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SIMPLE AH-ALGEBRAS OF REAL RANK ZERO

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ABSTRACT. Let A be a unital simple AH-algebra with real rank zero. It is shown that if A satisfies a so-called fundamental comparison property, then A has tracial topological rank zero. Combining some previous results, it is shown that a unital simple AH-algebra with real rank zero, stable rank one and weakly unperforated $K_0(A)$ must have slow dimension growth.

1. Introduction

Tracial topological rank for C^*-algebras was introduced in [15] (see also [13]). It is an attempt to formulate a new workable notion of rank for noncommutative C^*-algebras. It turns out this rank works very well in classification of simple nuclear C^*-algebras. In this short note we show that with the so-called fundamental comparison property, one can give a fairly short and elementary proof that all unital simple AH-algebra with real rank zero have tracial topological rank zero. Recall that an AH-algebra has the form $A = \lim_n A_n$, where each $A_n = P_n M_{k(n)}(C(X_n))P_n$, each $P_n \in M_{k(n)}(C(X_n))$ is a projection and X_n is a finite CW complex. It has been shown that a unital simple AH-algebra with real rank zero and with slow dimension growth has tracial topological rank zero. It was first shown in [7] (implicitly) that if $A = \lim_n A_n$, where each A_n is a unital corner of $M_{k(n)}(C(X_n))$, where X_n is assumed to be a finite CW complex with dimension no more than 3, then A has tracial topological rank zero. Then, by a reduction theorem (see [6] and [9]), this holds for any simple AH-algebra with slow dimension growth. The results presented in this note show that one can certainly save the reduction step of Gong and Dadarlat to achieve the necessary structural result, namely, simple AH-algebras of slow dimension growth have tracial topological rank. This fact alone significantly simplifies the proof of the classification of simple AH-algebras with real rank zero and slow dimension growth (see [7], [6] and [9]). But the result in this note also gives an abstract characterization of simple AH-algebras with real rank zero and slow dimension growth. It is known that a unital simple C^*-algebra with real rank zero, stable rank one and weakly unperforated K_0-group has the so-called fundamental comparison property (see [3] and Definition 1.1 below). It has been shown ([2]) that a unital simple AH-algebra with slow dimension growth and with real rank zero has stable rank one and has weakly unperforated K_0. But the converse was not

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known. This note establishes the converse. In particular, we show that if a unital simple AH-algebra has real rank zero, stable rank one and weakly unperforated K_0, then it has slow dimension growth. Slow dimension growth is not only a technical condition, it is a condition imposed on the structure of inductive limits, i.e., how the AH-algebra is formed. Stable rank one and weakly unperforated K_0 on the other hand are conditions on A itself and do not refer to how the inductive limit should be formed. By a result in [4], a unital simple AH-algebra which is approximately divisible has stable rank one as well as weakly unperforated K_0-group. Therefore the results in this note also shows that a unital simple AH-algebra which is approximately divisible and its projections separate its traces has slow dimension growth. We found that these abstract characterizations of the condition of slow dimension growth are interesting and useful.

Definition 1.1. A C^*-algebra A is said to satisfy (Blackadar's) fundamental comparison property if for any two projections $p\,q \in M_k(A)$ $(k = 1, 2, ...)$, $\tau(p) > \tau(q)$ for all traces τ of A implies that $p \gtrsim q$, i.e., there is a partial isometry $u \in M_k(A)$ such that $u^*u = q$ and $uu^* \le p$.

One should note that here we use traces instead of quasitraces. For AH-algebras, it is easy to see that every quasitrace is a trace.

Definition 1.2. Let A be a unital simple AH-algebra. We say that A has slow dimension growth if $A = \lim_n A_n$, where $A_n = \bigoplus_{j=1}^{l(n)} P_{(n,j)} M_{n(j)}(X_{(n,j)}) P_{(n,j)}$, $X_{(n,j)}$ is a connected finite CW complex and

$$\lim_{n\to\infty} \max_j \{ \frac{\dim X_{(n,j)}}{\mathrm{rank} P_{(n,j)}} \} = 0.$$

If $\dim X_{(n,j)}$ is bounded, then A is said to have bounded dimension. If A has bounded dimension, then A has slow dimension growth.

Let A be a C^*-algebra, G a subset of A and $x \in A$. In what follows we write $x \in_\varepsilon G$ if there is $y \in B$ such that $\|x - y\| < \varepsilon$.

Definition 1.3. A simple unital C^*-algebra A is said to have tracial topological zero (written $TR(A) = 0$) if for any $\varepsilon > 0$ and finite subset $\mathcal{F} \subset A$ containing a nonzero $a \in A_+$ there exists a finite-dimensional C^*-subalgebra $B \subset A$ with $1_B = p$ such that

(1) $\|px - xp\| < \varepsilon$ for all $x \in \mathcal{F}$,
(2) $pxp \in_\varepsilon B$ for all $x \in \mathcal{F}$ and
(3) $1 - p \preceq q$ for some $q \in \overline{aAa}$.

It is shown in [15] that a unital simple C^*-algebra A with $TR(A) = 0$ is quasidiagonal, has real rank zero, stable rank one, and weakly unperforated $K_0(A)$. It also has the Blackadar's fundamental comparison property.

2. The results

We present the following results:

Theorem 2.1. *Let A be a unital simple AH-algebra of real rank zero. If A satisfies the fundamental comparison property, then $TR(A) = 0$.*

Theorem 2.2. *Let A be a unital simple AH-algebra of real rank zero. Then the following are equivalent:*

(a) *A has fundamental comparison property;*
(b) *A has stable rank one and weakly unperforated $K_0(A)$;*
(c) *A has slow dimension growth.*
(d) $TR(A) = 0$.

Proof. It follows from [3] that (a) $\Leftrightarrow$ (b) and it follows from [2] that (c) $\Rightarrow$ (b). By [15], (d) implies (a) and (b). It follows from Theorem 2.1 that (a) implies (d). It remains to show that (d) and (b) imply (c). Since A is an AH-algebra, A satisfies the so-called Universal Coefficient Theorem. It follows from 4.3 (and 4.4) of [14] that A is pre-classifiable. Since $TR(A) = 0$, by [15] (or from (b) and [19]) $K_0(A)$ satisfies the Riesz decomposition property. It follows from 3.7 in [14] and 4.18 in [7] that A is isomorphic to unital AH-algebra with bounded dimension. Thus (d) implies (c). $\square$

Remark 2.3. In the proof of (d) $\Rightarrow$ (c) we used the results in classification of simple nuclear C^*-algebras. We can use a more general result in [17] instead of [14]. Given a weakly unperforated simple ordered group G with the Riesz property and a countable abelian group F, one should note that it is straightforward to construct a unital simple AH-algebra $A = \lim_n A_n$ with $A_n = P_n M_{k(n)}(C(X_n))P_n$, where X_n is a finite CW complex with $\dim X_n \leq 3$ and $TR(A) = 0$ such that $K_0(A) = G$ and $K_1(A) = F$. One does not need to use KK-theory but uses standard construction of topological spaces such as attaching a disk to a circle (via degree n map) and suspension of the resulting spaces. The construction then follows closely from a construction of Goodearl [10].

We will present two consequences at the end of this note.

Definition 2.4. Let I^n be the n-dimensional unit cube. Fix $k > 0$. A $1/k$-frame Ω of I^n is a closed subset I^n which is a union of hyperplanes (intersecting with I^n) satisfying the following:

$$\Omega = X_1 \times I^{n-1} \cup I \times X_2 \times I^{n-2} \cup \cdots \cup I^{n-1} \times X_n,$$

where $X_i = \{t_0^i, t_1^i, t_2^i, ..., t_{k+2}^i\}$ with $t_0^i = 0$ and $t_{k+2}^i = 1$ such that $1/2k < |t_j^i - t_{j+1}^i| < 1/k$, $j = 0, 1, ..., k+1$. Let $d > 0$. Define

$$\Omega^d = \{x \in I^n : \mathrm{dist}(x, \Omega) \leq d\}.$$

A $1/k$-frame is called *standard* if each $X_i = \{0, 1/(k+1), 2/(k+1), ..., 1\}$.

The proof of the following is standard.

Proposition 2.5. *Fix $k > 0$, $1/8k > d > 0$ and a $1/k$-frame Ω of I^n. Let $N > \max\{(8k)^2, 8/d\}$ be a positive integer. There is $\eta > 0$ and finitely many $(1/k)$-frames $\{\Omega_1, ..., \Omega_L\}$ satisfying the following:*

(i) $\Omega_i^{4\eta} \subset \Omega^{d/2}$, $i = 1, 2, ..., L$,

(ii) *for any $1/k$-frame Ω_0 with $\Omega_0^{4/N} \subset \Omega^{d/2}$, there is Ω_j with $1 \leq j \leq L$ such that*

$$\Omega_j^{\eta} \subset \Omega_0^{1/N}.$$

Lemma 2.6. *Let $k > 0$, $1/8k > d > 0$ and Ω a $1/k$-frame of I^n. For any $\sigma > 0$, there exists an integer $N > (8k)^2$ such that, for any normalized Borel measure μ on I^n, there is a $(1/k)$-frame Ω_0 which satisfies:*

(a) $\Omega_0^{2/N} \subset \Omega^{d/2}$ *and*

(b) $\mu(\Omega_0^{1/N}) < \sigma/2$.

Proof. Let $\delta = \min\{d, \sigma\}$. To save notation without loss of generality, we may assume that $k > 2$ and $X_i = \{0, 1/(k+2), 2/(k+2), ..., 1\}$ for each i. Choose an integer $L > 2n(k+2)/\delta$. Consider L disjoint close sub-intervals $J_1^{(i,j)}, ..., J_L^{(i,j)}$ of $[(j/k+2) - \delta/8, (j/k+2) + \delta/8]$ each of which has length $\delta/16L$. Put $F(1, j, m) = J_m^{(1,j)} \times I^{n-1}, ..., F(n, j, m) = I^{n-1} \times J_m^{(n,j)}$ $(m = 1, ..., L)$. For each i and j, there is at least one of $F(i, j, m)$ that has measure no more than $1/L$. We may assume that $\mu(F(i, j, 1)) \le 1/L$. Set $F = \bigcup_{i=1}^n \bigcup_{j=1}^{k+2} F(i, j, 1)$. Then $\mu(F) \le n(k+2)/L < \delta/2 < \sigma/2$. Choose $N > 64L/\delta$. We note that there is a $1/k$ frame $\Omega_0 \subset F$ such that $\Omega_0^{1/N} \subset F$. It is also clear that (a) holds. $\square$

The following is taken from 3.2 of [12] (see also 4.4 of [8]).

Lemma 2.7. *Let A be a unital C^*-algebra and $\phi : C(X) \to A$ be a homomorphism, where X is a compact metric space. For any $\varepsilon > 0$ and any finite subset $\mathcal{F} \subset C(X)$, there is $\delta > 0$ such that if*

(1) *$S_1, S_2, ..., S_n$ are disjoint open subsets of X such that* $\operatorname{dist}(x, x') < \delta$ *for any $x, x' \in S_i$ $(i = 1, ..., n)$;*

(2) *$\lambda_i \in S_i$, $i = 1, 2, ..., n$;*

(3) *p_i is a projection in $Her(a_i)$, where $a_i = \phi(x_i)$ and $x_i \in C(X)_+$ such that $x_i(t) = 0$ if $t \notin S_i$;*

(4) *$y_f = (1 - \sum_{i=1}^n p_i)\phi(f)(1 - \sum_{i=1}^n p_i)$,*

then

$$\|\phi(f) - (y_f + \sum_{i=1}^n f(\lambda_i)p_i)\| < \varepsilon \qquad \text{and} \quad \|(1 - \sum_{i=1}^n p_i)\phi(f) - \phi(f)(1 - \sum_{i=1}^n p_i)\| < \varepsilon$$

for all $f \in \mathcal{F}$.

Proof of Theorem 2.1. Write $A = \lim_n(A_n, \phi_n)$. Fix a finite subset $\mathcal{F} \subset A$, $\varepsilon > 0$ and $a \in A_+ \setminus \{0\}$. Without loss of generality, we may assume that there is a finite subset $\mathcal{G} \subset A_1$ such that $\phi_{1,\infty}(\mathcal{G}) = \mathcal{F}$. To save notation, we may further assume that $\mathcal{F}$ and $\mathcal{G}$ are in the unit balls of A and A_1, respectively. We will show that, for any nonzero $a \in A_+$, there is a finite-dimensional C^*-subalgebra $C \subset A$ with $1_C = p$ such that

(1) $\|px - xp\| < \varepsilon$,

(2) $pxp \in_\varepsilon C$ for all $x \in \mathcal{F}$ and

(3′) $1 - p$ is equivalent to a projection in $Her(a)$.

Since $RR(A) = 0$, there is a nonzero projection $q \in Her(a)$. Thus, we may replace a by a projection q, and so we may replace (3′) by

(3) $\tau(1 - p) < \sigma$ for all $\tau \in T(A)$, where $\sigma > 0$ is previously given.

Let $p_1, ..., p_m$ be central projections of A_1. By considering each

$$\phi_{1,\infty}(p_i)A\phi_{1,\infty}(p_i),$$

we can reduce the general case to the case in which $A_1 = PM_l(C(X))P$, where X is a connected finite CW complex.

We first consider the case that $A_1 = M_l(C(X))$. If (1), (2) and (3) can be established for the case that $A_1 = C(X)$, then it is clear that (1), (2) and (3) can be established for the case that $A_1 = M_l(C(X))$. Therefore we reduce to the case in which $A_1 = C(X)$.

So let $A_1 = C(X)$. There is an integer n such that $X \subset I^n$. There is a surjective map $\psi : C(I^n) \to C(X)$. We may further assume that $\mathcal{G} \subset C(I^n)$.

Now we choose δ in Lemma 2.7 for $\varepsilon/2$ and the finite subset $\mathcal{G}$ with $X = I^n$.

We will apply Lemma 2.6 and Proposition 2.5. Fix $k > 0$ so that $1/k < \delta/4$, $d = 1/(8k+1)$ and a standard $1/k$-frame Ω of I^n. Let N be in Lemma 2.6 for the above k, d and σ. Let $\eta > 0$ be in Proposition 2.5 and $\Omega_1, ..., \Omega_L$ be finitely many $1/k$-frames of I^n satisfying (i) and (ii) in Proposition 2.5.

Let $1 \geq f_i \geq 0$ be in $C(I^n)$ such that $f_i(t) = 1$ on $\Omega_i^{\eta/2}$ and $f_i(t) = 0$ on $X \setminus \Omega_i^{\eta}$, $i = 1, 2, ..., L$. Since $RR(A) = 0$, there are mutually orthogonal projections $\{p_{i,1}, ..., p_{i,l(i)}\} \subset A$ such that

$$\|\phi_{1,\infty}(f_i) - \sum_{s=1}^{l(i)} \lambda(i,s) p_{i,s}\| < \sigma/16$$

for all $1 \leq i \leq L$, where $0 \leq \lambda(i,s) \leq 1$ are positive numbers. By choosing a large m, there are, for each i, mutually orthogonal projections $\{q_{i,1}, ..., q_{i,l(i)}\} \subset A_m$ such that

$$\|\phi_{1,\infty}(f_i) - \sum_{s=1}^{l(i)} \lambda(i,s) \phi_{m,\infty}(q_{i,s})\| < \sigma/8$$

for all $1 \leq i \leq L$. For an even larger m, we may assume that

$$\|\phi_{1,m}(f_i) - \sum_{s=1}^{l(i)} \lambda(i,s) q_{i,s}\| < \sigma/4, \quad 1 \leq i \leq L. \tag{e 2.1}$$

We may assume that $A_m = PM_K(C(Y))P$, where $P \in M_K(C(Y))$ is a projection and Y is a disjoint union of $Y_1, Y_2, ..., Y_J$, where each Y_j is a connected finite CW complex ($1 \leq j \leq J$). Let $y_j \in Y_j$ and let $\tau_j = tr \circ p_{y_j} \circ \phi_{1,m}$, $j = 1, 2, ..., J$, where tr is the the normalized trace on $M_{s(j)}$ and p_{y_j} is the point-evaluation at y_j. Let $P_j = P|_{Y_j}$, $s(j)$ be its rank and let μ_j be the normalized measure induced by τ_j.

We now fix j. Let $B_j = \phi_{m,\infty}(P_j) A \phi_{m,\infty}(P_j)$. By applying Lemma 2.6 and Proposition 2.5, there is a $1/k$-frame such that $\Omega_{0,j}$ such that

$$\mu_j(\Omega_{0,j}^{1/N}) < \sigma/2 \quad \text{and} \quad \Omega_{0j}^{2/N} \subset \Omega^{d/2}.$$

Therefore by Proposition 2.5, there is an $i(j)$ such that

$$\Omega_{i(j)}^{\eta} \subset \Omega_{0j}^{1/N}.$$

It follows that $\tau_j(f_{i(j)}) < \sigma/2$. Let $\tau_\xi = tr \circ p_\xi \circ \phi_{1,m}$, where ξ is any other point on Y_j. Since Y_j is connected, each $\tau_\xi(q_{i,j}) = \tau_j(q_{i,j})$. By (e 2.1) this implies that $\tau_\xi(f_{i(j)}) < \sigma/2 + \sigma/2 = \sigma$ for all $\xi \in Y_j$. Since for any tracial state τ_{B_j} on B_j there is a normalized measure μ on Y_j such that $\tau_{B_j}(f_{i(j)}) = \int_{Y_j} \tau_\xi(f_{i(j)}) d\mu$, we conclude that $\tau_{B_j}(f_{i(j)}) < \sigma$.

Let $I^n \setminus \Omega^{\eta/16} = \bigcup_{i=1}^K O_i$, where O_i are mutually disjoint open subsets with diameter $< \delta$. Let $0 \leq h_i, h_i' \leq 1$ be in $C_0(O_i) \subset C(I^n)$ such that $h_i(t) = 1$ on $O_i \cap (I^n \setminus \Omega^{\eta/2})$ and $h_i(t) = 0$ in $\Omega_{\eta/4}$; $h_i'(t) = 1$ if $O_i \cap (I^n \setminus \Omega^{\eta/4})$ and $h_i'(t) = 0$

in $\Omega_{\eta/8}$. Note that $h_i h_i' = h_i$. Since $RR(B_i) = 0$, by [5] there is a projection $e_i \in Her(\phi_{1,\infty}(h_i'))$ such that $e_i \phi_{i,\infty}(h_i) = \phi_{i,\infty}(h_i)$. Let $Q_j = \sum_{i=1}^{K} e_i$. Then $1_{B_j} - Q_j \le \phi_{1,\infty}(f_{i(j)})$.

It follows from Lemma 2.7 and $1_{B_j} - Q_j \le \phi_{1,\infty}(f_{i(j)})$ that

(i) $\|Q_j x - x Q_j\| < \varepsilon$, for all $x \in \phi_{1,\infty}(\mathcal{G})$,

(ii) $Q_j x Q_j \in_\varepsilon C_j$, where C_j is the finite-dimensional C^*-algebras generated by $e_1, ..., e_K$,

(iii) $\tau(1_{B_j} - Q_j) \le \sigma \tau(1_{B_j})$ for all tracial states τ on A.

Applying this to each j, we obtain a finite-dimensional C^*-subalgebra $C \subset A$ with $1_C = p$ such that

(1) $\|pz - zp\| < \varepsilon$,

(2) $pzp \in_\varepsilon C$ for all $z \in \phi_{1,\infty}(G)$ and

(3) $\tau(1-p) < \sigma$ for all $\tau \in T(A)$.

Now we consider the case in which $A_1 = PM_l(C(X))P$. By 8.12 of [11] (see also 6.10.3 of [1]), there is K and a projection $Q \in M_K(PM_l(C(X))P)$ such that $QM_K(PM_l(C(X))P)Q \cong M_L(C(X))$ for some L. Let $e = 1_{A_1}$ be identified with a projection in $M_L(C(X))$. Let $\mathcal{F}_1 = \{e\} \cup \mathcal{F}$. If (1), (2) and (3) can be established for the case in which $A_1 = M_L(C(X))$, then (1), (2) and (3) can be established in $\phi_{1,\infty}(Q) M_K(A) \phi_{1,\infty}(Q)$ for $\mathcal{F}_1$ and $\varepsilon/32 < 1/64$. In particular, $\|pe - ep\| < \varepsilon/32$ and $pep \in_{\varepsilon/32} C$. Thus there is a projection $p' \in e\phi_{1,\infty}(Q)M_K(A)\phi_{1,\infty}(Q)e = A$ such that $\|p' - pep\| < \varepsilon/16$. There is a projection $q \in C$ such that $\|q - p'\| < \varepsilon/8$. There is a unitary $u \in A$ such that $\|u - 1\| < \varepsilon/4$ such that $u^* q u = p'$. Set $C_1 = u^*(qCq)u$. Then C_1 is a finite-dimensional C^*-subalgebra and $1_{C_1} = p'$. Moreover, since $\|(e - p') - (e - pep)\| < \varepsilon/16$, $e - p'$ is equivalent to a projection in $1 - p$. Now we have

(1) $\|p'x - xp'\| < \varepsilon/4$,

(2) $p'xp' \in_{\varepsilon/2} C_1$

(3) $\tau(e - p') < \sigma$ for all $\tau \in T(A)$. □

Remark 2.8. It is perhaps the right time to point out to the reader that Villadsen gave an example of a simple unital AH-algebra with stable rank one which does not have weakly unperforated K_0 (see [18]). In particular, it does not have slow dimension growth.

Corollary 2.9. *Let A be a unital simple AH-algebra. The following are equivalent:*

(i) *A is approximately divisible and projections in A separate the traces,*

(ii) $TR(A) = 0$,

(iii) *A has slow dimension growth and projections in A separate the traces,*

(iv) *A has real rank zero, stable rank one and weakly unperforated $K_0(A)$.*

From a more recent result in [17], we have the following:

Corollary 2.10. *Let A be a unital separable simple C^*-algebra. Then the following are equivalent:*

(i) *A is an AH-algebra with stable rank one, real rank zero and with weakly unperforated $K_0(A)$;*

(ii) *A is a C^*-algebra in $\mathcal{N}$ with $TR(A) = 0$.*

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