EXTENDIBILITY OF HOMOGENEOUS POLYNOMIALS ON BANACH SPACES

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ABSTRACT. We study the n -homogeneous polynomials on a Banach space X that can be extended to any space containing X . We show that there is an upper bound on the norm of the extension. We construct a predual for the space of all extendible n -homogeneous polynomials on X and we characterize the extendible 2-homogeneous polynomials on X when X is a Hilbert space, an $\mathcal{L}_1$ -space or an $\mathcal{L}_\infty$ -space.

1. INTRODUCTION

There is no Hahn-Banach theorem for n -homogeneous polynomials on a Banach space when $n > 1$. For example, the 2-homogeneous polynomial $P(x) = \sum_k x_k^2$ on l_2 cannot be extended to any $C(K)$ space containing l_2 , as the presence of the Dunford-Pettis property on $C(K)$ would force P to be weakly sequentially continuous [1]. Even when a homogeneous polynomial can be extended to a larger space, it can happen that the norm cannot be preserved [2, 9]. The Aron-Berner extension process for homogeneous polynomials on a Banach space X [2] avoids these obstacles by requiring that the extension be defined only for a very restricted class of spaces containing X —essentially, the bidual X^{**} and spaces closely related to it (see also [1, 4, 7, 10, 13]). A similar observation applies to the ultrapower method of Dineen-Timoney and Lindström-Ryan [6, 8].

We propose to study those homogeneous polynomials on a Banach space X that extend to *every* space containing X . We do not require that the norm be preserved. We show that, if a homogeneous polynomial, P , on X can be extended to every space containing X , then it is possible to put an upper bound on the norms of the extensions. This enables us to put a natural norm on the space $\mathcal{P}_e(^nX)$ of “extendible” n -homogeneous polynomials. We show that $\mathcal{P}_e(^nX)$ is complete in this norm. We then construct a predual for the Banach space $\mathcal{P}_e(^nX)$. We conclude with some special classes of spaces. We characterize the extendible 2-homogeneous polynomials on X when X is a Hilbert space, an $\mathcal{L}_1$ -space or an $\mathcal{L}_\infty$ -space.

All the Banach spaces considered can be taken over the real or complex numbers. $\mathcal{P}(^nX; Y)$ denotes the Banach space of bounded, n -homogeneous polynomials from X into Y . Thus if $P \in \mathcal{P}(^nX; Y)$, then there exists a unique bounded symmetric multilinear mapping $A : X^n \rightarrow Y$ such that $P(x) = A(x, \dots, x)$ for every $x \in X$.

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When Y is the scalar field we denote this space by $\mathcal{P}(^n X)$. For $P \in \mathcal{P}(^n X)$ we can define a bounded linear operator $T : X \rightarrow \mathcal{P}(^{n-1} X)$ by $T(x)(y) = A(x, y, \dots, y)$, where A is the multilinear form on X^n that generates P as described above. We shall refer to T as the linear operator associated with P .

2. EXTENDIBILITY OF LINEAR OPERATORS AND HOMOGENEOUS POLYNOMIALS

We begin with the description of a process that will enable us to “paste together” a family of spaces containing X in a coherent way. Let $j_\alpha : X \rightarrow Z_\alpha$ be a family of embeddings, indexed by $\alpha \in A$. By an *amalgamation* of this family we shall mean a triple consisting of a Banach space Z , an embedding $j : X \rightarrow Z$ and a family of bounded linear operators $i_\alpha : Z_\alpha \rightarrow Z$ with $\|i_\alpha\| \leq 1$, such that $i_\alpha \circ j_\alpha = j$ for every $\alpha \in A$. Where there is no danger of confusion, we shall refer to the amalgamation $(Z, j, \{i_\alpha\})$ simply as “the amalgamation Z ”.

An amalgamation may be constructed as follows: first, form the l_1 -sum $(\sum_\alpha Z_\alpha)_1$ and let k_α denote the usual embedding of Z_α into this space. Let

$$N = \left\{ (j_\alpha x_\alpha) \in \left(\sum_\alpha Z_\alpha \right)_1 : x_\alpha \in X, \sum_\alpha x_\alpha = 0 \right\}.$$

It is easy to see that N is a closed subspace of $(\sum_\alpha Z_\alpha)_1$. Let Z be the quotient space:

$$Z = \left(\sum_\alpha Z_\alpha \right)_1 / N,$$

and let π be the quotient mapping of $(\sum_\alpha Z_\alpha)_1$ onto Z . For each $\alpha \in A$ let $i_\alpha = \pi \circ k_\alpha$. Thus, we have $\|i_\alpha\| \leq 1$ for every α . To define an embedding of X into Z we note first that for all $\alpha, \beta \in A$, we have $k_\alpha j_\alpha(x) - k_\beta j_\beta(x) \in N$ for every $x \in X$. Therefore $\pi k_\alpha j_\alpha(x) = \pi k_\beta j_\beta(x)$ and we can define

$$j(x) = \pi k_\alpha j_\alpha(x) \quad \text{for any } \alpha \in A.$$

To see that j is an embedding, let $x \in X$. It is clear from the definition of $j(x)$ that $\|j(x)\| \leq \|x\|$. On the other hand, if we choose any $\alpha \in A$, we have, by the definition of the quotient norm,

$$\|j(x)\| = \inf\{\|k_\alpha j_\alpha(x) + u\| : u \in N\}.$$

Now if $u \in N$, then $u = (j_\beta x_\beta)$ where $\sum_\beta x_\beta = 0$ and so

$$\begin{aligned} \|k_\alpha j_\alpha(x) + u\| &= \|j_\alpha(x) + j_\alpha(x_\alpha)\| + \sum_{\beta \neq \alpha} \|j_\beta(x_\beta)\| \\ &= \|x + x_\alpha\| + \sum_{\beta \neq \alpha} \|x_\beta\| \geq \left\| x + x_\alpha + \sum_{\beta \neq \alpha} x_\beta \right\| \\ &= \left\| x + \sum_{\beta} x_\beta \right\| = \|x\|, \end{aligned}$$

since $\sum x_\beta = 0$. Therefore $\|j(x)\| = \|x\|$ for every $x \in X$. Finally, it follows from the definition of j that $i_\alpha \circ j_\alpha = j$ for every α . Thus $(Z, j, (j_\alpha))$ meets all the requirements for an amalgamation of the family of embeddings (j_α) .

We shall say that a bounded n -homogeneous polynomial $P : X \rightarrow Y$ is *extendible* if, for every embedding of X into a Banach space Z , there exists an extension of P to a bounded n -homogeneous polynomial $Q : Z \rightarrow Y$. With the help of amalgamations, we can show that we can control the norm of the extension independently of the space Z :

Proposition 1. *If $P \in \mathcal{P}(^n X; Y)$ is extendible, then there exists $C > 0$ such that for every embedding of X into a Banach space Z , there is an extension $Q \in \mathcal{P}(^n Z; Y)$ of P with $\|Q\| \leq C$.*

Proof. If this were false, then for every $n \in \mathbf{N}$ there would exist a Banach space Z_n and an embedding $j_n : X \rightarrow Z_n$ such that every extension of P to Z_n has norm at least n . Let $(Z, j, (i_n))$ be an amalgamation of this sequence of embeddings. Now P has a bounded extension, Q , defined on Z . But then $Q \circ i_n$ is an extension of P to Z_n , and hence

$$\|Q\| \geq \|Q \circ i_n\| \geq n \quad \text{for every } n \in \mathbf{N},$$

which is impossible. This concludes the proof.

Let $\mathcal{P}_e(^n X; Y)$ denote the subset of $\mathcal{P}(^n X; Y)$ consisting of all the extendible n -homogeneous polynomials. $\mathcal{P}_e(^n X; Y)$ is a vector space on which we can define a norm by letting $e(P)$ be the smallest positive real number C with the property that for every space Z containing X there is an extension $Q \in \mathcal{P}(^n Z; Y)$ of P with $\|Q\| \leq C$. We have $\|P\| \leq e(P)$ for every $P \in \mathcal{P}_e(^n X; Y)$.

Proposition 2. *$(\mathcal{P}_e(^n X; Y), e)$ is complete.*

Proof. It suffices to show that every absolutely summable series is convergent. Accordingly, suppose that $\sum_k e(P_k) < \infty$, where $P_k \in \mathcal{P}_e(^n X; Y)$ for every k . Then $\sum P_k$ is absolutely summable in $\mathcal{P}^n(X; Y)$ and hence it converges in this space to a bounded n -homogeneous polynomial P . Let Z be a space that contains X . For each k there exists an extension Q_k of P_k to Z , with $\|Q_k\| \leq e(P_k)$. It follows that the series $\sum Q_k$ converges in $\mathcal{P}(^n Z; Y)$ to a polynomial Q . It is clear that Q extends P , and so $P \in \mathcal{P}_e(^n X; Y)$. Furthermore, we have $\|Q\| \leq \sum \|Q_k\| \leq \sum e(P_k)$. Finally, we have

$$e\left(P - \sum_{k=1}^N P_k\right) = e\left(\sum_{k>N} P_k\right) \leq \sum_{k>N} e(P_k),$$

and hence $\sum P_k$ converges to P in $(\mathcal{P}_e(^n X; Y), e)$. This concludes the proof.

We shall see that the uniform norm is equivalent to the norm e on $\mathcal{P}_e(^n X)$ if and only if every polynomial in $\mathcal{P}(^n X)$ is extendible.

3. THE PREDUAL OF THE SPACE OF EXTENDIBLE POLYNOMIALS

We begin with the construction of a predual for the space $\mathcal{L}_e(^2 X \times Y)$ of extendible bilinear forms on $X \times Y$. We can define a complete norm, e , on this space exactly as in the previous section. We shall denote the projective norm on the tensor product $X \otimes Y$ by $\pi_{X,Y}$. We refer the reader to [3] for further details on tensor products. Now if i, j are embeddings of X, Y into W, Z respectively, then $i \otimes j$ is an algebraic embedding of $X \otimes Y$ into $W \otimes Z$. Thus we may identify $X \otimes Y$ with a vector subspace of $W \otimes Z$. For $u \in X \otimes Y$ we have $\pi_{W,Z}(u) \leq \pi_{X,Y}(u)$. The norms $\pi_{W,Z}$ and $\pi_{X,Y}$ are equivalent on $X \otimes Y$ if and only if every bounded

bilinear form on $X \times Y$ has a bounded extension to $W \times Z$. For an element u of $X \otimes Y$, we define

$$\eta(u) = \inf\{\pi_{W,Z}(u) : X \hookrightarrow W, Y \hookrightarrow Z\},$$

the infimum being taken over all pairs of embeddings $X \hookrightarrow W, Y \hookrightarrow Z$. We claim that η is a reasonable crossnorm on $X \otimes Y$. Obviously, we have $\eta(\lambda u) = |\lambda|\eta(u)$. To see that η is subadditive, let $u_1, u_2 \in X \otimes Y$ and let $\varepsilon > 0$. For each $r = 1, 2$ there exists a pair of embeddings $i_r : X \hookrightarrow W_r, j_r : Y \hookrightarrow Z_r$, such that $\pi_{W_r, Z_r}(u_r) \leq \eta(u_r) + \varepsilon/2$. Let $(W, i, (k_r)), (Z, j, (l_r))$ respectively be amalgamations of these embeddings. Then we have $i \otimes j = (k_r \otimes l_r) \circ (i_r \otimes j_r)$ for each r and since $\|k_r\|, \|l_r\| \leq 1$ it follows that $\pi_{W,Z} \leq \pi_{W_r, Z_r}$ for each r . Therefore

$$\eta(u_1 + u_2) \leq \pi_{W,Z}(u_1 + u_2) \leq \pi_{W,Z}(u_1) + \pi_{W,Z}(u_2) \leq \eta(u_1) + \eta(u_2) + \varepsilon,$$

and hence $\eta(u_1 + u_2) \leq \eta(u_1) + \eta(u_2)$. Next, suppose that $\eta(u) = 0$ for some $u \in X \otimes Y$. Then there is a sequence of pairs of embeddings $X \hookrightarrow W_r, Y \hookrightarrow Z_r$ such that $\pi_{W_r, Z_r}(u) < 1/r$. Arguing as above, let W, Z be amalgamations of these sequences. Then we have $\pi_{W,Z}(u) \leq \pi_{W_r, Z_r}(u) < 1/r$ for every k and so $\pi_{W,Z}(u) = 0$, whence $u = 0$. This shows that η is a norm. To see that η is a reasonable crossnorm, note first that if $x \in X, y \in Y$, then $\pi_{W,Z}(x \otimes y) = \|x\| \|y\|$ for all W, Z and so $\eta(x \otimes y) = \|x\| \|y\|$. Secondly, let $\varphi \in X^*, \psi \in Y^*$. Let $u \in X \otimes Y$ with $\eta(u) < 1$. Choose embeddings $X \hookrightarrow W, Y \hookrightarrow Z$ such that $\pi_{W,Z}(u) < 1$ and let φ_W, ψ_Z be Hahn-Banach extensions of φ, ψ to W, Z respectively. Then

$$|\varphi \otimes \psi(u)| = |\varphi_W \otimes \psi_Z(u)| \leq \|\varphi_W\| \|\psi_Z\| = \|\varphi\| \|\psi\|.$$

It follows that η is a reasonable crossnorm.

Proposition 3. $(X \hat{\otimes}_\eta Y)^*$ is isometrically isomorphic to $\mathcal{L}_e(^2X \times Y)$.

Proof. Every bounded linear functional on $X \hat{\otimes}_\eta Y$ is the linearization, $\tilde{T}$, of a bounded bilinear form T on $X \times Y$. Since $\tilde{T}$ is η -continuous, we have $|\tilde{T}(u)| \leq \eta^*(\tilde{T})\eta(u) \leq \eta^*(\tilde{T})\pi_{W,Z}(u)$ for every $u \in X \otimes Y$ and every pair of embeddings $X \hookrightarrow W, Y \hookrightarrow Z$. Hence $\tilde{T}$ is continuous on $X \otimes Y$ for the norm induced by $W \otimes_\pi Z$ and it follows from the Hahn-Banach theorem that T extends to a bilinear form S on $W \times Z$ with $\|S\| \leq \eta^*(\tilde{T})$. Therefore T is extendible and $e(T) \leq \eta^*(\tilde{T})$. Conversely, if $T \in \mathcal{L}_e(^2X \times Y)$ and $X \hookrightarrow W, Y \hookrightarrow Z$, then T extends to $S \in \mathcal{L}(^2W \times Z)$ with $\|S\| \leq e(T)$. It follows that $|\tilde{T}(u)| = |\tilde{S}(u)| \leq e(T)\pi_{W,Z}(u)$ for $u \in X \otimes Y$ and hence $\tilde{T}$ is η -continuous, with $\eta^*(\tilde{T}) \leq e(T)$. Therefore $\eta^*(\tilde{T}) = e(T)$. This completes the proof.

We now construct a predual for the space of extendible n -homogeneous polynomials. The dual of the n -fold symmetric tensor product $\widehat{\bigotimes}_{s,\pi}^n X$ is the Banach space $\mathcal{L}_s(^nX)$ of symmetric, continuous n -linear forms on X . Since this latter space is isomorphic to $\mathcal{P}(^nX)$, it follows that $\widehat{\bigotimes}_{s,\pi}^n X$ can be considered as an “isomorphic” predual of $\mathcal{P}(^nX)$. We prefer to renorm $\widehat{\bigotimes}_{s,\pi}^n X$ so this becomes an isometry, as follows: for $u \in \widehat{\bigotimes}_{s,\pi}^n X$, we will define

$$\|u\|_\pi = \sup\{|A(u)| : A \in \mathcal{L}_s(^nX) \text{ and } \|\hat{A}\| = 1\},$$

where $\hat{A}$ denotes the n -homogeneous polynomial associated with A . This norm is equivalent to the projective norm on $\widehat{\bigotimes}_{s,\pi}^n X$. Let us write x^n for the tensor

$x \otimes x \otimes \cdots \otimes x$. Then, if u belongs to the uncompleted symmetric tensor product $\bigotimes_{s,\pi}^n X$, it follows from the polarization formula that u can be expressed as a linear combination of tensors of the form x^n . We then have another formula for $\|u\|_\pi$:

$$\|u\|_\pi = \inf \left\{ \sum_{j=1}^m |\lambda_j| \|x_j\|^n : u = \sum_{j=1}^m \lambda_j x_j^n \right\}.$$

We shall denote by $\widehat{X}_\pi^{(n)}$ the space $\widehat{\bigotimes_{s,\pi}^n X}$ with this equivalent norm. Then $\mathcal{P}^n(X)$ is the dual space of $\widehat{X}_\pi^{(n)}$. The mapping $x \mapsto x^n$ is a “universal” continuous n -homogeneous polynomial on X : for every $P \in \mathcal{P}^n(X)$ there is a unique $\tilde{P} \in (\widehat{X}_\pi^{(n)})^*$ with the same norm, such that

$$P(x) = \tilde{P}(x^n) \quad \text{for every } x \in X.$$

We refer to [11, 12] for further details.

Now the space $\widehat{X}_\pi^{(n)}$ suffers the same defect as the projective tensor product $X \otimes Y$ in that its norm does not respect embeddings of X into larger spaces. Proceeding as we did for $X \otimes Y$, we can define a norm, η , on $X^{(n)}$ by

$$\eta(u) = \inf \left\{ \sum_{j=1}^m |\lambda_j| \|w_j\|^n : u = \sum_{j=1}^m \lambda_j w_j^n, w_j \in W, X \subset W \right\},$$

the infimum being taken over all embeddings $X \hookrightarrow W$. We denote by $X_\eta^{(n)}$ the homogeneous product space $X^{(n)}$ with this norm, and we denote the completion of this space by $\widehat{X}_\eta^{(n)}$. We then have $(\widehat{X}_\eta^{(n)})^* = \mathcal{P}_e^n(X)$:

Proposition 4. *$P \in \mathcal{P}^n(X)$ is extendible if and only if the linear form $\tilde{P}$ on $X^{(n)}$ is continuous with respect to the norm η . The correspondence $P \leftrightarrow \tilde{P}$ is an isometric isomorphism of $(\mathcal{P}_e^n(X), e)$ with $(\widehat{X}_\eta^{(n)})^*$.*

The proof is similar to the proof of Proposition 3. D. Cardano and I. Zalduendo have shown that integral polynomials are extendible. This also follows from the above proposition—the n -homogeneous polynomial P is integral precisely when the linear form $\tilde{P}$ is continuous with respect to the injective norm ε and since η is a reasonable crossnorm, we have $\varepsilon \leq \eta$.

Corollary 5. *The norm e is equivalent to the uniform norm on $\mathcal{P}_e^n(X)$ if and only if every n -homogeneous polynomial on X is extendible.*

Proof. If $\mathcal{P}^n(X) = \mathcal{P}_e^n(X)$, then, since $\|\cdot\| \leq e$, it follows that these norms are equivalent. Conversely, suppose these norms are equivalent on $\mathcal{P}_e^n(X)$. Since the uniform norm is the dual of the projective norm and e is the dual of η , it follows that the projective norm is equivalent to η on $X^{(n)}$. Therefore these norms yield the same dual, hence $\mathcal{P}^n(X) = \mathcal{P}_e^n(X)$. This concludes the proof.

Thus, if every n -homogeneous polynomial on X is extendible, then there exists a positive constant C , depending only on X and n , such that for every embedding $X \hookrightarrow Z$, every $P \in \mathcal{P}^n(X)$ has an extension to Z whose norm is at most $C\|P\|$. On the other hand, if X supports an n -homogeneous polynomial that is not extendible, then, using an amalgamation argument, we see that there is an embedding $X \hookrightarrow Z$ and a sequence of n -homogeneous polynomials P_k on X such that every extension of P_k to Z has norm at least equal to k .

4. EXAMPLES

We begin with a simple observation:

Lemma 6. *If $P \in \mathcal{P}({}^2X)$ is extendible, then the associated linear operator $T : X \rightarrow X^*$ is 2-summing.*

Proof. There exists an indexing set I and an embedding $j : X \rightarrow l_\infty^I$. Let $Q \in \mathcal{P}({}^2l_\infty^I)$ be an extension of P and let S be the associated linear operator. Since every bounded linear operator from an $\mathcal{L}_\infty$ -space into an $\mathcal{L}_1$ -space is 2-summing, S is 2-summing. Hence $T = j^* \circ S \circ j$ is 2-summing. This concludes the proof.

In general, this condition is not sufficient for P to be extendible. However, for $\mathcal{L}_1$ -spaces, it is enough:

Proposition 7. *Suppose X is an $\mathcal{L}_1$ -space. Then $P \in \mathcal{P}({}^2X)$ is extendible if and only if the associated linear operator $T : X \rightarrow X^*$ is 2-summing.*

Proof. If T is 2-summing, then there exists a probability measure μ and a factorization of T as $u \circ J_2 \circ v$, where $v \in \mathcal{L}(X, L_\infty(\mu))$, $u \in \mathcal{L}(L_2(\mu), X^*)$ and $J_2 : L_\infty(\mu) \rightarrow L_2(\mu)$ is the canonical inclusion. Now let $j : X \rightarrow Z$ be an embedding. Since $L_\infty(\mu)$ is injective, v extends to an operator $\tilde{v} : Z \rightarrow L_\infty(\mu)$ and then $S = u \circ J_2 \circ \tilde{v}$ extends T . Consider the restriction of S^* to X . We have $S^*|_X = \tilde{v}^* \circ J_2^* \circ u^*|_X$. Now, since X is an $\mathcal{L}_1$ -space, the operator $u^*|_X : X \rightarrow L_2(\mu)$ is 1-summing, and hence 2-summing. Therefore this operator extends to an operator $w : Z \rightarrow L_2(\mu)$. Let $R : Z \rightarrow Z^*$ be the composition $\tilde{v}^* \circ J_2^* \circ w$ and let Q be the 2-homogeneous polynomial on l_∞^I associated with R . Then Q extends P . This concludes the proof.

Next, we consider linear operators and 2-homogeneous polynomials on a Hilbert space H . The situation concerning extendibility of linear operators on Hilbert spaces is particularly simple. In general, 2-summing operators are extendible, since they factor through $L_\infty(\mu)$ spaces [5]. Conversely, suppose that $T : H \rightarrow H$ is extendible. Let $j : H \rightarrow l_\infty^I$ be an embedding. T extends to an operator $S : l_\infty^I \rightarrow H$. But every such operator is 2-summing, and hence T is 2-summing. Therefore, the spaces of extendible and 2-summing operators on H coincide.

The situation with 2-homogeneous polynomials is different. Let P be an extendible 2-homogeneous polynomial on H and let $T : H \rightarrow H$ be the associated linear operator. Let $j : H \rightarrow l_\infty^I$ be an embedding. P extends to a 2-homogeneous polynomial Q on l_∞^I which has an associated linear operator $S : l_\infty^I \rightarrow (l_\infty^I)^*$ with the property that $j^* \circ S \circ j = T$. Since $(l_\infty^I)^*$ is an $\mathcal{L}_1$ -space, $S : l_\infty^I \rightarrow (l_\infty^I)^*$ and $j^* : (l_\infty^I)^* \rightarrow H$ are both 2-summing, and hence $j^* \circ S$ is nuclear [5, p. 119]. Therefore T is nuclear and it follows that P is also nuclear. Thus every extendible 2-homogeneous polynomial on H is nuclear. On the other hand, it follows easily from the Hahn-Banach theorem that nuclear polynomials are extendible. In summary, we have:

Proposition 8. *Let H be a Hilbert space.*

- (a) *A bounded linear operator from H into H is extendible if and only if it is 2-summing.*
- (b) *A bounded 2-homogeneous polynomial on H is extendible if and only if it is nuclear.*

Finally, let X be an $\mathcal{L}_\infty$ -space. Then X^{**} is injective and so, by the Aron-Berner extension theorem [2] we have:

Proposition 9 (Aron and Berner). *Suppose X is an $\mathcal{L}_\infty$ -space. Then every $P \in \mathcal{P}({}^n X)$ is extendible.*

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