ is a countable set. For each n enumerate $\mathcal{U}_n$ bijectively as $(U_m^n \colon m = 1, 2, 3, \dots)$. Define, for each finite nonempty subset F of κ, the set

$$D_F = \{f \in {}^{\mathbf{N}}\mathbf{N} : \text{for each } n,\ F \not\subseteq U_{f(n)}^n\}.$$

Then each D_F is a closed, nowhere dense subset of ${}^{\mathbf{N}}\mathbf{N}$, and so there is an element of ${}^{\mathbf{N}}\mathbf{N}$ not in $\bigcup \{D_F : F \subseteq \kappa \text{ nonempty and finite}\}$. Letting f be such an element we see that $\mathcal{V} = \{U_{f(n)}^n : n = 1, 2, 3, \dots\}$ is an ω-cover of κ, and has the required properties. □

Here is how this lemma could also be obtained from Canjar's Lemma 7: Let $\kappa < \mathrm{cov}(\mathcal{M})$ as well as a descending sequence $\mathcal{U}_1 \supset \cdots \supset \mathcal{U}_n \supset \cdots$ of ω-covers of κ be given. Enumerate $\mathcal{U}_1$ bijectively as $(U_n : n < \omega)$. For each n and each $\alpha < \kappa$ define $A_\alpha^n = \{m < \omega : \alpha \in U_m \text{ and } U_m \in \mathcal{U}_n\}$. Then the family $\{A_\alpha^n : 0 < n < \omega, \alpha < \kappa\}$ of infinite subsets of ω has the finite intersection property. Define a function $f : \omega \to \omega$ such that for each n we have $f(n) = m$ if $U_n \in \mathcal{U}_m \setminus \mathcal{U}_{m+1}$. Then select, by [2], Lemma 7, an infinite subset A of ω such that $\{A\} \cup \{A_\alpha^n : n < \omega, \alpha < \kappa\}$ has the finite intersection property and on A either f is bounded, or else f is one-to-one.

Case 1: *f is bounded.* Let κ be an upper bound for f. Then the ω-cover $\mathcal{V} = \{U_n : n \in A\}$ is contained in $\mathcal{U}_1 \setminus \mathcal{U}_{k+1}$. But $A \cap A_\alpha^{k+1}$ is non-empty, meaning that $\mathcal{V} \cap \mathcal{U}_{k+1}$ is non-empty. This contradiction shows that Case 1 doesn't occur.

Case 2: *f is one-to-one.* Then the ω-cover $\mathcal{V} = \{U_n : n \in A\}$ has the property that, for each $n, \mathcal{V} \setminus \mathcal{U}_n$ has at most n elements, and we are done.

We are now ready to prove the implication $1 \Rightarrow 2$ of Theorem 1. Assume that κ is less than $\mathrm{cov}(\mathcal{M})$, let $\mathcal{U}$ be an ω-cover of κ, and let $f : [\mathcal{U}]^2 \to \{0, 1\}$ be given. Enumerate $\mathcal{U}$ bijectively as $(U_n : n = 1, 2, 3 \dots)$.

Define sequences $(i_n : n = 1, 2, 3, \dots)$ (of 0's and 1's) and $\mathcal{U}_1 \supset \mathcal{U}_2 \supset \cdots \supset \mathcal{U}_n \supset \cdots$ of ω-covers of κ such that

1. $\mathcal{U}_1 = \{V \in \mathcal{U} : f(\{U_1, V\}) = i_1\}$, and
2. $\mathcal{U}_{n+1} = \{V \in \mathcal{U}_n : f(\{U_{n+1}, V\}) = i_{n+1}\}$ for each n.

Then, choose an ω-cover $\mathcal{V} \subset \mathcal{U}_1$ such that for each n the set $\mathcal{V}\backslash\mathcal{U}_{n+1}$ is finite. Put $\mathcal{V}_1 = \mathcal{V}\backslash\mathcal{U}_2$, and for each n greater than 1 put $\mathcal{V}_n = \mathcal{V}\backslash(\mathcal{U}_{n+1} \cup \mathcal{V}_1 \cup \cdots \cup \mathcal{V}_{n-1})$. We consider two cases:

1. either $\bigcap_{n=1}^{\infty} \mathcal{U}_n$ is an ω-cover of κ,
2. or else $\bigcup_{n=1}^{\infty} \mathcal{V}_n$ is an ω-cover of κ.

In the first case, write $\mathcal{W} = \bigcap_{n=1}^{\infty} \mathcal{U}_n$ and list $\mathcal{W}$ as $(U_{n_k} : k = 1, 2, 3, \dots)$, using the enumeration which we have fixed earlier on, so that $n_1 < n_2 < \cdots < n_k < \cdots$. Now note that for each k, either $i_{n_k} = 0$, or else $i_{n_k} = 1$. Partition $\mathcal{W}$ into two disjoint pieces according to the values of the i_{n_k}'s; one of these is an ω-cover of κ. We may assume that the one which is an ω-cover of κ has $i_{n_k} = 1$ for each k. Again, this part of $\mathcal{W}$ may now be named $\mathcal{W}$, and we may assume that the enumeration $(U_{n_k} : k = 1, 2, 3, \dots)$ from above enumerates this $\mathcal{W}$.

Then choose $l_1 < l_2 < \cdots < l_n < \cdots$ such that

- $l_1 = n_1$, and
- $l_{n+1} > l_n$ is so large that if $n_j \leq l_n$ and $n_k \geq l_{n+1}$, then $U_{n_k} \in \mathcal{U}_{n_j}$.

Then, set:

$$\mathcal{E} = \{U_{n_j} : \text{for an even } k,\ l_k \leq n_j < l_{k+1}\}$$

and

$$\mathcal{O} = \{U_{n_j} : \text{for an odd } k,\ l_k \leq n_j < l_{k+1}\}.$$

As $\mathcal{W}$ is an ω-cover of κ, and as it is the union of the two disjoint sets $\mathcal{E}$ and $\mathcal{O}$, at least one of these sets is an ω-cover, say $\mathcal{E}$ (the argument for $\mathcal{O}$ proceeds similarly).

Put $\mathcal{E}_k = \{U_{n_j} : l_{2\cdot k} \leq n_j < l_{2\cdot k+1}\}$. Then this partitions the ω-cover $\mathcal{E}$ of κ into pairwise disjoint finite sets. Let $\mathcal{S}$ be a subset of $\mathcal{E}$ which is an ω-cover of κ, and meets each $\mathcal{E}_k$ in at most one point. Then $\mathcal{S}$ is a subset of $\mathcal{U}$ which is monochromatic for the coloring f, and is an ω-cover of κ. This gives the argument in the first case.

In the second case, $\bigcup_{n=1}^{\infty} \mathcal{V}_n$ is an ω-cover of κ; call it $\mathcal{W}$. Now for each n, if m is larger than n, then $\mathcal{V}_m$ is a subset of $\mathcal{U}_n$. Choose $k_1 < k_2 < k_3 < \cdots < k_n < \cdots$ such that for each n, if U_i is an element of $\mathcal{V}_n$, then $i \leq k_n$. Then choose $l_1 < l_2 < \cdots l_n < \cdots$ such that for each n:

1. if j is at least as large as l_1, then $\mathcal{V}_j \subset \mathcal{U}_{k_1}$, and if U_i is in $\mathcal{V}_j$, then $i < k_1$,
2. if j is at least as large as l_{n+1}, then $\mathcal{V}_j \subseteq \mathcal{U}_{k_{l_n}}$, and if U_i is in $\mathcal{V}_j$, then i is larger than k_{l_n}, and
3. $k_1 + \cdots + k_{l_n} + l_1 + \cdots + l_n < l_{n+1}$.

Then define $g\colon \mathbf{N} \to \mathbf{N}$ so that $g(1) = l_1$ and, for each n, $g(n+1) > g(n)$ is so large that if $j \geq g(n+1)$ and U_i is in $\mathcal{V}_j$, then U_i is in $\mathcal{U}_{k_{g(n)}}$ and $i > k_{g(n)}$.

We now put $\mathcal{P}_1 = \mathcal{V}_1 \cup \cdots \cup \mathcal{V}_{g(1)-1}$, and, for each n, $\mathcal{P}_{n+1} = \mathcal{V}_{g(n)} \cup \cdots \cup \mathcal{V}_{g(n+1)-1}$. Define $\mathcal{E} = \bigcup_{n=1}^{\infty} \mathcal{P}_{2\cdot n}$ and $\mathcal{O} = \bigcup_{n=1}^{\infty} \mathcal{P}_{2\cdot n-1}$. Then $\mathcal{E}$ or $\mathcal{O}$ is an ω-cover of κ; we may assume that $\mathcal{E}$ is an ω-cover (the argument for $\mathcal{O}$ is similar).

The sequence $(\mathcal{P}_{2\cdot n} : n = 1, 2, 3, \dots)$ is a partition of the ω-cover $\mathcal{E}$ of κ. Since κ is a Q-point cardinal number, we find a subset $\mathcal{S}$ of $\mathcal{E}$ which is an ω-cover of κ, and which meets each $\mathcal{P}_{2\cdot n}$ in at most one point.

For each n, let V_n be the element of $\mathcal{S}$ in $\mathcal{P}_{2\cdot n}$. Then V_n is an element of $\mathcal{V}_{j_n}$, where now, for each n, $g(2\cdot n-1) \leq j_n < g(2\cdot n)$. We may further write $V_n = U_{i_n}$ where $U_{i_n} \in \mathcal{V}_{j_n}$. By the choice of g we then have:

1. $U_{i_n} \in \mathcal{U}_{k_{g(2\cdot n-1)}}$ and $i_n > k_{g(2\cdot n-1)}$, and
2. $i_n < k_{j_n} < k_{g(2\cdot n)} < k_{g(2\cdot(n+1)-1)}$.

Consequently we have $k_{g(2\cdot n-1)} < i_n < k_{g(2\cdot(n+1)-1)}$. But then we have that, for each $m > n$, $U_{i_m} \in \mathcal{U}_{i_n}$. Then we extract from $\{U_{i_m} : m = 1, 2, 3, \dots\}$ a homogeneous set for f as before.

We have now shown that 1 of Theorem 1 implies the partition relation

$$\Omega \to (\Omega)^2_2. \tag{1}$$

This partition relation by itself is already enough to imply 1 of the theorem. In the "Remarks" section below I'll outline my original proof of this fact. Professor Andreas Blass pointed out that there is a more efficient proof of $2 \Rightarrow 1$ which uses the methods of [2], Lemma 10, and Bartoszynski's characterization [1] of $\mathrm{cov}(\mathcal{M})$. Here is the argument given by Blass, slightly paraphrased:

Suppose that statement 2 of Theorem 1 holds. Consider an arbitrary family H of κ functions from ω to ω. We seek a single function that agrees with each of them somewhere. Proceed as in Canjar's proof of his Lemma 10 to define a family F of countable sets as follows:

1. First, let $(B_n : n < \omega)$ be a partition of ω such that for each n we have $|B_n| = n^2$. Then, for each n, let $(L^n_k \colon k = 1, 2, \dots, n)$ be a list of n pairwise disjoint n-element subsets of B_n. For each n we let X_n be the set of functions from B_n to ω, and then we let X be the union of the X_n's.
2. Next, let $N \colon X \to \omega$ be the function which is defined so that
$$N(x) = n \Leftrightarrow x \in X_n.$$
3. For $h \in H$ define the set A_h to be the set of p in X such that for each k between 1 and $N(p)$, there is a y in $L^{N(p)}_k$ at which $h(y)$ is equal to $p(y)$.

Then we put

$$F = \{A_h \colon h \in H\} \cup \{\{p \in X \colon N(p) > j\} : j < \omega\}.$$

Canjar shows that F has the finite intersection property. Moreover, F has cardinality no larger than κ. Now F is a subset of the powerset of X. Putting, for each x in X, $S_x = \{B \in F : x \in B\}$, we get an ω-cover $\{S_x \colon x \in X\}$ of F. Define a partition

$$\Phi \colon [\{S_x \colon x \in X\}]^2 \to \{0, 1\}$$

so that

$$\Phi(\{S_x, S_y\}) = \begin{cases} 0 & \text{if there is an } n \text{ with } x, y \in X_n, \\ 1 & \text{otherwise.} \end{cases}$$

By the partition hypothesis, there is an $i \in \{0, 1\}$ and a subset E of X such that $\{S_x \colon x \in E\}$ is an ω-cover of X, and is homogeneous of color i for Φ.

We first see that $i = 1$: If on the contrary we had $i = 0$, then we would have some n, fixed for the remainder of the argument, such that $E \subseteq X_n$. This would mean that for $j > n$ we have $E \cap \{p \in X : N(p) > j\} = \varnothing$, so that no element of $\{S_x \colon x \in E\}$ would cover any of these elements of F—this contradicts the fact that $\{S_x \colon x \in E\}$ is a cover of F.

Now that we have established that $i = 1$, we see that for each n, $|E \cap X_n| \leq 1$. For each n for which this is possible, let q_n denote the element of $E \cap X_n$. Then define $g\colon \omega \to \omega$ so that for each relevant n, if b is in the domain of q_n, then $g(b) = q_n(b)$. We shall see that g is as required.

Let h be an element of H. Since we have an ω-cover of F, we see that for every finite subset G of H, there is an element of E belonging to the intersection $\bigcap_{g \in G} A_g$. In particular, there is an element of E belonging to A_h, say q_n (in the choice of indices made above). By the definition of A_h, there is an element x in the domain of q_n where $h(x) = q_n(x)$. But then this x is a point where h and g agree.

2. Remarks

In the original proof of $2 \Rightarrow 1$ of Theorem 1, I argued as follows: 2 implies that κ is both a P-point cardinal number and a Q-point cardinal number. This attribute of a cardinal number κ then implies that every set of real numbers having cardinality κ has Rothberger's property C''. But then recall the fact that $\mathrm{cov}(\mathcal{M})$ is the least cardinality of a set of real numbers which does not have Rothberger's property C''.

This particular proof, though longer than the one given by Blass, had the other virtue that in order to prove the full partition relation stated in 2 of Theorem 1 (i.e. for all finite superscripts and subscripts), one could proceed as in the usual proof of Ramsey's theorem where one inducts on the superscripts and subscripts, and uses the proven instances of the partition relations. In this inductive proof for the partition relation for ω-covers, one at some point uses the P-point and the Q-point properties as well as the already established instances of the partition relation to extract a homogeneous ω-cover.

Blass also pointed out that the theorem could be reformulated as a theorem regarding small filter-bases (see 6 below). The proof of $2 \Rightarrow 1$ of the reformulated version would go even smoother, since now we don't have to translate back and forth between the filter terminology and the omega-cover terminology. In summary, then, using the methods of this paper and Canjar's one finds the following statements are equivalent:

Theorem. *For an infinite cardinal number κ, the following are equivalent*:

1. $\kappa < \mathrm{cov}(\mathcal{M})$.
2. $\Omega \to (\Omega)^2_2$.
3. *κ is a P-point cardinal number and a Q-point cardinal number.*
4. *For all positive integers m and n, $\Omega \to (\Omega)^n_m$.*
5. $\Omega \to (\Omega, 4)^3$.
6. *For every κ-generated nontrivial filter F on ω and every partition of $[\omega]^n$ into k pieces, there is a homogeneous set that meets every set in F.*

One can generalize from the situation of cardinal numbers to topological spaces: Instead of considering ω-covers by arbitrary subsets, consider ω-covers by open subsets of the space. In the specific situation where the space is a subspace of the real line, one finds a class of subsets of the real line which is, in general, a proper subcollection of the C''-sets of Rothberger, and which is characterized by the partition relation of Theorem 1 for such ω-covers. These matters are pursued in the papers [5] and [9].

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