COMPACT QUANTUM GROUPS ASSOCIATED WITH MONOIDAL FUNCTORS

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ABSTRACT. We provide a C^* -algebra structure on the bialgebra associated with a monoidal linear $*$ -functor. The C^* -algebra obtained in this way is a compact quantum group in the sense of Baaĵ and Skandalis. We show that the category of finite dimensional unitary corepresentations of this C^* -algebra is equivalent to the given category.

INTRODUCTION

Woronowicz ([W]) and Baaĵ and Skandalis ([BS] and [Sk]) defined compact quantum groups as the C^* -algebras generated by the matrix elements of their corepresentations. On the other hand, monoidal categories now form the right framework for the study of quantum groups; see [JS], [JS2], [K], [KT] and [Y]. In [JS], Joyal and Street constructed a bialgebra $\text{End}^\vee(X)$ from a given monoidal functor X satisfying suitable conditions. This bialgebra plays an important role in the modern treatment of Tannaka reconstruction; see [JS], [Sc] and [D].

In this paper we establish the relationship between these two methods. We provide a C^* -algebra structure on $\text{End}^\vee(X)$, and then show that the completion of $\text{End}^\vee(X)$ is a compact quantum group in the Baaĵ-Skandalis sense. We also show that the category of finite dimensional unitary corepresentations of this C^* -algebra is equivalent to the given C^* -category. This generalizes the result of [W2, Theorem 1.3]. Here concrete monoidal C^* -categories are replaced by abstract monoidal C^* -categories equipped with monoidal linear $*$ -functors.

§1. THE BIALGEBRA $\text{End}^\vee(X)$

Throughout this section, $\mathcal{C}$ is a monoidal linear category. We denote by $\mathcal{Vect}_f$ the monoidal category of finite dimensional vector spaces; here the monoidal product is the usual tensor product, and the monoidal unit is the complex numbers $\mathbf{C}$. We assume that $X : \mathcal{C} \longrightarrow \mathcal{Vect}_f$ is a monoidal linear functor.

Recall from [JS, §3] that $\text{End}^\vee(X)$ is defined as the common coequalizer of the maps

$$S \longmapsto (SX(\mu), r), \quad S \longmapsto (X(\mu)S, s),$$

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from $\text{Hom}(X(s), X(r))$ into $\sum^{\oplus} \{ \text{End}(X(r)) : r \in \text{Ob}(\mathcal{C}) \}$, for all $\mu \in \text{Hom}(r, s)$. Thus $\text{End}^{\vee}(X)$ is the quotient of $\sum^{\oplus} \{ \text{End}(X(r)) : r \in \text{Ob}(\mathcal{C}) \}$ by the subspace $\mathcal{V}$ generated by elements of the form

$$(SX(\mu), r) - (X(\mu)S, s),$$

for all $S \in \text{Hom}(X(s), X(r))$ and all $\mu \in \text{Hom}(r, s)$. We write $[S] = (S, r) + \mathcal{V}$ for all $S \in \text{End}(X(r))$.

For each object r of $\mathcal{C}$, we pick a basis $\{e_i^r\}$ for $X(r)$. Put $e_{i,j}^r = e_i^{r*} \otimes e_j^r$. Then $\{e_{i,j}^r\}$ is a basis for $\text{End}(X(r))$. We define a linear map $\gamma_r : X(r) \rightarrow X(r) \otimes \text{End}^{\vee}(X)$ by

$$\gamma_r(u) = \sum_{i,j} e_{i,j}^r(u) \otimes [e_{j,i}^r], \quad \forall u \in X(r).$$

Then each γ_r is a comodule structure on $X(r)$, and $\gamma : X \rightarrow X \otimes \text{End}^{\vee}(X)$ is a monoidal-preserving natural transformation. The vector space $\text{End}^{\vee}(X)$ is a bialgebra with comultiplication Δ_0 , counit ϵ_0 and multiplication given by

$$\begin{aligned} \Delta_0([e_{i,j}^r]) &= \sum_k [e_{i,k}^r] \otimes [e_{k,j}^r], \\ \epsilon_0([S]) &= \text{Tr}(S), \quad [S][T] = [S \otimes T]. \end{aligned}$$

Let $\text{Comod}_f(\text{End}^{\vee}(X))$ denote the monoidal category of finite dimensional right $\text{End}^{\vee}(X)$ -comodules. Using the natural transformation γ , we obtain a monoidal linear functor

$$\widehat{X} : \mathcal{C} \rightarrow \text{Comod}_f(\text{End}^{\vee}(X)),$$

given by $\widehat{X}(r) = (X(r), \gamma_r)$. We recall the following standard result from Theorem 3 of [JS, §7].

Theorem 1.1. *If $\mathcal{C}$ is abelian and X is exact and faithful, then $\widehat{X}$ is an equivalence of categories.*

For each object r of $\mathcal{C}$, put

$$\Gamma_r = \sum_{i,j} e_{i,j}^r \otimes [e_{j,i}^r], \quad \forall u \in X(r).$$

Then Γ_r is a corepresentation of the bialgebra $\text{End}^{\vee}(X)$, that is,

$$(id \otimes \Delta_0)(\Gamma_r) = (\Gamma_r \otimes I)(id \otimes \tau)(\Gamma_r \otimes I),$$

where τ is the twist map $\tau(a \otimes b) = b \otimes a$. Since γ is a monoidal-preserving natural transformation, it follows that

$$\begin{aligned} \Gamma_r \diamond \Gamma_s &= \Gamma_{rs}, \\ \Gamma_s(X(\mu) \otimes I) &= (X(\mu) \otimes I)\Gamma_r. \end{aligned}$$

for all $\mu \in \text{Hom}(r, s)$ and all objects r, s in $\mathcal{C}$. Also note that

$$\begin{aligned} \gamma_r(u) &= \Gamma_r(u \otimes I), \quad \forall u \in X(r), \\ [S] &= (Tr \otimes id)(\Gamma_r(S \otimes I)), \quad \forall S \in \text{End}(X(r)). \end{aligned}$$

Suppose that $\mathcal{C}$ is left autonomous. We recall from [JS2, §7] that a monoidal category $\mathcal{C}$ is said to be left autonomous if for any object r , there is an object $\tilde{r}$ and there are arrows $\vartheta_r \in \text{Hom}(\iota, r\tilde{r})$ and $\bar{\vartheta}_r \in \text{Hom}(\tilde{r}r, \iota)$ such that

$$\begin{aligned}(I_r \diamond \bar{\vartheta}_r)(\vartheta_r \diamond I_r) &= I_r, \\ (\bar{\vartheta}_r \diamond I_{\tilde{r}})(I_{\tilde{r}} \diamond \vartheta_r) &= I_{\tilde{r}},\end{aligned}$$

where $\diamond$ is the monoidal product, and ι is the monoidal unit of $\mathcal{C}$. We call $\tilde{r}$ a left dual of r , and we refer to the pair $(\vartheta_r, \bar{\vartheta}_r)$ as an adjunction between $\tilde{r}$ and r . We then deduce that there is a bijective antilinear map $J_r : X(r) \rightarrow X(\tilde{r})$ such that

$$\begin{aligned}X(\vartheta_r)(1) &= \sum_i e_i^r \otimes J_r e_i^r, \\ X(\bar{\vartheta}_r)(J_r u_2 \otimes u_1) &= \langle u_1, u_2^* \rangle, \quad \forall u_1, u_2 \in X(r),\end{aligned}$$

where u_2^* is the image of u_2 in the dual space $X(r)^*$. For each $S \in \text{End}(X(r))$, put

$$S^\sharp = (X(\bar{\vartheta}_r) \otimes I_{\tilde{r}})(I_{\tilde{r}} \otimes S \otimes I_{\tilde{r}})(I_{\tilde{r}} \otimes X(\vartheta_r)).$$

Then $\text{End}^\vee(X)$ admits an antipode ν (see [JS, §9]) given by $\nu([S]) = [S^\sharp]$.

§2. COMPACT QUANTUM GROUPS ASSOCIATED WITH MONOIDAL FUNCTORS

Throughout this section, $\mathcal{R}$ is a strict monoidal abelian C^* -category; we assume that the monoidal unit ι is irreducible. Note that since $\mathcal{R}$ is abelian, it follows that $\mathcal{R}$ has subobjects and direct sums in the sense of [DR, §1]. We denote by $\mathcal{Hil}_f$ the strict monoidal C^* -category of finite dimensional Hilbert spaces; here the monoidal product is the usual tensor product, and the monoidal unit is the complex numbers $\mathbf{C}$. If Q is a compact quantum group, we denote by $\mathcal{Ucorep}_f(Q)$ the strict monoidal C^* -category of finite dimensional unitary corepresentations of Q .

Observe that if $\mathcal{R}$ is left autonomous, then it is also right autonomous. If $(\vartheta_r, \bar{\vartheta}_r)$ is an adjunction between $\tilde{r}$ and r , then $(\bar{\vartheta}_r^*, \vartheta_r^*)$ is an adjunction between r and $\tilde{r}$. We will choose $\vartheta_{\tilde{r}} = \bar{\vartheta}_r^*$ and $\bar{\vartheta}_{\tilde{r}} = \vartheta_r^*$.

The category $\mathcal{Hil}_f$ is left autonomous in the following way. For each object V of $\mathcal{Hil}_f$, let W be an object of $\mathcal{Hil}_f$ equipped with a bijective antilinear map $J : V \rightarrow W$. Pick an orthonormal basis $\{e_i\}$ for V , and define

$$\begin{aligned}t_J(1) &= \sum_i e_i \otimes J e_i, \\ \bar{t}_J(J v_2 \otimes v_1) &= \langle v_1 | v_2 \rangle, \quad \forall v_1, v_2 \in V.\end{aligned}$$

Then W is a left dual of V with an adjunction $(t_J, \bar{t}_J)$. Also V is a left dual of W with an adjunction $(\bar{t}_J^*, t_J^*)$. Note that $\bar{t}_J^* = t_{J^{-1}}$ and $t_J^* = \bar{t}_{J^{-1}}$, with respect to an orthonormal basis $\{f_i\}$ of W .

If Q is a compact quantum group, then the category $\mathcal{Ucorep}_f(Q)$ is left autonomous in the following way. For each object (α, V_α) of $\mathcal{Ucorep}_f(Q)$, put $\tilde{\alpha} = \alpha^{J \otimes *}$, where $J : V_\alpha \rightarrow \tilde{V}_\alpha$ is the canonical antilinear map. Since α is unitary, it follows that $t_J \in \text{Hom}(\iota, \alpha \diamond \tilde{\alpha})$ and $\bar{t}_J \in \text{Hom}(\tilde{\alpha} \diamond \alpha, \iota)$. Thus $(\tilde{\alpha}, \tilde{V}_\alpha)$ is a left dual of (α, V_α) with an adjunction $(t_J, \bar{t}_J)$.

Theorem 2.1. *Suppose that $\mathcal{R}$ is a left autonomous strict monoidal abelian C^* -category. Assume that there is an exact faithful monoidal linear $*$ -functor $X :$*

$\mathcal{R} \longrightarrow \mathcal{Hil}_f$. Then there is a C^* -norm on $\text{End}^\vee(X)$, and the completion Q of $\text{End}^\vee(X)$ under this norm is a compact quantum group.

Let r be any object of $\mathcal{R}$. Since $X : \mathcal{R} \longrightarrow \mathcal{Hil}_f$ is faithful, $\text{Hom}(r, r)$ is a finite dimensional C^* -algebra. Hence there are minimal projections $\phi_i \in \text{Hom}(r, r)$ such that $\phi_i \phi_j = \delta_{ij}$ and $\sum_i \phi_i = I_r$. Since $\mathcal{R}$ has subobjects, there are arrows $\mu_i \in \text{Hom}(r_i, r)$ and $\mu_i^* \in \text{Hom}(r, r_i)$ such that $\mu_i \mu_i^* = \phi_i$ and $\mu_i^* \mu_i = I_{r_i}$. Since the ϕ_i are minimal, the r_i are irreducible. Since $\phi_i \phi_j = \delta_{ij}$, we have $\mu_i^* \mu_j = \delta_{ij}$. Therefore $r = \sum_i^\oplus r_i$.

Put $t_r = X(\vartheta_r)$ and $\bar{t}_r = X(\bar{\vartheta}_r)$. Since X is monoidal, $(t_r, \bar{t}_r)$ is an adjunction between $X(\tilde{r})$ and $X(r)$. Let $\tilde{J} : X(r) \longrightarrow \widetilde{X(r)}$ be the canonical antilinear map. Then $(\bar{t}_r \otimes I)(I \otimes t_{\tilde{r}})$ is a bijective linear map from $X(\tilde{r})$ onto $\widetilde{X(r)}$, and hence $\dim(X(\tilde{r})) = \dim(\widetilde{X(r)})$. Therefore there is a bijective antilinear map $J : X(r) \longrightarrow X(\tilde{r})$ such that $\langle Ju_1 | Ju_2 \rangle = \langle u_2 | u_1 \rangle$ for all $u_1, u_2 \in X(r)$. Put $\psi = (\bar{t}_r \otimes I)(I \otimes t_{\tilde{r}})$; then it is a bijective linear map on $X(\tilde{r})$. We then deduce that

$$t_r = (I \otimes \psi^{-1})t_{\tilde{r}}, \quad \bar{t}_r = \bar{t}_{\tilde{r}}(\psi \otimes I).$$

Put $J_r = \psi^{-1}J$. Then we have

$$t_r = t_{J_r}, \quad \bar{t}_r = \bar{t}_{J_r}.$$

Put $t_{\tilde{r}} = \bar{t}_r^*$ and $\bar{t}_{\tilde{r}} = t_r^*$. Since X is a $*$ -functor, we get

$$t_{\tilde{r}} = X(\vartheta_{\tilde{r}}), \quad \bar{t}_{\tilde{r}} = X(\bar{\vartheta}_{\tilde{r}}).$$

For each object r of $\mathcal{R}$, we pick an orthonormal basis $\{e_i^r\}$ for $X(r)$.

Proposition 2.2. *For any $r_1, \dots, r_n \in \mathcal{J}(\mathcal{R})$, the set*

$$\{ [e_{ij}^{r_k}] : i, j = 1, \dots, \dim(X(r_k)), k = 1, \dots, n \}$$

is linearly independent.

Proof. Let W be the vector space generated by the set

$$\{ [e_{ij}^{r_k}] : i, j = 1, \dots, \dim(X(r_k)), k = 1, \dots, n \}.$$

To prove this proposition, it is sufficient to show that for any finite set of complex numbers

$$\{ c_{ij}^k : i, j = 1, \dots, \dim(X(r_k)), k = 1, \dots, n \},$$

there is a linear functional $\rho \in W^*$ such that $\rho([e_{ij}^{r_k}]) = c_{ij}^k$ for all i, j and k . Let $B = \sum_k^\oplus \text{End}(X(r_k))$, and put

$$A = \{ \sum_k^\oplus (id \otimes \rho)(\Gamma_{r_k}) : \rho \in W^* \}.$$

Then A is a subalgebra of B . Since the functor $\widehat{X}$ of Theorem 1.1 is fully faithful, it follows that $\Gamma_{r_1}, \dots, \Gamma_{r_n}$ are pairwise nonequivalent irreducible. We then deduce that $X(r_1), \dots, X(r_n)$ are pairwise nonisomorphic simple A -modules. Therefore

$$A' = \text{End}_A\left(\sum_k^\oplus X(r_k)\right) = \sum_k^\oplus \mathbf{C}I_{r_k}.$$

Thus B is contained in the bicommutant A'' . By the Jacobson density theorem, for any $T = \sum_k^\oplus T_k \in B$, there is an element $a = \sum_k^\oplus (id \otimes \rho)(\Gamma_{r_k}) \in A$ such that

$ae_i^{r_k} = Te_i^{r_k}$ for all i and k . For each $k = 1, \dots, n$, if we take $T_k = \sum_{ij} c_{ji}^k e_{ij}^{r_k}$, then we get

$$\sum_{ij} \rho([e_{ji}^{r_k}]) e_{ij}^{r_k} = \sum_{ij} c_{ji}^k e_{ij}^{r_k}. \quad \blacksquare$$

Proposition 2.3. Put $Q_0 = \sum_{r \in \mathcal{J}(\mathcal{R})}^{\oplus} [\text{End}(X(r))]$. Then $\text{End}^{\vee}(X) = Q_0$.

Proof. Let $S \in \text{End}(X(s))$. Choose a decomposition $I_s = \sum_i \mu_i \mu_i^*$ with $\mu_i \in \text{Hom}(s_i, s)$ and $s_i \in \mathcal{J}(\mathcal{R})$. Then we have

$$[S] = \sum_i [X(\mu_i^*) S X(\mu_i)].$$

Therefore $\text{End}^{\vee}(X)$ is generated by

$$\{ [R] : R \in \text{End}(X(r)), r \in \mathcal{J}(\mathcal{R}) \}.$$

We deduce from Proposition 2.2 that

$$\{ [e_{ij}^r] : i, j = 1, \dots, \dim(X(r)), r \in \mathcal{J}(\mathcal{R}) \}$$

is a basis for $\text{End}^{\vee}(X)$, and this proves the proposition. $\blacksquare$

We define an involution operation $*$ on $\text{End}^{\vee}(X)$ by

$$[S]^* = [S^{*\sharp}] = [\tilde{S}].$$

Then $\text{End}^{\vee}(X)$ becomes a unital $*$ -algebra.

For each object r of $\mathcal{R}$, we have

$$\tilde{\Gamma}_r = \sum_{ij} \widetilde{e_{ij}^r} \otimes [\widetilde{e_{ji}^r}] = \sum_{ij} e_{ij}^{\tilde{r}} \otimes [e_{ji}^{\tilde{r}}] = \Gamma_{\tilde{r}}.$$

Since $\vartheta_r \in \text{Hom}(\iota, r\tilde{r})$ and $\bar{\vartheta}_r \in \text{Hom}(\tilde{r}r, \iota)$, it follows that

$$(\Gamma_r \diamond \tilde{\Gamma}_r)(t_r \otimes I) = t_r \otimes I,$$

$$(\bar{t}_r \otimes I)(\tilde{\Gamma}_r \diamond \Gamma_r) = \bar{t}_r \otimes I.$$

Therefore Γ_r is a unitary.

We define a linear functional h_0 on Q_0 by

$$h_0([S_r]) = \begin{cases} 1, & \text{if } [S_r] = [1]; \\ 0, & \text{if } S_r \in \text{End}(X(r)), r \neq \iota. \end{cases}$$

Proposition 2.4. Let $r, s \in \mathcal{J}(\mathcal{R})$ and $r \neq s$.

(i) For any $S_r \in \text{End}(X(r))$ and $S_s \in \text{End}(X(s))$,

$$h_0([S_s]^* [S_r]) = 0.$$

(ii) For any $S_r, T_r \in \text{End}(X(r))$,

$$h_0([T_r]^* [S_r]) = \dim(X(r))^{-1} \sum_m \langle S_r e_m^r | T_r e_m^r \rangle.$$

(iii) For any nonzero $a = \sum_r^{\oplus} [S_r] \in Q_0$,

$$h_0(a^* a) = \sum_r \dim(X(r))^{-1} \sum_m \|S_r e_m^r\|^2 > 0.$$

Proof. (i) Choose a decomposition $I_{\tilde{r}s} = \sum_i \mu_i \mu_i^*$ with $\mu_i \in \text{Hom}(r_i, \tilde{r}s)$ and $r_i \in \mathcal{J}(\mathcal{R})$. We then have

$$\begin{aligned} h_0([S_s]^*[S_r]) &= h_0([\tilde{S}_s \otimes S_r]) \\ &= \sum_{r_i=\iota} h_0([X(\mu_i^*)(\tilde{S}_s \otimes S_r)X(\mu_i)]). \end{aligned}$$

Note that for any objects r, s and t , $\mu \mapsto (I_{\tilde{r}} \diamond \mu)(\vartheta_{\tilde{r}} \diamond I_t)$ is a bijective linear map from $\text{Hom}(rt, s)$ onto $\text{Hom}(t, \tilde{r}s)$. Since $\text{Hom}(r, s) = \{0\}$, it follows that $X(\mu_i) = 0$ for all i with $r_i = \iota$. This proves (i).

(ii) By similar arguments as in (i) with $r = s$, we get

$$h_0([T_r]^*[S_r]) = \sum_{r_i=\iota} h_0([X(\mu_i^*)(\tilde{T}_s \otimes S_r)X(\mu_i)]).$$

Since $\text{Hom}(\iota, \tilde{r}r) = \mathbf{C}\bar{\vartheta}_r^*$ and $\text{Hom}(\tilde{r}r, \iota) = \mathbf{C}\bar{\vartheta}_r$, it follows that

$$\sum_{r_i=\iota} X(\mu_i)X(\mu_i^*) = cX(\bar{\vartheta}_r^*)X(\bar{\vartheta}_r),$$

for some scalar c . For any i with $r_i \neq \iota$, we have $\mu_i^* \bar{\vartheta}_r^* \in \text{Hom}(\iota, r_i) = \{0\}$. Hence

$$\begin{aligned} \bar{t}_r^* &= X(\bar{\vartheta}_r^*) = X\left(\sum_i \mu_i \mu_i^* \bar{\vartheta}_r^*\right) = X\left(\sum_{r_i=\iota} \mu_i \mu_i^* \bar{\vartheta}_r^*\right) \\ &= cX(\bar{\vartheta}_r^*)X(\bar{\vartheta}_r)X(\bar{\vartheta}_r^*) = c\bar{t}_r^* \bar{t}_r \bar{t}_r^*. \end{aligned}$$

Thus $c\bar{t}_r \bar{t}_r^*(1) = 1$. Observe that

$$\begin{aligned} \bar{t}_r \bar{t}_r^*(1) &= \sum_i \langle J_r^{-1} e_i^{\tilde{r}} | J_r^{-1} e_i^{\tilde{r}} \rangle = \dim(X(r)), \\ \bar{t}_r(\tilde{T}_r \otimes S_r) \bar{t}_r^*(1) &= \sum_m \langle S_r e_m^r | T_r e_m^r \rangle. \end{aligned}$$

Now we have

$$\begin{aligned} h_0([T_r]^*[S_r]) &= h_0\left(\left[\sum_{r_i=\iota} X(\mu_i)X(\mu_i^*)(\tilde{T}_s \otimes S_r)\right]\right) \\ &= ch_0\left([X(\bar{\vartheta}_r^*)X(\bar{\vartheta}_r)(\tilde{T}_s \otimes S_r)]\right) \\ &= ch_0\left([\bar{t}_r(\tilde{T}_s \otimes S_r)\bar{t}_r^*]\right) \\ &= ch_0\left(\left[\sum_m \langle S_r e_m^r | T_r e_m^r \rangle 1\right]\right) \\ &= \dim(X(r))^{-1} \sum_m \langle S_r e_m^r | T_r e_m^r \rangle. \end{aligned}$$

(iii) It is a consequence of (i) and (ii). ■

Proof of Theorem 2.1. For each $a \in Q_0$, we put

$$\|a\| = \sup\{\|\pi(a)\| : \pi \text{ is a nondegenerate representation on Hilbert spaces}\}.$$

Since each Γ_r is unitary, $\|a\|$ is finite and hence $\|\cdot\|$ is a C^* -seminorm on Q_0 . The ideal $\mathcal{I}_0$ of Q_0 consisting of elements of seminorm zero is closed under the involution operation $*$. The canonical quotient map $q : Q_0 \rightarrow Q_0/\mathcal{I}_0$ is a unital $*$ -homomorphism. Let Q denote the completion of $Q_0/\mathcal{I}_0$. We represent the C^* -algebra Q on a Hilbert space by a faithful nondegenerate representation, and then see that

$$\|(q \otimes q) \circ \Delta_0(a)\| \leq \|a\|, \quad \forall a \in Q_0.$$

We then deduce that there is a comultiplication $\Delta : Q \rightarrow Q \otimes Q$ such that

$$\Delta(q(a)) = (q \otimes q) \circ \Delta_0(a), \quad \forall a \in Q_0.$$

Each $\beta_r = (id \otimes q)(\Gamma_r)$ is a unitary corepresentation of (Q, Δ) , and the matrix elements of the family $\{\beta_r\}$ generate Q . By Remark 2.6(b) of [B], there exists a unique Haar measure h for $(Q, \Delta, \{\beta_r\})$. Let $r \in \mathcal{J}(\mathcal{R})$ with $r \neq \iota$. Since the unit I of Q_0 is not in $\mathcal{I}_0$, it follows that $q(I)$ is not in the linear span of $q([e_{ij}^r])$ for all i, j . By the Hahn-Banach theorem, we can choose $\eta \in Q^*$ such that $\eta(q(I)) = 1$ and $\eta(q([e_{ij}^r])) = 0$ for all i, j . Using Theorem 2.3 of [B], we get

$$\begin{aligned} h(q([e_{ij}^r])) &= (\eta * h)(q([e_{ij}^r])) \\ &= \sum_k \eta(q([e_{ik}^r]))h(q([e_{kj}^r])) = 0. \end{aligned}$$

Therefore $h \circ q = h_0$. If π is the cyclic representation of Q induced by the positive linear functional h , then it follows Proposition 2.4(iii) that $\pi \circ q$ is a faithful nondegenerate representation of Q_0 . Therefore $\mathcal{I}_0 = \{0\}$ and $\|\cdot\|$ is a C^* -norm. ■

The following theorem contains Theorem 1.3 of [W2] as a special case.

Theorem 2.5. *The functor $\Pi : \mathcal{R} \rightarrow \mathcal{Ucorep}_f(Q)$, given by $\Pi(r) = \Gamma_r$ and $\Pi(\mu) = X(\mu)$, is an equivalence of monoidal categories.*

Proof. Since $\widehat{X}$ is fully faithful, so is Π . To prove Π is an equivalence, let α be any object of $\mathcal{Ucorep}_f(Q)$. We need to prove that α is equivalent to $\Pi(r)$ for some r in $\mathcal{R}$. By Theorem 2.4 of [B], α is the direct sum of irreducible unitary subcorepresentations α_i . The family $\mathcal{L} = \{\Gamma_r : r \in \mathcal{J}(\mathcal{R})\}$ consists of mutually nonequivalent irreducible corepresentations of Q , and the matrix elements of $\mathcal{L}$ generate Q_0 . By Theorem 2.5 of [B], each α_i is equivalent to an element Γ_{r_i} of $\mathcal{L}$. If we put $r = \sum^\oplus r_i$, then $\Pi(r) = \Gamma_r$ is equivalent to $\sum^\oplus \Gamma_{r_i}$. Hence α is equivalent to $\Pi(r)$. ■

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