A NOTE ON THE COHOMOLOGICAL BELOW 1/4-PINCHING THEOREM

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ABSTRACT. By making use of a theorem of Toda, we establish a sharper version of the below 1/4-pinching theorem of Abresch and Meyer.

By a δ -pinched manifold, we mean a complete Riemannian manifold with sectional curvature $0 < \delta \leq K \leq 1$. The celebrated Berger's Rigidity Theorem (see e.g. [AM2]) deals with the 1/4-pinching problem. As for the case of $\delta < 1/4$, Abresch and Meyer [AM1] proved the following result.

Theorem ([AM1, AM2]). *There exists a constant $\delta_{ev} \in (0, \frac{1}{4})$, such that, for any even-dimensional, compact, simply connected Riemannian manifold M^n with δ_{ev} -pinched sectional curvature, the cohomology rings $H^*(M^n; F)$ with coefficients $F \in \{Q, Z_2\}$ are isomorphic to the corresponding cohomology rings of one of the compact, rank-one, symmetric spaces S^n , $CP^{n/2}$, $HP^{n/4}$, or CaP^2 , or to the rings $H^*(M^n; F)$ which are truncated polynomial rings generated by an element of degree 8.*

Recall that $H^*(CaP^2; F) = F[\xi]/(\xi^3)$, where $\deg \xi = 8$. Then, it is a natural problem to decide whether there exists a manifold M^n such that $H^*(M; F) = F[\xi]/(\xi^{m+1})$, where $\deg \xi = 8$ and $m > 2$.

The purpose of this note is to point out that the exceptional alternative “or the rings $H^n(M^n; F)$ which are truncated polynomial rings generated by an element of degree 8” can be eliminated due to the following Theorem of Toda.

Theorem ([To]). *If a simply connected space X has the truncated polynomial ring $Z_p[\xi]/(\xi^{m+1})$ as its mod p cohomology ring with $p = 2$ or 3 and $m > 2$, then $\deg \xi = 2$ or 4 .*

It should be pointed out that the condition of simple-connectivity in the preceding theorem can be eliminated [Hu, p. 304] by using K -theory $K(X) \otimes Z_{(2)}$ and the Adams operations.

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