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MAXIMAL ELEMENTS IN NONCOMPACT SPACES WITH APPLICATION TO EQUILIBRIA

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ABSTRACT. A new maximal theorem for L_S-majorized correspondences in noncompact spaces is presented and applied to obtain an equilibrium existence theorem for noncompact abstract economies. The corresponding results of Borglin and Keiding (1976), Yannelis and Prabhakar (1983), Ding and Tan (1993), Yuan and Tarafdar (1996), and Ding and Yuan (1998) are generalized by our results.

1. Introduction

The existence of a Cournot-Nash equilibrium for a normal form game was proved by Nash (1951). The notion of an abstract economy (social system) was introduced by Debreu (1952) which contains the normal game form as a special case and proved the existence of an equilibrium. The equilibrium result of Debreu not only provides as a corollary the Nash existence theorem, but it was also the instrument used by Arrow-Debreu (1954) to prove the existence of a Walrasian equilibrium. The Debreu theorem was generalized by Shafer-Sonnenschein (S-S) and Borglin-Keiding (B-K) to allow for preferences which need not be representable by utility functions. All the above results are for finite dimensions and do not allow for an infinite number of agents. Most importantly however, as was remarked by Yannelis-Prabhakar (Y-P), the proofs of S-S and B-K fail in the presence of an infinite number of commodities or an infinite number of agents, and new arguments are required. To this end not only were new mathematical results proved in Y-P, but also a new class of correspondences was introduced called L-class.

In this paper, we extend the L-class further and provide new results on maximal elements and equilibrium by dispensing with the assumption of compact strategy sets which generalizes the results in Y-P. We mention that such attempts were made by Yannelis (1985) for maximal elements over noncompact subsets of linear topological spaces.

The corresponding results of Fan (1962), Borglin and Keiding (1976), Yannelis and Prabhakar (1983), Yannelis (1985), Chang (1989) (1990), Kim (1992), Ding and Tan (1993), Yuan and Tarafdar (1996), and Ding and Yuan (1998) are generalized by our results.

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2. Notation and definitions

(1) 2^A denotes the set of all subsets of A.
(2) coA denotes the convex hull of the set A.
(3) A/B denotes the difference of sets A and B.
(4) clA, $intA$ denote the closure and interior of the set A.
(5) $\phi : X \to 2^Y$ is a correspondence, that is, $\phi(x)$ is a subset of Y for each $x \in X$.
(6) Let $X = \prod_{i\in I} X_i$ and $\pi_j : X \to X_j$ be called the projection of X onto X_j, if $\pi_j(x) = x_j$ for each $x = (x_i)_{i\in I} \in X$.

Let I be a (possibly uncountable) set. For each agent $i \in I$, let its choice set or strategy set X_i be a nonempty set in a topological vector space. Let $X = \prod_{j\in I} X_j$. Following Gale and Mas-Colell [9], $\Gamma = (X_i, P_i)_{i\in I}$ is a qualitative game if for each player $i \in I, X_i$ is the strategy set of player i, and $P_i : X = \prod_{j\in I} X_j \to 2^{X_i}$ is a preference correspondence of player i which is irreflexive [i.e. $\pi_i(x) \notin P_i(x)$ for each $x \in X$]; also, a point $\tilde{x} \in X$ is said to be an equilibrium point of the game $\Gamma = (X_i, P_i)_{i\in I}$ if $P_i(\tilde{x}) = \emptyset$ for each $i \in I$. A generalized model is as follows:

Definition 1. Let I denote the set of agents. For each $i \in I$, let X_i be a nonempty set and $X = \prod_{j\in I} X_j$. Following Ding et al. [6], an *abstract economy* or (generalized game) $G = (X_i, A_i, B_i, P_i)_{i\in I}$ is defined as a family of quadruples (X_i, A_i, B_i, P_i), where $A_i, B_i : X \to 2^{X_i}$ are feasible correspondences of agent i and $P_i : X \to 2^{X_i}$ are preference correspondences of agent i. An *equilibrium point* for G is an $\tilde{x} \in X$ where $\tilde{x} = (\tilde{x}_i)$ satisfies

(1) $\tilde{x}_i \in B_i(\tilde{x})$,
(2) $P_i(\tilde{x}) \cap A_i(\tilde{x}) = \emptyset$,

for each $i \in I$. When $A_i = B_i$ for each $i \in I$ our definition coincides with the standard definition, e.g. in Borglin and Keiding [2] or in Yannelis and Prabhakar [13].

Definition 2. Let X be a topological space, Y a nonempty subset of a vector space E, $\theta : X \to E$ a map and $\phi : X \to 2^Y$ a correspondence. The following notions were introduced in [2], [6]–[10], [12]–[15]:

(1) A set $G \subset X$ is said to be compactly open in X if for each compact set K in X the set $G \cap K$ is open in K.
(2) ϕ is said to have *compactly open lower sections* in X if for each $y \in Y$ the set $\phi^{-1}(y) = \{x \in X \mid y \in \phi(x)\}$ is compactly open in X.
(3) ϕ is said to have *locally compactly open lower section* at x if there is an open set W_x containing x such that for each $y \in Y$ the set $\phi^{-1}(y) \cap W_x$ is compactly open in X.
(4) ϕ is said to be of class $L_{\theta,S}$ if for every $x \in X, \theta(x) \notin co\phi(x)$, and ϕ has compactly open lower sections in X.
(5) A correspondence $\phi_x : X \to 2^Y$ is said to be an $L_{\theta,S}$-majorant of ϕ at $x \in X$ if there exists an open neighborhood N_x of in X such that (a) for each $z \in N_x, \phi(z) \subset \phi_x(z)$ and $\theta(z) \notin co\phi_x(z)$, (b) for each $y \in Y, \phi_x^{-1}(y)$ is compactly open in X;
(6) Suppose $X' \subset X$. ϕ is $L_{\theta,S}$-majorized in X' if for each $x \in X'$ with $\phi(x) \neq \emptyset$ there exists an $L_{\theta,S}$-majorant of ϕ at x in X.

In this paper, we deal mainly with the case (I) $X = Y$ which is a nonempty convex subset of the topological vector space E and $\theta = I_X$, the identity map on X, or the case (II) $X = \prod_{i\in I} X_i$ and $\theta = \pi_j : X \to X_j$ is the projection of X onto X_j and $Y = X_j$ is a nonempty convex subset of a topological vector space. In both cases (I) and (II), we write L_S in place of $L_{\theta,S}$.

3. The existence of maximal elements in noncompact spaces

Let X be a nonempty subset of a topological space and U be a preference correspondence on X, that is, for each $x \in X$ associates a set $U(x) \subset X$, which may be interpreted as the set of those objects in X that are "better", "larger" or "after" x. An element $x \in X$ is U-maximal if $U(x) = \emptyset$.

Theorem 5.1 of Yannelis and Prabhakar [14] is needed in order to prove the existence of maximal elements in noncompact spaces. This is given below in Theorem 1.

Theorem 1. *Let X be a compact convex set in a Hausdorff topological vector space and let $U : X \to 2^X$ satisfy the following conditions:*

(1) *$x \notin coU(x)$ for all $x \in X$.*

(2) *$U^{-1}(x) = \{y \in X \mid x \in U(y)\}$ is open in X for each $x \in X$.*

Then $\{x \in X \mid U(x) = \emptyset\}$ is nonempty and compact.

The following result is a new theorem for the existence of maximal elements for L_S-majorized correspondences in noncompact spaces.

Theorem 2. *Let X be a convex subset of a Hausdorff topological vector space E and let $U : X \to 2^X$ be everywhere L_S-majorized. Suppose there exists a compact set L of X such that for each finite subset S of X, there is a compact convex set K containing S and satisfying*

$$K / \bigcup_{x\in K} U^{-1}(x) \subset L.$$

Then $\{x \in X \mid U(x) = \emptyset\}$ is a nonempty subset of L.

Proof. Suppose that $U(x) \neq \emptyset$ for each $x \in L$. Then by the hypothesis, $U(x) \neq \emptyset$ for each $x \in X$. For each $x \in X$, choose an open neighborhood N_x of x and $\phi_x : X \to 2^X$ such that (1) for each $x' \in N_x, U(x') \subset \phi_x(x'), x' \notin co\phi_x(x')$ and (2) for each $y \in X, \phi_x^{-1}(y)$ is compactly open. Let $\mathcal{U} = \{N_x | x \in X\}$. For each $x \in L$, choose $\tilde{N}_x$ to be a closed neighborhood of x such that $\tilde{N}_x \subset N_x$. Since L is compact, there exists a finite cover $\{\tilde{N}_{x_1}, \cdots, \tilde{N}_{x_n}\}$ such that L is contained in the interior of $\bigcup_{i=1}^n \tilde{N}_{x_i}$. Let $F_1 = \bigcup_{i=1}^n \tilde{N}_{x_i}$. Define $\psi_i : X \to 2^X, i = 1, \cdots, n$, by

$$\psi_i(x) = \begin{cases} \phi_{x_i}(x), & \text{if } x \in \tilde{N}_{x_i}; \\ X, & \text{otherwise.} \end{cases}$$

Define P_1 on X by $P_1(x) = \bigcap_{i=1}^n \psi_{x_i}(x)$. Then (1) for each $x' \in F_1, U(x') \subset P_1(x'), x' \notin coP_1(x')$ and (2) for each $y \in X, P_1^{-1}(y)$ is compactly open in X. Choose $y_1, \cdots, y_m \in X$ such that

$$(*) \qquad L \subset \bigcup_{i=1}^m P_1^{-1}(y_i).$$

According to the hypothesis, there is a compact convex subset K containing $y_1, \cdots, y_m$ such that

$$(**) \qquad K / \bigcup_{x \in K} U^{-1}(x) \subset L.$$

Let $X' = co(L \cup K)$ and O be an open set containing L such that its closure $\overline{O} \subset F_1$. Then X' is paracompact. For each $x \in X'/O$, there exists a closed neighborhood $\tilde{N}_x$ of x such that $\tilde{N}_x \subset N_x$ where $N_x \in \mathcal{U}$ and $\tilde{N}_x \bigcap L = \emptyset$. Since X'/O is paracompact, we can assume that there is a subset $D \subset X'/O$ such that $\{\tilde{N}_x \mid x \in D\}$ is a locally finite closed cover of X'/O. Let $I(y) = \{x \in D \mid y \in \tilde{N}_x\}$. Define $P_2 : [X'/O] \to X$ by $P_2(y) = \bigcap_{x \in I(y)} \phi_x(y)$. Then (1) for each $x' \in [X'/O], U(x') \subset P_2(x'), x' \notin coP_2(x')$ and (2) for each $y \in X, P_2^{-1}(y)$ is compactly open in $[X'/O]$. Define $P : X' \to 2^X$ by

$$P(x) = \begin{cases} P_1(x) \cap P_2(x), & \text{if } x \in F_1 \cap [X'/O]; \\ P_1(x), & \text{if } x \in O \cap X'; \\ P_2(x), & \text{if } x \in [X'/F_1]. \end{cases}$$

Then (1) for each $x' \in X', U(x') \subset P(x')$, $x' \notin coP(x')$ and (2) for each $y \in X$, $P^{-1}(y)$ is compactly open in X'. Furthermore, for each $x \in L, P(x) = P_1(x)$ and hence by $(*)$ for each $x \in L$,

$$(* * *) \qquad P(x) \cap K \neq \emptyset.$$

Then let $P_K : K \to 2^K$ be defined by $P_K(x) = P(x) \cap K$ for each $x \in K$. Then using Theorem 1, there exists $x_0 \in K$ such that $P_K(x_0) = \emptyset$. Since $K \cap (\bigcup_{x \in K} U^{-1}(x)) \subset \bigcup_{x \in K} P_K^{-1}(x)$, it follows that

$$K / \bigcup_{x \in K} P_K^{-1}(x) \subset K / \bigcup_{x \in K} U^{-1}(x)$$

and hence by $(**)$, $x_0 \in L$. This contradicts $(* * *)$, therefore there is $x \in L$ such that $U(x) = \emptyset$.

Corollary 1. *Let X be a convex subset of a Hausdorff topological vector space E and let $U : X \to 2^X$ be everywhere L_S-majorized. Suppose there exists a compact set L and a compact convex set K_0 of X such that for each $x \in X$*

$$co(K_0 \cup \{x\}) / \bigcup_{x' \in co(K_0 \cup \{x\})} U^{-1}(x') \subset L.$$

Then $\{x \in X \mid U(x) = \emptyset\}$ is a nonempty subset of L.

Remark 1. Corollary 1 is an extension of Corollary 5.1 of Yannelis and Prabhakar [14] from compact spaces to noncompact spaces. It also extends Theorem 1 of Kim [10] from intersecting one fixed point to intersecting some compact convex set, and it extends Theorem 1 of Ding and Tan [7] and Theorem 2.3 of Ding and Yuan [8] from paracompact spaces to noncompact spaces.

4. Equilibria of the generalized game

For an application of Theorem 2, we prove the following existence theorem of equilibrium for an abstract economy with an infinite number of agents in noncompact topological vector spaces.

Theorem 3. *Let $(X_i, A_i, B_i, P_i)_{i\in I}$ be an abstract economy and D be a compact subset of $X = \prod_{i\in I} X_i$. Suppose the following conditions are satisfied:*

(1) *X_i is a nonempty convex subset of a Hausdorff topological vector space for each $i \in I$;*
(2) *for each $x \in X, A_i(x)$ is nonempty and $coA_i(x) \subset B_i(x)$ for each $i \in I$;*
(3) *$\triangle_i := \{x \in X \mid x_i \in B_i(x)\}$ is closed in X for each $i \in I$;*
(4) *$A_i : X \to 2^{X_i}$ has locally compactly open lower sections in $\triangle_i$ and is L_S-majorized in $X/\triangle_i$ for each $i \in I$;*
(5) *$A_i \cap P_i : X \to 2^{X_i}$ is L_S-majorized in $\triangle_i$ for each $i \in I$;*
(6) *$\bigcap_{i\in I}\{x \in \triangle_i \mid (P_i \cap A_i)(x) = \emptyset\} = \bigcap_{i\in I} cl\{x \in \triangle_i \mid (P_i \cap A_i)(x) = \emptyset\}$. Moreover, for each finite set $S \subset X$, there exists a compact convex set $K = \prod_{i\in I} K_i$ containing S such that for each $i \in I$:*
   (i) *for each $x \in [K \cap \triangle_i]/\prod_{i\in I} D_i,\ A_i(x) \cap P_i(x) \cap K_i \neq \emptyset$;*
   (ii) *for $x \in [K/\prod_{i\in I} D_i],\ A_i(x) \cap K_i \neq \emptyset$.*

Then an equilibrium point for the game exists.

Proof. For each $i \in I$, define $\phi_i : X \to 2^{X_i}$ by

$$\phi_i(x) = \begin{cases} P_i(x) \cap A_i(x), & \text{if } x \in \triangle_i; \\ A_i(x), & \text{otherwise.} \end{cases}$$

Fix i and $x \in \triangle_i$ where $(P_i \cap A_i)(x) \neq \emptyset$. There exists an open neighborhood N_x of x and a correspondence $\psi_x : X \to 2^{X_i}$ such that (1) for each $x' \in N_x, x'_i \notin co\psi_x(x')$ and $P_i(x') \cap A_i(x') \subset \psi_x(x'),$ (2) for each $y \in X_i, \psi_x^{-1}(y)$ is compactly open. Since A_i has locally compactly open lower section at x, there is an open set W_x containing x such that $A_i^{-1}(y) \cap W_x$ is compactly open for each $y \in X_i$. Let $O_x = N_x \cap W_x$. Define $\varphi_x : X \to 2^{X_i}$ by

$$\varphi_x(x') = \begin{cases} \psi_x(x') \cap A_i(x'), & \text{if } x' \in \triangle_i \cap O_x; \\ A_i(x'), & \text{if } x' \in O_x/\triangle_i; \\ \emptyset, & \text{if } x' \in X/O_x. \end{cases}$$

Then φ_x is of class L_S and is an L_S-majorant of ϕ_i at x. For each $x \in X/\triangle_i$, since A_i is L_S-majorized at x and $\triangle_i$ is closed, it follows that ϕ_i is L_S-majorized at x. Thus ϕ_i is L_S-majorized everywhere.

For each $i \in I$, define $\phi'_i : X \to 2^X$ by $\phi'_i(x) = \{x' \in X \mid x'_i \in \phi_i(x)\}$. Since ϕ_i is L_S-majorized everywhere, ϕ'_i is L_S-majorized everywhere for each $i \in I$.

Define $\Lambda : X \to 2^I$ by $\Lambda(x) = \{i \in I \mid \phi'_i(x) \neq \emptyset\}$. Define $U : X \to 2^X$ by $U(x) = \bigcap_{i\in\Lambda(x)} \phi'_i(x)$ for each $x \in X$. Suppose $U(x) \neq \emptyset$. From assumption (6), choose $i \in \Lambda(x)$ with $x \in int_X\{x \in X \mid (P_i \cap A_i)(x) \neq \emptyset\}$. Then $\phi'_i(x) \neq \emptyset$ and there exists an open neighborhood G_x of x and $T_x : X \to 2^X$ such that (1) for each $x' \in G_x, \phi'_i(x') \subset T_x(x'), x' \notin coT_x(x')$ and (2) for each $y \in X, T_x^{-1}(y)$ is compactly open. Thus there exists an open subset $O \subset G_x$ containing x such that $\phi'_i(x') \neq \emptyset$ for each $x' \in O$. Then U is majorized by T_x on O. This implies that U is L_S-majorized in X.

Moreover, for each finite set $S \subset X$ there exists a compact convex set $K = \prod_{i\in I} K_i$ containing S such that for each $i \in I$: If $x \in [K \cap \triangle_i]/\prod_{i\in I} D_i$, then $A_i(x) \cap P_i(x) \cap K_i \neq \emptyset$; if $x \in [K/\prod_{i\in I} D_i]$, then $A_i(x) \cap K_i \neq \emptyset$. From above, we have $\phi_i(x) \cap K_i \neq \emptyset$ for each $x \in [K/\prod_{i\in I} D_i]$ and each $i \in I$.

Thus $U(x)\cap\prod_{i\in I}K_i\neq\emptyset$ for each $x\in[K/\prod_{i\in I}D_i]$. By Theorem 2, there exists $\tilde{x}\in\prod_{i\in I}D_i$ such that $U(\tilde{x})=\emptyset$. Then $\phi_i(\tilde{x})=\emptyset$ for each $i\in I$. From assumption (2), $\tilde{x}\in\bigcap_{i\in I}\triangle_i$. Hence $\tilde{x}_i\in B_i(\tilde{x})$, and $P_i(\tilde{x})\cap A_i(\tilde{x})=\emptyset$ for each $i\in I$.

Corollary 2. *Let $X=\prod_{i\in I}X_i$ and $(X_i,A_i,B_i,P_i)_{i\in I}$ be an abstract economy. Suppose the following conditions are satisfied:*

(1) *X_i is a nonempty convex subset of a Hausdorff topological vector space E_i for each $i\in I$;*
(2) *for each $x\in X, A_i(x)$ is nonempty and $coA_i(x)\subset B_i(x)$ for each $i\in I$;*
(3) *$\triangle_i:=\{x\in X\mid x_i\in B_i(x)\}$ is closed in X for each $i\in I$;*
(4) *$A_i:X\to 2^{X_i}$ has compactly open lower sections for each $i\in I$;*
(5) *$A_i\cap P_i:X\to 2^{X_i}$ is L_S-majorized in $\triangle_i$ for each $i\in I$;*
(6) *$\bigcap_{i\in I}\{x\in\triangle_i\mid(P_i\cap A_i)(x)=\emptyset\}=\bigcap_{i\in I}cl\{x\in\triangle_i\mid(P_i\cap A_i)(x)=\emptyset\}$;*
(7) *there exist a nonempty compact convex subset K_0 of X and a nonempty compact subset D of X such that for each $y\in X/D$ there is an $x\in co(K_0\cup\{y\})$ with $x_i\in co(A_i(y)\cap P_i(y))$ for all $i\in I$.*

Then an equilibrium point for the game exists.

Remark 2. Assumptions (3), (5) and (6) in Corollary 2 improve the relative assumptions in Theorem 4 of Ding and Tan [7], Theorem 3.5 of Ding and Yuan [8], and Theorem 2.3 of Yuan and Tarafdar [16]. Also assumption (7) in Corollary 2 generalizes these theorems without paracompactness or one-point intersection.

In Corollary 2, let $A_i(x)=B_i(x)=X_i$ for each $x\in X$. Then we have the following:

Corollary 3. *Let $X=\prod_{i\in I}X_i$ and $(X_i,P_i)_{i\in I}$ be a qualitative game. Suppose the following conditions are satisfied:*

(1) *X_i is a nonempty convex subset of a Hausdorff topological vector space E_i for each $i\in I$;*
(2) *$P_i:X\to 2^{X_i}$ is L_S-majorized for each $i\in I$;*
(3) *$\bigcap_{i\in I}\{x\in X\mid P_i(x)=\emptyset\}=\bigcap_{i\in I}cl\{x\in X\mid P_i(x)=\emptyset\}$;*
(4) *there exist a nonempty compact convex subset K_0 of X and a nonempty compact subset D of X such that for each $y\in X/D$ there is an $x\in co(K_0\cup\{y\})$ with $x_i\in co(P_i(y))$ for all $i\in I$.*

Then an equilibrium point for the game exists.

Let $\Gamma=\{N,(X_i)_{i\in N},(p_i)_{i\in N}\}$ be an n-person game in normal form, where $N=\{1,2,\cdots,n\}$ is the set of agents, X_i is a nonempty compact convex subset of a Hausdorff topological vector space, X_i is the set of strategies of the ith agent, p_i is a function from the product $X=\prod_{i\in N}X_i$ into the real numbers R, and p_i represents the utility (payoff or cost) function of ith agent. For each i, let $X_{-i}=\prod_{j\neq i}X_j$. For $x\in X$ and $i\in N$, $x=(x_i,x_{-i})$. We say that a vector x is a pure-strategy Nash equilibrium if, for each $i\in N, x_i\in X_i$ and for each $t\in X_i$,

$$p_i(x_i,x_{-i})\leq p_i(t,x_{-i}).$$

For each i, define a correspondence $P_i:X\to 2^{X_i}$ by $P_i(x)=\{y_i\in X_i\mid p_i(y_i,x_{-i})<p_i(x)\}$. If $p_i(.,x_{-i})$ is quasiconvex on X_i, then $P_i(x)$ is convex and $x_i\notin P_i(x)$ for all $x\in X$. If p_i is continuous, then P_i has an open graph. Thus we can replace preference correspondences $\{P_i\}_{i\in N}$ in Corollary 3 by cost functions $\{p_i\}_{i\in N}$ and have the following:

Corollary 4. *Let* $X = \prod_{i \in N} X_i$ *and* $\{N, (X_i)_{i\in N}, (p_i)_{i\in N}\}$ *be an n-person game. Suppose the following conditions are satisfied:*

(1) X_i *is a nonempty convex subset of a Hausdorff topological vector space* E_i *for each* $i \in N$*;*

(2) *the cost function* $p_i : X \to R$ *is continuous in* X *and the function* $p_i(., x_{-i})$ *is quasiconvex on* X_i *for each* x *and each* $i \in N$*;*

(3) *there exist a nonempty compact convex subset* K_0 *of* X *and a nonempty compact subset* D *of* X *such that for each* $y \in X/D$ *there is an* $x \in co(K_0 \cup \{y\})$

$$sup_{i\in N}(p_i(x_i, y_{-i}) - p_i(y)) < 0.$$

Then the game admits a pure-strategy Nash equilibrium.

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