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ON HYPERCYCLIC OPERATORS ON BANACH SPACES

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Abstract. We provide in this paper a direct and constructive proof of the following fact: for a Banach space X there are bounded linear operators having hypercyclic vectors if and only if X is separable and $\dim X = \infty$. This is a special case of a recent result, which in turn solves a problem proposed by S. Rolewicz.

1. Introduction

In this paper $(X, ||\cdot||)$ will denote a Banach space on $\mathbb{K} = \mathbb{R}$ (the real line) or $\mathbb{C}$ (the complex plane). $\mathbb{N}$ is the set of positive integers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Let $B(X)$ be the set of bounded linear operators from X into itself. If $T \in B(X)$, then the *orbit* of a vector $x \in X$ is the set $\mathrm{Orb}\,(T, x) = \{T^n x : n \in \mathbb{N}_0\}$. A vector $x \in X$ is called *hypercyclic* for T if $\mathrm{Orb}\,(T, x)$ is dense in X or, in other words, there is no proper closed T–invariant subset of X containing x. T is called *hypercyclic* if it has a hypercyclic vector. Of course, if X supports such an operator, then X must be separable. In addition, X must be infinite-dimensional; indeed, if T^n is compact for some $n \in \mathbb{N}$, then T is not hypercyclic ([Ki, Theorem 4.2]; see also [Ro]). A vector $x \in X$ is said to be *cyclic* (*supercyclic*) for an operator $T \in B(X)$ if the linear span of $\mathrm{Orb}\,(T, x)$ (if the set $\{\lambda y : y \in \mathrm{Orb}\,(T, x),\ \lambda \in \mathbb{K}\}$) is dense in X. An operator $T \in B(X)$ is *cyclic* (*supercyclic*) if it has a cyclic (a supercyclic) vector. It is evident that hypercyclicity implies supercyclicity and this, in turn, implies cyclicity. S. Rolewicz [Ro] was the first to isolate the concept of hypercyclicity. He showed in 1969 that on l_p $(1 \leq p \leq \infty)$ or c_0 there is a hypercyclic operator. He proved in fact that any multiple of the backward shift on Hilbert space by a scalar of modulus larger than one has hypercyclic vectors. Rolewicz also considered hypercyclicity on some Fréchet spaces.

An easy consequence of Baire's theorem is that a hypercyclic operator possesses a dense G_δ (so residual) set of hypercyclic vectors (see [Ki, Theorem 2.1] and [GeS]). In [GeS], R.M. Gethner and J.H. Shapiro studied hypercyclicity on Fréchet spaces and, as a consequence of a useful criterion ([GeS, Theorem 2.2]; see also [Ki, Theorem 1.4] and [Gr2]), obtained two classical theorems on entire functions; namely, Birkhoff's translation theorem [Bi] and MacLane's differentiation theorem [Ma]. In

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[GoS], G. Godefroy and J.H. Shapiro obtained several results about cyclicity of generalized backward shifts, supercyclicity of nonscalar adjoint multiplication operators, hypercyclicity of certain operators in the Fréchet space $H(\mathbb{C}^n)$ of entire functions on $\mathbb{C}^n$, and many more.

C. Read [Re] has constructed a Banach space operator such that every nonzero vector is hypercyclic. It is not known if such an example can exist on Hilbert space. B. Beauzamy [Be1], [Be2] and [Be3] constructed examples of linear operators on Hilbert spaces having dense, invariant linear manifolds all of whose nonzero elements are hypercyclic. P. S. Bourdon [Bo] has recently proved that any hypercyclic operator T on a Banach space has a dense T-invariant linear manifold consisting, except for zero, entirely of hypercyclic vectors. I. Halperin, C. Kitai and P. Rosenthal [HaKR] showed that any linearly independent denumerable set of vectors in a separable infinite-dimensional Hilbert space is contained in the orbit of a hypercyclic vector. D.A. Herrero [He] proved that the algebra $B(H)$ of all bounded linear operators acting on a complex separable infinite-dimensional Hilbert space H includes a large supply of chaotic (so hypercyclic) operators.

In 1969, S. Rolewicz [Ro, Problem 1] proposed the following problem: Does an infinite-dimensional separable *Banach* space support a *hypercyclic* operator? In [Hz], G. Herzog has characterized those (necessarily separable) Banach spaces X which have *supercyclic* operators: $\dim X \in \{0,1,2\}$ or $\dim X = \infty$ if $\mathbb{K} = \mathbb{R}$, and $\dim X \in \{0,1\}$ or $\dim X = \infty$ if $\mathbb{K} = \mathbb{C}$. To prove this, he used methods of the theory of universal functions developed by K.G. Große-Erdmann [Gr1]. He also cleverly used a result due to R.I. Ovsepian and A. Pelczyński (see Theorem 2 below) about biorthogonal systems on Banach spaces. The operator furnished by G. Herzog in [Hz] is compact, so it cannot be hypercyclic. The answer to Rolewicz's problem has been recently given in the affirmative in [An2]. In fact, much more is proved there. A corollary of the main result in [An2] is that any infinite-dimensional Fréchet space admits a hypercyclic operator. The main result is demonstrated by introducing the concepts of l^1-complete space, locally l^1-convex space and quasi-extension of l^1-operators, and then "translating" the problem to $l^1(\mathbb{N})$, for which H. N. Salas [Sa, Section 3] showed that identity plus a unilateral backward weighted shift with positive weights is hypercyclic. Nevertheless, we believe that a direct and constructive proof of the solution of Rolewicz's problem (at least for Banach spaces) can be of some interest for Banach analysts. Our technique will be similar to that used by H. N. Salas [Sa, Section 3]. Our result has been obtained independently of [An2].

To finish this section, we state the result to be proved:

Theorem 1. *Let X be an infinite-dimensional separable Banach space. Then there is $T \in B(X)$ such that T is hypercyclic.*

2. Preliminary results

We will need two known facts for the proof of Theorem 1. The first one can be found in [OP]. The second one deals with Toeplitz matrices and is proved in the previously mentioned paper of Salas [Sa, Lemma 3.2].

Theorem 2. *If X is an infinite-dimensional separable Banach space, then there is a sequence $\{e_p\}_0^\infty \subset X$ and a sequence $\{\varphi_q\}_0^\infty \subset X^*$ with the following properties:*

a) $\varphi_q(e_p) = \delta_{pq}\ \forall p, q \in \mathbb{N}_0$.

b) $\overline{\operatorname{span}\{e_p : p \in \mathbb{N}_0\}} = X$.

c) If $\varphi_q(x) = 0\ \forall q \in \mathbb{N}_0$, *then* $x = 0$.
d) $||e_p|| = 1\ \forall p \in \mathbb{N}_0$ *and* $\sup_{q\in\mathbb{N}_0} ||\varphi_q|| = C < \infty$.

Lemma 3. *Let* $C_n = (c_{ij}(n))$ *be the* $2^k \times 2^k$ *matrix whose entries* $c_{ij}(n)$ *are the combinatory numbers* $\binom{n}{2^k+j-i}$. *Let* $B_n = (b_i(n))$ *be a column vector such that* $b_i(n)$ *is a polynomial in* n *of degree at most* $2^k - i$ $(i = 1, 2, ..., 2^k)$. *Then for* n *large enough there is a solution* $X_n = (x_i(n))$ *of the equation* $B_n = C_n X_n$ *and the entries* $x_i(n)$ *satisfy* $|x_i(n)| \le P/n^i$, *where* $P = P(k)$ *is a constant.*

3. Proof of the main result

We now prove Theorem 1. Fix a sequence $\{a_p\}_1^\infty$ of positive real numbers such that the series $\sum_1^\infty a_p$ converges. Define the self-mapping $S : X \to X$ by

$$Sx = \sum_{p=0}^{\infty} a_{p+1}\varphi_{p+1}(x)e_p \quad \forall x \in X$$

where $\{e_p\}_0^\infty$ and $\{\varphi_p\}_0^\infty$ are furnished by Theorem 2. From property d) in that theorem, it follows that $S \in B(X)$ (in fact $||S|| \le C\sum_1^\infty a_p$). Let I be the identity operator on X. We will show that the operator $T = I + S$ is hypercyclic.

From property b) in Theorem 2, we can select a dense sequence $\{z_k\}_1^\infty$ in X of the form $z_k = \sum_{i=0}^{2^k-1} z_{i,k}e_i$ $(k \in \mathbb{N})$ for suitable scalars $z_{i,k}$ $(k \in \mathbb{N};\ i = 0, 1, ..., 2^k - 1)$. We proceed by induction in order to construct an increasing sequence $\{n_j\}_1^\infty$ and vectors $y_j = \sum_{i=2^j}^{2^{j+1}-1} b_i e_i \in X$ $(j \in \mathbb{N})$ satisfying

$$||y_j|| \le 2^{-j}(1 + ||T||)^{-n_{j-1}} \tag{1}$$

and

$$||T^{n_j}(\sum_{p=1}^{j} y_p) - z_j|| \le 2^{-j}. \tag{2}$$

Assume just for a moment that the integers n_j and the vectors y_j $(j \in \mathbb{N})$ have been found. Define $y = \sum_{j=1}^\infty y_j$. We are going to show that y is hypercyclic for T. Indeed, since $\{z_k\}$ is dense in X, it suffices to prove that $\lim_{k\to\infty} ||T^{n_k}y - z_k|| = 0$. But this follows from (1), (2) and the next chain of inequalities:

$$\begin{aligned} ||T^{n_k}y - z_k|| &\le ||T^{n_k}(\sum_{j=1}^{k} y_j) - z_k|| + ||\sum_{j=k+1}^{\infty} T^{n_k}y_j|| \\ &\le 2^{-k} + \sum_{j=k+1}^{\infty} ||T^{n_k}|| \cdot ||y_j|| \\ &\le 2^{-k} + \sum_{j=k+1}^{\infty} 2^{-j}(1 + ||T||)^{n_k}(1 + ||T||)^{-n_{j-1}} \le 2^{-k+1}. \end{aligned}$$

Thus the theorem would be proved.

Firstly, observe that $T^n = \sum_{r=0}^n \binom{n}{r} S^r$, where $S^0 = I$. Moreover, note that, by property a) in Theorem 2, $Se_i = a_i e_{i-1}$ for $i \in \mathbb{N}$ and $Se_0 = 0$, so $S^m e_s = (\prod_{l=s-m+1}^{s} a_l)e_{s-m}$ if $m \le s$ and $= 0$ otherwise, for each $m \in \mathbb{N}$. Let us try to find

numbers $x_1, x_2 \in \mathbb{K}$ and $n \in \mathbb{N}$ such that $||T^n(x_1e_2 + x_2e_3) - z_1|| \leq 1/2$. We have that

$$T^n(x_1e_2 + x_2e_3) - z_1 = (x_1 + na_3x_2)e_2 + x_2e_3$$
$$+ [\binom{n}{2}a_1a_2x_1 + \binom{n}{3}a_1a_2a_3x_2 - z_{0,1}]e_0 + [\binom{n}{1}a_2x_1 + \binom{n}{2}a_2a_3x_2 - z_{1,1}]e_1 = 0.$$

We now impose for both square brackets to be zero or, equivalently,

$$\varphi_p(T^n(x_1e_2 + x_2e_3) - z_1) = 0 \quad (p = 0, 1).$$

Then we get the linear system

$$\binom{n}{2}x_1 + \binom{n}{3}a_3x_2 = \frac{z_{0,1}}{a_1a_2},$$

$$\binom{n}{1}x_1 + \binom{n}{2}a_3x_2 = \frac{z_{1,1}}{a_2}.$$

Its matrix with respect to x_1, u_2 (where $u_2 = a_3x_2$) is the matrix C_n of Lemma 3 for $k = 1$. Then there is a constant P such that $|x_1| \leq P/n$ and $|u_2| \leq P/n^2$ for n large enough. Thus, we can take $n = n_1 \in \mathbb{N}$ satisfying $||x_1e_2 + x_2e_3|| \leq 1/2$, $||(x_1+n_1a_3x_2)e_2+x_2e_3|| \leq 1/2$ and, consequently, $||T^{n_1}(x_1e_2+x_2e_3)-z_1|| \leq 1/2$, as required. Set $b_2 = x_1$, $b_3 = x_2$ and $y_1 = b_2e_2+b_3e_3$. Then $||y_1|| \leq (1/2)(1+||T||)^{-n_0}$ and $||T^{n_1}(y_1) - z_1|| \leq 1/2$, where $n_0 = 0$. This completes the first step of the induction.

In order to make the process clearer, we also describe the second step of the induction. We now try to find numbers $x_1, x_2, x_3, x_4 \in \mathbb{K}$ and $n \in \mathbb{N}$ such that

$$||T^n(y_1) + \sum_{i=1}^{4} x_ie_{4+i-1} - z_2|| \leq 1/4 \tag{3}$$

and

$$||\sum_{i=1}^{4} x_ie_{4+i-1}|| \leq (1/4)(1 + ||T||)^{-n_1}. \tag{4}$$

With this we will define $y_2 = \sum_{i=1}^{4} x_ie_{4+i-1}$. Again we want

$$\varphi_p(T^n(y_1 + y_2) - z_2) = 0 \tag{5}$$

for $p = 0, 1, 2, 3$. Then

$$T^n(y_1 + y_2) - z_2 = x_1e_4 + x_2[e_5 + \binom{n}{1}a_5e_4] + x_3[e_6 + \binom{n}{1}a_6e_5 + \binom{n}{2}a_6a_5e_4]$$

$$+x_4[e_7 + \binom{n}{1}a_7e_6 + \binom{n}{2}a_7a_6e_5 + \binom{n}{3}a_7a_6a_5e_4].$$

Thus

$$||T^n(y_1 + y_2) - z_2|| \leq \sum_{i=1}^{4} |x_i||Q_i(n)| \tag{6}$$

where $Q_i(n)$ is a polynomial of degree $i-1$ in n. Condition (5) is equivalent to the system of linear equations

$$\binom{n}{4}x_1+\binom{n}{5}a_5x_2+\binom{n}{6}a_6a_5x_3+\binom{n}{7}a_7a_6a_5x_4=d_1(n),$$

$$\binom{n}{3}x_1+\binom{n}{4}a_5x_2+\binom{n}{5}a_6a_5x_3+\binom{n}{6}a_7a_6a_5x_4=d_2(n),$$

$$\binom{n}{2}x_1+\binom{n}{3}a_5x_2+\binom{n}{4}a_6a_5x_3+\binom{n}{5}a_7a_6a_5x_4=d_3(n),$$

$$\binom{n}{1}x_1+\binom{n}{2}a_5x_2+\binom{n}{3}a_6a_5x_3+\binom{n}{4}a_7a_6a_5x_4=d_4(n),$$

where

$$d_1(n)=(1/a_1a_2a_3a_4)[-b_2\binom{n}{2}a_2a_1-b_3\binom{n}{3}a_3a_2a_1-z_{0,2}],$$

$$d_2(n)=(1/a_2a_3a_4)[-b_2\binom{n}{1}a_2-b_3\binom{n}{2}a_3a_2-z_{1,2}],$$

$$d_3(n)=(1/a_3a_4)[-b_2-b_3\binom{n}{1}a_3-z_{2,2}],$$

$$d_4(n)=(1/a_4)[-b_3-z_{3,2}].$$

Set $x_1=u_1$, $x_2a_5=u_2$, $x_3a_6a_5=u_3$, $x_4a_7a_6a_5=u_4$, and observe that $d_i(n)$ is a polynomial of degree $4-i$, thus Lemma 3 can be applied for $k=2$. Therefore $|u_i|\le P/n^i$ and consequently $|x_i|\le Q/n^i$ for another constant. By choosing n large enough, we have that (4) is satisfied. Since we also have (5), it follows that (3) and (6) are also satisfied. Now set $x_1=b_4$, $x_2=b_5$, $x_3=b_6$, $x_4=b_7$ and we have $y_2=\sum_{i=4}^{7}b_ie_i$.

Define, for the sake of convenience, $b_0=b_1=0$. Assume now that $n_0=0<n_1<\ldots<n_{k-1}$ and $y_1,\ldots,y_{k-1}$ have been chosen such that (1) and (2) hold. Consider the linear system

$$\varphi_p\left(T^n(\sum_{j=1}^{k-1}y_j+\sum_{i=1}^{2^k}x_ie_{2^k+i-1})-z_k\right)=0\quad(p=0,1,\ldots,2^k-1). \tag{7}$$

Observe that, for each $p\in\{0,1,\ldots,2^k-1\}$, the left-hand side of (3) is

$$\sum_{j=1}^{2^k}x_j\varphi_p(T^ne_{2^k+j-1})-\varphi_p(z_k)+\sum_{j=1}^{k-1}\sum_{q=2^j}^{2^{j+1}-1}\varphi_p(T^nb_qe_q)=\sum_{j=1}^{2^k}x_j[\varphi_p(e_{2^k+j-1})$$

$$+\sum_{q=1}^{n}\binom{n}{q}\varphi_p(S^qe_{2^k+j-1})]-z_{p,k}+\sum_{q=2}^{2^k-1}b_q[\varphi_p(e_q)+\sum_{r=1}^{n}\binom{n}{r}\varphi_p(S^re_q)]$$

(use now that $p\le 2^k-1$ and take $n\ge 2^{k+1}$)

$$= \sum_{j=1}^{2^k} x_j \sum_{q=1}^{2^k+j-1} \binom{n}{q} \Big(\prod_{r=2^k+j-q}^{2^k+j-1} a_r \Big) \cdot \varphi_p(e_{2^k+j-1-q}) - z_{p,k} + b_p$$

$$+ \sum_{q=2}^{2^k-1} b_q \sum_{r=1}^{q} \binom{n}{r} \Big(\prod_{l=q-r+1}^{q} a_l \Big) \delta_{p,q-r}$$

$$= \sum_{j=1}^{2^k} x_j \binom{n}{2^k+j-1-p} \Big(\prod_{r=p+1}^{2^k+j-1} a_r \Big) - z_{p,k} + b_p + \alpha_p,$$

where $\alpha_p = 0$ if $p = 2^k - 1$ and $\alpha_p = \sum_{q=p+1}^{2^k-1} b_q \binom{n}{q-p} \prod_{l=p+1}^{q} a_l$ if $p < 2^k - 1$. By putting $i = p + 1$ for $i \in \{1, ..., 2^k\}$, the matrix of (3) with respect to $x_1, ..., x_{2^k}$ is the $2^k \times 2^k$ matrix with entries $d_{ij} = \binom{n}{2^k+j-i} \cdot \prod_{r=i}^{2^k+j-1} a_r$. As shown in [Sa], the finite products of weights a_r do not affect the estimation of solutions $x_1, ..., x_{2^k}$, so Lemma 3 can be applied. This is so because α_p $(= \alpha_{i-1})$ is a polynomial in n of degree not greater than $\max\{q - i + 1 : q \leq 2^k - 1\} = 2^k - i$.

We conclude that if n is large enough the solutions satisfy $|x_i| \leq P/n^i$, where $P = P(k)$ is a constant. We have arrived at the following:

$$T^n\Big(\sum_{j=1}^{k-1} y_j + \sum_{i=1}^{2^k} x_i e_{2^k+i-1}\Big) - z_k = \sum_{q=2}^{2^k-1} b_q \Big(\sum_{j=0}^{n} \binom{n}{j} S^j\Big) e_q$$

$$+ \sum_{q=1}^{2^k} \Big(\sum_{j=q}^{n} \binom{n}{j} S^j\Big) x_q e_{2^k+q-1} - \sum_{q=0}^{2^k-1} z_{q,k} e_q + \sum_{q=1}^{2^k} \Big(\sum_{j=0}^{q-1} \binom{n}{j} S^j\Big) x_q e_{2^k+q-1}$$

$$= \sum_{q=1}^{2^k} \Big(\sum_{j=0}^{q-1} \binom{n}{j} S^j\Big) x_q e_{2^k+q-1},$$

because, from (7), all coefficients in the $\{e_p\}_0^\infty$–expansion of the left-hand side are zero for $p = 0, 1, ..., 2^k - 1$. Thus, from property d) in Theorem 2, it is derived that

$$||T^n\Big(\sum_{j=1}^{k-1} y_j + \sum_{i=1}^{2^k} x_i e_{2^k+i-1}\Big) - z_k|| \leq \sum_{q=1}^{2^k} \Big(\sum_{j=0}^{q-1} \binom{n}{j} ||S||^j\Big) ||x_q|| \leq Q/n$$

for another constant $Q = Q(k)$.

Choose $n_k \in \mathbb{N}$ large enough in such a way that $|x_i(n_k)| \leq 4^{-k}(1 + ||T||)^{-n_{k-1}}$ $(i = 1, ..., 2^k - 1)$ and $n_k > \max\{2^k Q, n_{k-1}\}$. Set $x_i = b_{2^k+i-1}$ $(i = 1, ..., 2^k)$ and $y_k = \sum_{i=2^k}^{2^{k+1}-1} b_i e_i$. Then $||y_k|| \leq 2^{-k}(1+||T||)^{-n_{k-1}}$ and $||T^{n_k}(\sum_{j=1}^{k} y_j) - z_k|| \leq 2^{-k}$. This is (1) and (2) for $j = k$. The theorem is now completely proved.

Corollary 4. *Let X be an infinite-dimensional separable Banach space and $\{e_p\}_0^\infty$, $\{\varphi_q\}_0^\infty$ sequences as in Theorem 2. Assume that $\{a_p\}_1^\infty \subset (0, +\infty)$ is a sequence*

satisfying $\sum_1^\infty a_p < \frac{1}{C}$. *Then the operator* $L \in B(X)$ *given by*

$$Lx = x + \sum_{n=1}^{\infty}\sum_{k=0}^{\infty}(-1)^n(\prod_{j=k+1}^{k+n} a_j)\varphi_{k+n}(x)e_k$$

is hypercyclic.

Proof. If $T = I+S$ is the operator defined in the proof of Theorem 1, then $||S|| < 1$, so T is invertible. Then T^{-1} is also hypercyclic (see [Ki] and [HeK]). But $T^{-1} = (I-(-S))^{-1} = I + \sum_{n=1}^{\infty}(-1)^n S^n = L$, and we are done. □

4. Final remarks and open problems

Note that the operator $T = I + S$ of Theorem 1 is a compact perturbation of identity, because S is compact. Indeed, S is a norm limit in $B(X)$ of a sequence of operators with finite-dimensional range. Property c) of $\{\varphi_q\}$ (Theorem 2) has not been used in our proof. G. Herzog does not use it either in the proof of his theorem on supercyclicity [Hz]. We also remark that every power T^n ($n \in \mathbb{N}$) of our operator $T = I + S$ is also hypercyclic. This is due to a recent result of S. I. Ansari [An1]. To finish, we propose two up-to-date open problems:

1. What are the Banach spaces that admit hypercyclic operators *not* of the form $T = I + S$ with S compact?

2. Is it possible to do in Banach spaces what Halperin, Kitai and Rosenthal did in Hilbert spaces: Given a denumerable set of vectors, find an operator and a vector whose orbit contains such a set?

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