DOMAINS OF FINITE TYPE AND HÖLDER CONTINUITY OF THE PERRON-BREMERMANN FUNCTION

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ABSTRACT. Let Ω be a smoothly bounded domain in $\mathbb{C}^n$ such that $0 \in \partial\Omega$. We give a bound for the type of $\partial\Omega$ at 0 in terms of the Hölder exponent of its Perron-Bremermann function with simple boundary data. We then use this to show that a smoothly bounded domain in $\mathbb{C}^2$ is pseudoconvex of finite type if and only if its Perron-Bremermann function corresponding to Hölder continuous boundary data is Hölder continuous.

1. INTRODUCTION

Let Ω be a bounded domain in $\mathbb{C}^n$. A point $\zeta \in \partial\Omega$ is called a peak point for the class of plurisubharmonic functions if there exists a function $\rho \in PSH(\Omega) \cap C(\overline{\Omega})$ such that $\rho(\zeta) = 0$ and $\rho(z) < 0$ for all $z \in \overline{\Omega}$, $z \neq \zeta$. We now let $f \in C(\partial\Omega)$ and we denote by $U(\Omega, f)$ the set of plurisubharmonic functions v on Ω satisfying $\limsup_{z \rightarrow \zeta, z \in \Omega} v(z) \leq f(\zeta)$, for all $\zeta \in \partial\Omega$. The Perron-Bremermann function $\Psi_{\Omega, f}$ for Ω and f is defined by $\Psi_{\Omega, f}(z) = \sup\{v(z) : v \in U(\Omega, f)\}$, $z \in \Omega$ (see for example [9, p. 89]). We can extend $\Psi_{\Omega, f}$ to $\overline{\Omega}$ by setting $\Psi_{\Omega, f} = f$ on $\partial\Omega$. So, in what follows, whenever $\Psi_{\Omega, f}$ is referred to as a function defined on $\overline{\Omega}$ we assume that it is extended to $\partial\Omega$ in this way. If Ω is regular for the Dirichlet problem for the Laplace operator it is easy to see that $\Psi_{\Omega, f} \in U(\Omega, f)$. A classical theorem of Walsh [11] states that if $\lim_{z \rightarrow \zeta, z \in \Omega} \Psi_{\Omega, f}(z) = f(\zeta)$ for all $\zeta \in \partial\Omega$ then $\Psi_{\Omega, f}$ is actually continuous on $\overline{\Omega}$. This fact implies the following well known theorem (see also Theorem 2.3 in [10]).

Theorem. *Let Ω be a bounded domain in $\mathbb{C}^n$, regular for the Dirichlet problem for the Laplace operator. The following are equivalent:*

- (i) *every $\zeta \in \partial\Omega$ is a peak point for the class of plurisubharmonic functions;*
- (ii) *if $f \in C(\partial\Omega)$ then $\Psi_{\Omega, f} \in C(\overline{\Omega})$.*

In [2] it is shown (Theorem 6.2) that if Ω is a strongly pseudoconvex domain in $\mathbb{C}^n$ and f is a Hölder continuous function on $\partial\Omega$ then the function $\Psi_{\Omega, f}$ is Hölder continuous on $\overline{\Omega}$. Moreover, estimates for the modulus of continuity of $\Psi_{\Omega, f}$ in terms of the modulus of continuity of f are obtained, via different methods, in [7] and [1]. In this note we prove that a smoothly bounded domain Ω in $\mathbb{C}^2$ is pseudoconvex of finite type if and only if the function $\Psi_{\Omega, f}$ is Hölder continuous on $\overline{\Omega}$ whenever f is a Hölder continuous function on $\partial\Omega$.

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For $S \subset \mathbb{C}^n$ and α so that $0 < \alpha \leq 1$ we denote by $Lip^\alpha(S)$ the set of functions on S which are Hölder continuous with exponent α . If f is a real valued function defined on S we set $H(f) = \sup\{\alpha \in (0, 1] : f \in Lip^\alpha(S)\}$. Note that if this set is empty then, by definition, $H(f) = 0$. We also set, for a fixed $x_0 \in S$,

$$h(f, x_0) = \max \left\{ \liminf_{x \rightarrow x_0, x \in S} \frac{\log |f(x) - f(x_0)|}{\log \|x - x_0\|}, 0 \right\}.$$

It is then obvious that if f satisfies a Hölder condition at x_0 with exponent $\alpha > 0$ (i.e. $|f(x) - f(x_0)| \leq c\|x - x_0\|^\alpha$ for x in some neighborhood of x_0) then $\alpha \leq h(f, x_0)$. In particular $H(f) \leq h(f, x_0)$. Conversely, if $h(f, x_0) > 0$ then for any ϵ , $0 < \epsilon < h(f, x_0)$, f satisfies a Hölder condition at x_0 with exponent $h(f, x_0) - \epsilon$.

Let us briefly recall one of the definitions of the notion of finite type. Let Ω be a bounded domain in $\mathbb{C}^n$ with C^∞ smooth boundary and assume $0 \in \partial\Omega$. If X is a germ of a one-dimensional analytic variety at 0 we set

$$\tau(X) = \sup \left\{ s > 0 : \limsup_{z \rightarrow 0, z \in X \setminus \{0\}} \frac{d(z, \partial\Omega)}{\|z\|^s} < \infty \right\}$$

(d =euclidean distance). If r is a defining function for $\partial\Omega$ near 0, X is irreducible and $z = \beta(t)$, $t \in \mathbb{C}$, is a parametrization for X such that $\beta(0) = 0$, then it is easy to see that $\tau(X) = v(r \circ \beta)/v(\beta)$, where $v(r \circ \beta)$ and $v(\beta)$ are the vanishing orders of $r \circ \beta$ and β at 0. We say that $\partial\Omega$ is of type less than $2k$ at 0 if $\tau(X) \leq 2k$ for every germ X of a one-dimensional analytic variety at 0 (for more information see [3]). The domain Ω is said to be of finite type if $\partial\Omega$ is of finite type at all of its points.

2. MAIN RESULTS

For $z \in \mathbb{C}^n$ we write $z = (z', z_n) \in \mathbb{C}^{n-1} \times \mathbb{C}$, $z_n = x_n + iy_n$. Let Ω be a bounded domain in $\mathbb{C}^n$ with C^∞ smooth boundary and let $\zeta \in \partial\Omega$. Without loss of generality we may assume that $\zeta = 0$ and that the boundary of Ω near 0 is given by $y_n = \lambda(z', x_n)$, where λ is a smooth function satisfying $\lambda(0, 0) = 0$, $d\lambda(0, 0) = 0$.

We define $f_1(z) = -\|(z', x_n)\| = -(\|z'\|^2 + x_n^2)^{1/2}$, $z \in \mathbb{C}^n$, and we let $\psi_1(z) = \Psi_{\Omega, f_1}(z)$, $z \in \overline{\Omega}$, be the Perron-Bremermann function for Ω and f_1 . Note that $H(f_1) = 1$.

Theorem 1. *Let Ω and ψ_1 be as above and let X be a germ of a one-dimensional analytic variety at $0 \in \partial\Omega$. Then*

$$\tau(X) \leq 1/h(\psi_1, 0).$$

Theorem 2. *Let Ω be a bounded domain in $\mathbb{C}^2$ with C^∞ boundary. The following are equivalent:*

- (i) Ω is pseudoconvex of finite type;
- (ii) for any $f \in Lip^\alpha(\partial\Omega)$, $0 < \alpha \leq 1$, we have $\Psi_{\Omega, f} \in Lip^\gamma(\overline{\Omega})$, for some $\gamma = \gamma(f, \alpha)$, $0 < \gamma \leq \alpha$.

Moreover, if (i) holds and the type of Ω is $2k$ then γ can be chosen as $\gamma = \gamma(\alpha) = \alpha/(2k)$. In addition, if $\zeta \in \partial\Omega$ (wlog $\zeta = 0$) is a point of the highest type $2k$ and ψ_1 is as in Theorem 1, then $H(\psi_1) = 1/(2k)$.

3. PROOFS

We need the following lemma ([5, Proposition 3.1]):

Lemma. *Let Ω be a smoothly bounded domain in $\mathbb{C}^n$ with $0 \in \partial\Omega$ and let X be a germ of a one-dimensional analytic variety at 0. Assume that there exists a negative plurisubharmonic function ϕ on Ω such that for some $r > 0$ and for $z \in \Omega$, with $\|z\| < r$, ϕ satisfies $-c_1\|z\|^\gamma \leq \phi(z) \leq -c_2\|z\|^N$, where $c_1, c_2 > 0$, $0 < \gamma < N$. Then $\tau(X) \leq N/\gamma$.*

Proof of Theorem 1. We may assume that $h(\psi_1, 0) > 0$. We fix ϵ so that $0 < \epsilon < h(\psi_1, 0)$, and set $\gamma = \gamma(\epsilon) = h(\psi_1, 0) - \epsilon$. Note that ψ_1 is negative and plurisubharmonic in Ω and by the definition of $h(\psi_1, 0)$ there is a number $r > 0$ such that $\psi_1(z) \geq -\|z\|^\gamma$ for $z \in \Omega$ with $\|z\| < r$.

It is enough to prove that for every $\alpha < 1$ there is a ball U_α centered at 0 such that

$$(3.1) \quad \psi_1(z) \leq -c_\alpha \|z\|^{1/\alpha}$$

for $z \in \Omega \cap U_\alpha$. Indeed, then it would follow from the above lemma that $\tau(X) \leq 1/(\alpha\gamma(\epsilon))$, and hence $\tau(X) \leq 1/h(\psi_1, 0)$ by letting $\epsilon \rightarrow 0$ and $\alpha \rightarrow 1$.

To this end we introduce the functions $f_\alpha(z) = -\|(z', x_n)\|^\alpha = -(\|z'\|^2 + x_n^2)^{\alpha/2}$, $z \in \mathbb{C}^n$, and $\psi_\alpha(z) = \Psi_{\Omega, f_\alpha}(z)$, $z \in \overline{\Omega}$. Note that $f_\alpha \in Lip^\alpha(\mathbb{C}^n)$, $H(f_\alpha) = \alpha$ and ψ_α is negative and plurisubharmonic in Ω . As $\alpha < 1$ the function $v = -(-\psi_1)^\alpha$ is plurisubharmonic in Ω . Also since $\limsup_{z \rightarrow \zeta} \psi_1(z) \leq f_1(\zeta)$, $\zeta \in \partial\Omega$, we have $\limsup_{z \rightarrow \zeta} v(z) \leq -(-f_1(\zeta))^\alpha = f_\alpha(\zeta)$, so

$$(3.2) \quad v(z) = -(-\psi_1(z))^\alpha \leq \psi_\alpha(z), \quad z \in \Omega.$$

Thus (3.1) follows from (3.2) if we show that

$$(3.3) \quad \psi_\alpha(z) \leq -c_\alpha \|z\|,$$

for some constant $c_\alpha > 0$ and for $z \in \Omega \cap U_\alpha$, U_α being a suitable ball centered at 0.

We fix $\delta > 0$ small, to be specified later in terms of α alone, and write $\Omega = \Omega_1 \cup \Omega_2$ near 0, where $\Omega_1 = \{z \in \Omega : \|z\| < r(\delta), |y_n| \geq \delta(\|z'\|^2 + x_n^2)^{1/2}\}$, $\Omega_2 = \{z \in \Omega : \|z\| < r(\delta), |y_n| \leq \delta(\|z'\|^2 + x_n^2)^{1/2}\}$ and $r(\delta)$ is sufficiently small. By Hopf's lemma ([6, p. 57]) there is a constant $c > 0$ depending on Ω and ψ_α such that $\psi_\alpha(z) \leq -cd(z, \partial\Omega)$. But for $z \in \Omega_1$ we have $d(z, \partial\Omega) \geq c(\delta)\|z\|$ provided $r(\delta)$ is small enough; hence (3.3) holds if $z \in \Omega_1$.

Now let h be the harmonic function in Ω with boundary value f_α . Since $f_\alpha \in Lip^\alpha$ we have as a consequence of the results in [8] that $h \in Lip^\alpha(\overline{\Omega})$ (note that this is not necessarily true when $\alpha = 1$). Let $k(\alpha) > 0$ be such that $|h(z) - h(w)| \leq k(\alpha)\|z - w\|^\alpha$, $z, w \in \overline{\Omega}$. For $z = (z', x_n + iy_n) \in \Omega$ near 0 we define $\tilde{z} = (z', x_n + i\tilde{y}_n) \in \partial\Omega$. So $\tilde{y}_n = \lambda(z', x_n)$ and hence $|\tilde{y}_n| \leq l(\|z'\|^2 + x_n^2)$ for some constant $l > 0$ depending on λ . For $z \in \Omega_2$ we have:

$$\begin{aligned} \psi_\alpha(z) &\leq h(z) \leq f_\alpha(\tilde{z}) + k(\alpha)|y_n - \tilde{y}_n|^\alpha \\ &\leq -(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)|y_n|^\alpha + k(\alpha)|\tilde{y}_n|^\alpha \\ &\leq -(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)\delta^\alpha(\|z'\|^2 + x_n^2)^{\alpha/2} + k(\alpha)l^\alpha(\|z'\|^2 + x_n^2)^\alpha. \end{aligned}$$

We now choose $\delta = \delta(\alpha)$ small enough so that $k(\alpha)\delta^\alpha \leq 1/4$ and also choose $r(\delta)$ small enough so that $k(\alpha)l^\alpha(\|z'\|^2 + x_n^2)^{\alpha/2} \leq 1/4$. Thus by the above

$$\psi_\alpha(z) \leq -\frac{1}{2}(\|z'\|^2 + x_n^2)^{\alpha/2}$$

for $z \in \Omega_2$. Also, for $z \in \Omega_2$ we have

$$\|z\|^2 \leq (1 + \delta^2)(\|z'\|^2 + x_n^2) \leq 2(\|z'\|^2 + x_n^2).$$

Hence $\psi_\alpha(z) \leq -\frac{1}{4}\|z\|^\alpha \leq -\frac{1}{4}\|z\|$ for $z \in \Omega_2$, and (3.3) is proved. $\square$

Proof of Theorem 2. The implication (ii) $\Rightarrow$ (i) follows immediately from Theorem 1. Indeed, we pick a point on $\partial\Omega$ which without loss of generality may be assumed to be 0. With the notations in Theorem 1 we have $f_1 \in Lip^1$ so $\psi_1 \in Lip^\gamma(\overline{\Omega})$, for some $\gamma \in (0, 1)$, and hence $\tau(X) \leq 1/\gamma$ for any germ X of a one-dimensional analytic variety at 0. Also, for any $\zeta \in \partial\Omega$, we let $f(z) = -\|z - \zeta\|$ and note that by (ii) $\Psi_{\Omega,f} \in PSH(\Omega) \cap C(\overline{\Omega})$ peaks at ζ , so Ω is pseudoconvex.

To prove that (i) $\Rightarrow$ (ii) we follow the method of Bedford and Taylor ([2, Theorem 6.2]), and use a theorem of Fornaess and Sibony ([5, Theorem A]) on the existence of plurisubharmonic peak functions. Precisely, if Ω is a bounded pseudoconvex domain in $\mathbb{C}^2$ with smooth boundary and of finite type $2k$ then there exist a neighborhood U of $\overline{\Omega}$ and a constant C such that for each $\zeta \in \partial\Omega$ there is a plurisubharmonic function $\phi_\zeta(z)$ on U satisfying $|\phi_\zeta(z) - \phi_\zeta(w)| \leq C\|z - w\|$ for $z, w \in U$, $\phi_\zeta(z) \leq -\|z - \zeta\|^{2k}$ for $z \in \Omega$, and $\phi_\zeta(\zeta) = 0$.

Now let $f \in Lip^\alpha(\partial\Omega)$, $0 < \alpha \leq 1$, and let c_f be the smallest constant for which $|f(\zeta) - f(\xi)| \leq c_f\|\zeta - \xi\|^\alpha$, $\zeta, \xi \in \partial\Omega$. We fix $\zeta \in \partial\Omega$ and consider the function $v_\zeta(z) = -c_f(-\phi_\zeta(z))^{\alpha/(2k)} + f(\zeta)$. Since ϕ_ζ is negative and plurisubharmonic in Ω and $\alpha/(2k) < 1$ the function v_ζ is plurisubharmonic in Ω and continuous on $\overline{\Omega}$. For $\xi \in \partial\Omega$ we have

$$v_\zeta(\xi) \leq -c_f\|\xi - \zeta\|^\alpha + f(\zeta) \leq f(\xi),$$

so $v_\zeta(z) \leq \Psi_{\Omega,f}(z)$, for $z \in \overline{\Omega}$. Note that v_ζ satisfies a Hölder condition of the form $|v_\zeta(z) - v_\zeta(w)| \leq C'c_f\|z - w\|^{\alpha/(2k)}$, $z, w \in \overline{\Omega}$, where $C' = C^{\alpha/(2k)}$.

We next define $u(z) = \sup\{v_\zeta(z) : \zeta \in \partial\Omega\}$, $z \in \overline{\Omega}$. Then u satisfies the same Hölder condition as v_ζ so, in particular, u is continuous on $\overline{\Omega}$ and hence plurisubharmonic in Ω . Also $u(z) \leq \Psi_{\Omega,f}(z)$ for $z \in \overline{\Omega}$. As $v_\zeta(\xi) \leq f(\xi)$, $\xi \in \partial\Omega$, and $v_\zeta(\xi) = f(\xi)$ we see that $u = \Psi_{\Omega,f} = f$ on $\partial\Omega$. So if $z \in \overline{\Omega}$ and $\zeta \in \partial\Omega$ we have $-C'c_f\|z - \zeta\|^{\alpha/(2k)} \leq u(z) - u(\zeta) \leq \Psi_{\Omega,f}(z) - f(\zeta)$. Replacing f by $-f$ and proceeding in the same way we construct a function $\tilde{u}$ which satisfies the same Hölder condition as u in $\overline{\Omega}$, is plurisubharmonic in Ω and $\tilde{u} = -f$ on $\partial\Omega$. By the maximum principle $\Psi_{\Omega,f} + \tilde{u} \leq 0$ on $\overline{\Omega}$ so for $z \in \overline{\Omega}$ and $\zeta \in \partial\Omega$ we get $\Psi_{\Omega,f}(z) - f(\zeta) \leq -\tilde{u}(z) + \tilde{u}(\zeta) \leq C'c_f\|z - \zeta\|^{\alpha/(2k)}$. We conclude that

$$(3.4) \quad |\Psi_{\Omega,f}(z) - \Psi_{\Omega,f}(\zeta)| \leq C'c_f\|z - \zeta\|^{\alpha/(2k)},$$

for $z \in \overline{\Omega}$ and $\zeta \in \partial\Omega$.

We now show that the above inequality holds for all $z, w \in \overline{\Omega}$. For $y \in \mathbb{C}^n$ we set $\Omega_y = \Omega \cap (-y + \Omega)$ and consider the function

$$F_y(z) = \begin{cases} \max\{\Psi_{\Omega,f}(z), \Psi_{\Omega,f}(z+y) - C'c_f\|y\|^{\alpha/(2k)}\}, & \text{if } z \in \Omega_y, \\ \Psi_{\Omega,f}(z), & \text{if } z \in \Omega \setminus \Omega_y. \end{cases}$$

By (3.4) we have $\limsup_{z \rightarrow \zeta, z \in \Omega} F_y(z) \leq f(\zeta)$ for all $\zeta \in \partial\Omega$. Also if $z \in \partial\Omega_y \cap \Omega$ then $z + y \in \partial\Omega$ so by (3.4) $\Psi_{\Omega,f}(z) \geq \Psi_{\Omega,f}(z + y) - C'c_f\|y\|^{\alpha/(2k)}$ and F_y is plurisubharmonic. Hence $F_y(z) \leq \Psi_{\Omega,f}(z)$ for all $z \in \Omega$. If $w \in \Omega$ then $z \in \Omega_{w-z}$ so

$$\Psi_{\Omega,f}(w) - C'c_f\|w - z\|^{\alpha/(2k)} \leq F_{w-z}(z) \leq \Psi_{\Omega,f}(z)$$

and (3.4) holds for all $z, w \in \overline{\Omega}$.

Finally, we assume that $\zeta = 0 \in \partial\Omega$ is a point of type $2k$ and we let ψ_1 be the function in Theorem 1. Then $1/(2k) \leq H(\psi_1) \leq h(\psi_1, 0) \leq 1/(2k)$, the last inequality following from Theorem 1. Hence $H(\psi_1) = 1/(2k)$ and the proof is complete. $\square$

Remark. If $f \in C(\partial\Omega)$ then it is easy to see that $\|\Psi_{\Omega,f}\|_\infty \leq \|f\|_\infty$ (indeed, $-2\|f\|_\infty \leq \Psi_{\Omega,f} - \|f\|_\infty = \Psi_{\Omega,f} - \|f\|_\infty \leq 0$, by the maximum principle). So if Ω is a bounded pseudoconvex domain in $\mathbb{C}^2$ of finite type $2k$ it follows from the proof of the above theorem that there exists a constant C depending only on Ω such that, for $f \in Lip^\alpha(\partial\Omega)$, $0 < \alpha \leq 1$, we have $\|\Psi_{\Omega,f}\|_{\frac{\alpha}{2k}} \leq C\|f\|_\alpha$. Here $\|\cdot\|_\alpha$ denotes the standard norm on $Lip^\alpha(S)$, $\|f\|_\alpha = \|f\|_\infty + \sup\{|f(x) - f(y)|/\|x - y\|^\alpha : x \neq y \in S\}$.

We finally consider the case when Ω is a bounded pseudoconvex domain in $\mathbb{C}^n$ with real analytic boundary. It follows from Theorems 2 and 3 in [4] that there are constants C, γ and N depending only on Ω such that $C > 0$, $0 < \gamma < 1 < N$, and such that for every $\zeta \in \partial\Omega$ there is a function $\phi_\zeta \in PSH(\Omega) \cap C(\overline{\Omega})$ satisfying $|\phi_\zeta(z) - \phi_\zeta(w)| \leq C\|z - w\|^\gamma$ for $z, w \in \overline{\Omega}$, $\phi_\zeta(z) \leq -\|z - \zeta\|^N$ for $z \in \Omega$ and $\phi_\zeta(\zeta) = 0$. Using these plurisubharmonic peak functions one can proceed as in the proof of Theorem 2 to obtain the following

Theorem 3. *Let Ω be a bounded pseudoconvex domain in $\mathbb{C}^n$ with real analytic boundary. Then there exists a constant $m > 1$ depending only on Ω ($m = N/\gamma$, where γ and N are as above) such that if $f \in Lip^\alpha(\partial\Omega)$, $0 < \alpha \leq 1$, then $\Psi_{\Omega,f} \in Lip^{\alpha/m}(\overline{\Omega})$.*

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