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UNIFORM FACTORIZATION FOR COMPACT SETS OF OPERATORS

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Abstract. We prove a factorization result for relatively compact subsets of compact operators using the Bartle and Graves Selection Theorem, a characterization of relatively compact subsets of tensor products due to Grothendieck, and results of Figiel and Johnson on factorization of compact operators. A further proof, essentially based on the Banach-Dieudonné Theorem, is included. Our methods enable us to give an easier proof of a result of W.H. Graves and W.M. Ruess.

Introduction

The purpose of this note is to obtain a factorization result for relatively compact subsets of the Banach space of all compact weak*-weak continuous linear maps using a Banach space version of Michael's Selection Theorem which Bartle and Graves [BG] proved in the early fifties. Precisely, they showed that if X and Y are Banach spaces and u is a continuous linear map from Y onto X, then there exists a continuous map $f : X \to Y$ such that $f(x) \in u^{-1}(x)$ for all $x \in X$. In addition to the Selection Theorem, our approach is to use Grothendieck's characterization of relatively compact sets in the projective tensor product and factorization results of compact operators through a universal Banach space, due to Johnson [J] and Figiel [F]. We further present a second method of proof for our factorization result, based on the Banach-Dieudonné Theorem. From our main theorem we obtain extensions of results of Graves and Ruess [GR]. Their methods are different and, in our opinion, more complicated. We also obtain a new proof of a result of Toma, characterizing polynomials that are weakly uniformly continuous on bounded sets.

1. Preliminaries

Generally, our notation and terminology are standard and we refer to the books [Di] and [G]. For the definition and properties of $\mathcal{L}_p$-spaces the reader is referred to [LT].

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$L_{w*}(X',Y)$ $[K_{w*}(X',Y)]$ is the space of all [compact] weak*-weak continuous linear operators with the usual operator norm. There is an isometric isomorphism $K(X,Y) \simeq K_{w*}(X'',Y)$ $[\,W(X,Y) \simeq L_{w^*}(X'',Y)]$ given by $T \mapsto T''$, where $K(X,Y)$ $[W(X,Y)]$ denotes the space of all [weakly] compact operators from X into Y. The closed unit ball of a Banach space X is denoted B_X. X'_c $[X'_\tau\,]$ denotes the dual of X endowed with the topology $c(X',X)$ $[\,\tau(X',X)]$ of uniform convergence on the [weakly] compact subsets of X. $\mathcal{U}_c$ $[\,\mathcal{U}_\tau\,]$ denotes a c- $[\tau$-$]$ neighborhood base of zero consisting of closed absolutely convex sets (disks). If $U \in \mathcal{U}_c$ $[\mathcal{U}_\tau]$, we denote by $X'_{(U)}$ the completion of the quotient of X' by the nullspace of U, endowed with the norm defined by the Minkowski-functional of U. If $C \subset X$ is a closed bounded disk, we denote by X_C the span of C in X, endowed with the norm given by the Minkowski-functional of C.

In [F], [J], the authors proved that there is a universal Banach space Z such that every operator $T \in K(X,Y)$ can be factored as $T = v \circ u$, where $u \in K(X,Z)$ and $v \in K(Z,Y)$. In particular, Z can be chosen as $Z = (\sum_{W \subset C_p} W)_p$, $1 \le p \le \infty$, where W runs through the subspaces of C_p (and where, as usual, $p = \infty$ is the c_0-sum).

We can show that Z also serves as a universal factorization space for $K_{w*}(X',Y)$-operators. One way to see this is by an application of Johnson's factorization methods to the norm closure in $L(X',Y)$ of the finite-rank, weak*-weak continuous operators. Another way is the following approach, which makes use of [Ru1, Thm. 1.7 (e)]: if $T \in K_{w*}(X',Y)$, there exists $U \in \mathcal{U}_c$ such that $T(U)$ is relatively compact in Y. (For details, compare (2) – (6) of the *method 2* of proof of Theorem 1 below.) Defining $\tilde{T} : X'/U^{-1}(0) \to Y$ by $\tilde{T}(x' + U^{-1}(0)) := T(x')$, and extending to $X'_{(U)}$ by continuity, T can be decomposed in the following way:

$$X' \xrightarrow{id} X'_c \xrightarrow{\pi_U} X'_{(U)} \xrightarrow{\tilde{T}} Y. \tag{1.1}$$

Factoring $\tilde{T}$ through Z, the conclusion of the above is the following result.

(1) *For every pair of Banach spaces X, Y, and for every $T \in K_{w^*}(X',Y)$ there are operators $u \in K_{w^*}(X',Z)$ and $v \in K(Z,Y)$ such that $T = v \circ u$.*

If X or Y is an $\mathcal{L}_1$-space (resp. an $\mathcal{L}_\infty$-space), then Randtke [R2] (resp. Dazord [D], cf. also Randtke [R1], Johnson [J]) has shown that every operator in $K(X,Y)$ factors compactly through l_1 (resp. c_0).

2. The results

Given Banach spaces X, Y, let the universal Banach space Z be as above. According to (1), the continuous, bilinear map

$$\tau : K_{w^*}(X',Z) \times K(Z,Y) \to K_{w^*}(X',Y), \;\; \tau(u,v) = v \circ u,$$

is onto. The linearization $\hat{\tau} : K_{w^*}(X',Z)\hat{\otimes}_\pi K(Z,Y) \to K_{w^*}(X',Y)$ of τ is a continuous linear onto map. Therefore we can apply the Bartle and Graves selection theorem which asserts that there is a continuous map $\sigma : K_{w^*}(X',Y) \to K_{w^*}(X',Z)\hat{\otimes}_\pi K(Z,Y)$ such that $\hat{\tau} \circ \sigma = id_{K_{w^*}(X',Y)}$. We remark that the linearization argument is necessary if we wish to apply the Bartle and Graves selection theorem. Indeed, C. Fernandez [Fe] has recently shown that there are continuous bilinear surjections $\tau : X \times Y \to Z$ between Banach spaces X, Y and Z for which there is no one-sided inverse.

Theorem 1. *Let X and Y be Banach spaces. For every relatively compact subset H of $K_{w^*}(X',Y)$ there exist an operator $u \in K_{w^*}(X',Z)$, a relatively compact subset $\{B_T : T \in H\}$ of $K(Z)$ and an operator $v \in K(Z,Y)$ such that $T = v \circ B_T \circ u$ for all $T \in H$.*

Proof. Method 1. By continuity, the set $\sigma(H)$ is relatively compact in $K_{w^*}(X',Z) \hat{\otimes}_\pi K(Z,Y)$. Now, by Grothendieck [G, p. 51], there exist null sequences (r_i) in $K_{w^*}(X',Z)$ and (s_i) in $K(Z,Y)$ and a relatively compact subset K of l_1 such that for each $T \in H$ we can write $\sigma(T) = \sum_{i=1}^\infty \lambda_i^T r_i \otimes s_i$ where $\lambda^T = (\lambda_i^T) \in K$.

Define $r : X' \to c_0(Z)$ by $r(x') = (r_i(x'))$. That $r \in K_{w^*}(X', c_0(Z))$ follows directly from $\|r_i\| \to 0$. For each $T \in H$ define $A_T : c_0(Z) \to l_1(Z)$ by $A_T(z) = (\lambda_i^T z_i)$, $z = (z_i) \in c_0(Z)$. Since

$$\sum_{i=1}^\infty \left\|\lambda_i^T z_i\right\| \le \sup_i \|z_i\| \cdot \sum_{i=1}^\infty \left|\lambda_i^T\right|,$$

we have $A_T \in L(c_0(Z), l_1(Z))$. Now consider the continuous map $A : l_1 \to L(c_0(Z), l_1(Z))$ defined by $A(\lambda)z = (\lambda_i z_i)$. Since $A(\overline{K}) \supset \{A_T : T \in H\}$, it follows that the subset $\{A_T : T \in H\}$ of $L(c_0(Z), l_1(Z))$ is relatively compact. Now we define a compact operator $s : l_1(Z) \to Y$ by $s(w) = \sum_{i=1}^\infty s_i(w_i)$, $w = (w_i) \in l_1(Z)$. Compactness of s follows from $\|s_i\| \to 0$. Since $\hat{\tau} \circ \sigma = id_{K_{w^*}(X',Y)}$, we conclude that $T = \hat{\tau}(\sigma(T)) = \sum_{i=1}^\infty \lambda_i^T s_i \circ r_i$ and so $T = s \circ A_T \circ r$. Finally, we factor r and s through Z. Thus, there exist operators $u \in K_{w^*}(X',Z)$, $\alpha \in K(Z, c_0(Z))$, $\beta \in K(l_1(Z), Z)$ and $v \in K(Z,Y)$ such that $r = \alpha \circ u$ and $s = v \circ \beta$. Let $B_T = \beta \circ A_T \circ \alpha$ for each $T \in H$. Then $\{B_T : T \in H\}$ is a relatively compact subset of $K(Z)$ and $T = v \circ B_T \circ u$ for every $T \in H$.

Method 2: The following facts will be needed:

(2) Given any compact disk C in Y, there exists another such, C_1 say, with $C \subset C_1$ such that the C_1-topology of Y_{C_1} restricted to C is equal to $\|\cdot\|_Y \mid C$. (Simply take $C_1 = \bigcap_{n=1}^\infty (nC + (1/n)B_Y)$.)

(3) $c(X',X)$ is the finest locally convex topology on X' agreeing with $c(X',X)$ on all $nB_{X'}$, $n \in \mathbb{N}$. (Banach-Dieudonné Theorem.)

Now, if $H \subset K_{w^*}(X',Y)$ is relatively compact, then

(4) $H(B_{X'})$ is relatively compact in Y.

Since, accordingly, $H^*(B_{Y'}) \subset K_1$ a compact disk in X,

(5) $H(K_1^\circ) \subset B_Y$.

Let $U = \bigcap_{n=1}^\infty (nB_{X'} + (1/n)H^{(-1)}(B_Y))$; then $U \in \mathcal{U}_c$ by (3) and (5), and

(6) $H(U) \subset K_2$ a compact disk in Y. (This follows from (4) and $H(U) \subset nH(B_{X'}) + (1/n)B_Y$ for all $n \in \mathbb{N}$, and, actually, is a special case of [Ru1, Theorem 1.7 (e)].)

Choose a compact disk K in Y related to K_2 according to (2). Then we have:

(7) Every sequence $(T_n)_n \subset H$ has a subsequence that is uniformly Cauchy over U with respect to the K-topology. (This follows from operator-norm relative compactness of H, together with $U \subset nB_{X'} + (1/n)H^{(-1)}(B_Y)$ for all $n \in \mathbb{N}$, $H(U) \subset K_2$ and $K \mid K_2 = \|\cdot\|_Y \mid K_2$.)

Now, given $T \in H$, define $\tilde{T} : X'/U^{(-1)}(0) \to Y_K$ by $\tilde{T}(x' + U^{(-1)}(0)) = T(x')$, and extend continuously to $X'_{(U)}$. We then have the following uniform factorization for the operators $T \in H$:

$$X' \xrightarrow{id} X'_c \xrightarrow{\pi_U} X'_{(U)} \xrightarrow{\tilde{T}} Y_K \xrightarrow{id_K} Y. \tag{2.1}$$

Now, factor both (the compact weak*-weak-continuous map) $\pi_U \circ id$ through Z as $\pi_U \circ id = r \circ u$, $u \in K_{w*}(X', Z)$, $r \in K(Z, X'_{(U)})$, and (the compact map) id_K as $id_K = v \circ s$, $s \in K(Y_K, Z)$, $v \in K(Z, Y)$, and let $B_T = s \circ \tilde{T} \circ r \in K(Z)$. Then, by (7), $\{B_T \mid T \in H\}$ is relatively compact in $K(Z)$, and $T = v \circ B_T \circ u, T \in H$. This completes the proof.

The above *method 2* of proof also applies to the case of L_{w*}-operators, except that the factor space Z may depend on X and Y. Specifically, the following result holds.

Proposition 2. *Let X and Y be Banach spaces. There exists a reflexive Banach space $Z = Z(X, Y)$ such that, for every relatively compact subset H of $L_{w^*}(X', Y)$, there exist an operator $u \in L_{w^*}(X', Z)$, a relatively compact subset $\{B_T : T \in H\}$ of $W(Z)$ and an operator $v \in W(Z, Y)$ such that $T = v \circ B_T \circ u$ for all $T \in H$.*

Proof. Given the assumptions of Proposition 2, in *method 2* of proof above, statements (2) to (7) hold with C and C_1 in (2) weakly compact, $c(X', X)$ in (3) being replaced by $\tau(X', X)$ (the assertion then following from the fact that X'_τ is a gDF-space, cf. [Ru2]), and, in (4) – (6), "compact" being replaced by "weakly compact". Altogether, the corresponding reasoning thus leads to the following uniform factorization of the $T's$ in H:

$$X' \xrightarrow{id} X'_\tau \xrightarrow{\pi_U} X'_{(U)} \xrightarrow{\tilde{T}} Y_K \xrightarrow{id_K} Y, \tag{2.2}$$

for some $U \in \mathcal{U}_\tau$ and some weakly compact disk $K \subset Y$. At this point, let

$$Z = Z(X, Y) := (\sum_{C,U} R_{C,U})_2 \oplus_2 (\sum_K R_K)_2,$$

where U runs through an X'_τ-neighbourhood base, C runs through the weakly compact disks in $X'_{(U)}$, and K runs through the weakly compact disks in Y, and the corresponding R-spaces are the associated reflexive Banach spaces in the Davis-Figiel-Johnson-Pelczynski factorization [Di] for the range spaces $X'_{(U)}$ and Y, respectively. Applying now the factorizations corresponding to (1.1) and (2.1) to the mappings of (2.2) completes the proof.

Corollary 3. *Let X and Y be Banach spaces. For every relatively compact subset H of $K(X, Y)$ $[W(X, Y)]$, there exist an operator $u \in K(X, Z)$ $[W(X, Z)]$, a relatively compact subset $\{B_T : T \in H\}$ of $K(Z)$ $[W(Z)]$ and an operator $v \in K(Z, Y)$ $[W(Z, Y)]$ such that $T = v \circ B_T \circ u$ for all $T \in H$. (Here, the spaces Z are those of Theorem 1 and Proposition 2, respectively.)*

In the compact case, this corollary can be combined with factorization results through nice spaces for compact operators between special spaces. In the following result, we apply Corollary 3 when X is an $\mathcal{L}_1$-space or an $\mathcal{L}_\infty$-space and obtain Theorem 2.1 in [GR]. When Y is an $\mathcal{L}_1$-space or an $\mathcal{L}_\infty$-space, a similar result can be stated.

Corollary 4. *Assume that X is an $\mathcal{L}_1$-space (resp. an $\mathcal{L}_\infty$-space). For every relatively compact subset H of $K(X, Y)$ there exist an operator $p \in K(X, l_1)$ (resp. $p \in K(X, c_0)$) and a relatively compact subset $\{Q_T : T \in H\}$ of $K(l_1, Y)$ (resp. of $K(c_0, Y)$) such that $T = Q_T \circ p$ for all $T \in H$.*

Proof. Assume that X is an $\mathcal{L}_1$-space (resp. an $\mathcal{L}_\infty$-space). We apply Corollary 3; then, as we have pointed out in the preliminaries, u factors compactly through l_1 (resp. c_0) such that $u = q \circ p$. Put $Q_T := v \circ B_T \circ q$ and we are done. $\square$

Finally, we show how Corollary 3 yields a new proof of a result of E. Toma [T]. Recall that $\mathcal{P}(^nX)$ denotes the space of continuous n-homogeneous polynomials on X. Each such polynomial P is associated with a unique element A of the space $\mathcal{L}^s(^nX)$ of n-linear, symmetric mappings on X, satisfying $P(x) = A(x, ..., x)$ for each $x \in X$. The space of n-homogeneous polynomials that are weakly uniformly continuous on the unit ball of X is denoted $P_{wu}(^nX)$ and the corresponding space of symmetric n-linear forms is denoted $\mathcal{L}^s_{wu}(^nX)$. For each n-homogeneous polynomial P there is a linear operator $T_P : X \to \mathcal{L}^s(^{n-1}X)$, defined by $T_P(x_1)(x_2, ..., x_n) = A(x_1, x_2, ..., x_n)$. P belongs to $P_{wu}(^nX)$ if and only if the operator T_P is compact; furthermore, if $P \in P_{wu}(^nX)$, then T_P takes its values in the space $\mathcal{L}^s_{wu}(^{n-1}X)$ [AP]. The following corollary is shown in [T]; we offer a different proof, based on the above results.

Proposition 5. *Let X be a Banach space and $\mathcal{H}_n$ a relatively compact subset of the space $K(X, \mathcal{L}^s_{wu}(^{n-1}X))$. Then there is a compact subset K' of X' such that for all $T \in \mathcal{H}_n$ and all $x \in X$, $|T(x)(x, ..., x)| \leq \sup_{k' \in K'} |k'(x)|^n$.*

Proof. We proceed by induction on $n = 2, 3, ...$ Let $\mathcal{H}_2$ be a relatively compact subset of the space $K(X, X')$ of compact linear maps from X to $\mathcal{L}^s_{wu}(X, C) = X'$. By Corollary 2, there are a Banach space Z, a relatively compact subset $\{L_T : T \in \mathcal{H}_2\}$ of $K(X, Z)$, and an operator $w \in K(Z, X')$ such that $T = w \circ L_T$ for all $T \in \mathcal{H}_2$. Thus, for each $x \in X$ and for each $T \in \mathcal{H}_\in$, we have

$$|T(x)(x)| = |w \circ L_T(x)(x)| = |\langle L_T(x), w^t(x)\rangle|,$$

regarding $x \in X \subset X''$, and so $|T(x)(x)| \leq ||L_T(x)|| \cdot ||w^t(x)||$. Now,

$$||L_T(x)|| = \sup_{z' \in B_{Z'}} |\langle L_T(x), z'\rangle| = \sup_{z' \in B_{Z'}} |\langle x, L_T^t(z')\rangle| \leq \sup_{k' \in K_1'} |\langle x, k'\rangle|$$

where $K_1' = \overline{\{L_T^t(z') : T \in \mathcal{H}_2, z' \in B_{Z'}\}}$ is easily seen to be compact. Furthermore,

$$||w^t(x)|| = \sup_{z \in B_Z} |\langle w^t(x), z\rangle| = \sup_{z \in B_Z} |\langle x, w(z)\rangle| = \sup_{k' \in K_2'} |\langle x, k'\rangle|,$$

where $K_2' = \overline{w(B_Z)}$ is compact. Therefore, if $K' = K_1' \cup K_2'$, then

$$|T(x)(x)| \leq \sup_{k' \in K'} |k'(x)|^2$$

for all $T \in \mathcal{H}_2$ and all $x \in X$.

Assume now that the result is true for all $j < n$ and let $\mathcal{H}_n$ be a relatively compact subset of the space $K(X, \mathcal{L}^s_{wu}(^{n-1}X))$. As before, there are a Banach space Z, a relatively compact subset $\{L_T : T \in \mathcal{H}_n\}$ of $K(X, Z)$, and an operator $w \in K(Z, \mathcal{L}^s_{wu}(^{n-1}X))$ such that $T = w \circ L_T$ for all $T \in \mathcal{H}_n$. Thus, for each $x \in X$ and each $T \in \mathcal{H}_n$, we have $|T(x)(x, ..., x)| = |w \circ L_T(x)(x, ..., x)| = |\langle L_T(x), w^t(x, ..., x)\rangle| \leq ||L_T(x)|| \cdot ||w^t(x, ..., x)||$, where we are regarding $(x, ..., x)$ as an element of $\mathcal{L}^s_{wu}(^{n-1}X)'$. Hence, for a compact subset $K_1' \subset X'$, $||L_T(x)|| \leq \sup_{k' \in K_1'} |\langle x, k'\rangle|$. Next, we have

$$||w^t(x, ..., x)|| = \sup_{z \in B_Z} |\langle w^t(x, ..., x), z\rangle| = \sup_{z \in B_Z} |w(z)(x, ..., x)|.$$

Now $\{w(z):\ ||z|| \leq 1\} \equiv \mathcal{H}_{n-1}$ is a relatively compact subset of $\mathcal{L}^s_{wu}({}^{n-1}X)$, which by [AP] means that $\mathcal{H}_{n-1}$ is a relatively compact subset of $K(X, \mathcal{L}^s_{wu}({}^{n-2}X))$. By the induction hypothesis, there is a compact subset $K'_2 \subset X'$ such that for all $x \in X$ and all $z \in B_Z$, $|w(z)(x, ..., x)| \leq \sup_{k' \in K'_2} |k'(x)|^{n-1}$. Letting $K' = K'_1 \cup K'_2$, it follows that

$$|T(x)(x, ..., x)| \leq \sup_{k' \in K'} |k'(x)|^n$$

for all $T \in \mathcal{H}_n$ and all $x \in X$, and the result is proved.

Corollary 6 ([T])**.** *For any n, a continuous n-homogeneous polynomial P belongs to $\mathcal{P}_{wu}({}^nX)$ if and only if there is a compact subset K' of X' such that $|P(x)| \leq \sup_{k' \in K'} |k'(x)|^n$ for all $x \in X$.*

Proof. If $P \in \mathcal{P}_{wu}({}^nX)$, then $T_P : X \to \mathcal{L}^s_{wu}({}^{n-1}X)$ is compact and the result follows by applying the proposition to $\mathcal{H}_n \equiv \{T_P\}$.

Conversely, suppose there exists a compact subset K' of X' such that $|P(x)| \leq \sup_{k' \in K'} |k'(x)|^n$ for all $x \in X$. Let J be the polar of K' in X and let π be the canonical mapping of X onto the Banach space X_J associated with J. Now the dual of X_J is $(X')_{K'}$ and it follows that π is compact. Hence π is weakly uniformly continuous on the unit ball of X. But by the assumption P factors through π. Therefore P is weakly uniformly continuous on the unit ball of X.

Added in proof

The authors are indebted to K. Floret for pointing out that in Method 1 of the proof of Theorem 1, the Bartle-Graves selection theorem is not needed. In fact, by the lifting property of quotient mappings for compact sets, there is a compact set $L \in K_{w^*}(X', Z)\hat{\otimes}_\pi K(Z, Y)$ such that $\hat{\tau}(L) = \bar{H}$.

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