A NOTE ON SUBGAUSSIAN ESTIMATES FOR LINEAR FUNCTIONALS ON CONVEX BODIES

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ABSTRACT. We give an alternative proof of a recent result of Klartag on the existence of almost subgaussian linear functionals on convex bodies. If K is a convex body in $\mathbb{R}^n$ with volume one and center of mass at the origin, there exists $x \neq 0$ such that

$$|\{y \in K : |\langle y, x \rangle| \geq t \|\langle \cdot, x \rangle\|_1\}| \leq \exp(-ct^2 / \log^2(t+1))$$

for all $t \geq 1$, where $c > 0$ is an absolute constant. The proof is based on the study of the L_q -centroid bodies of K . Analogous results hold true for general log-concave measures.

1. INTRODUCTION

The purpose of this note is to provide an alternative proof of a recent result of Klartag (see [9]) on the existence of almost subgaussian linear functionals on convex bodies. Let K be a convex body in $\mathbb{R}^n$ with volume $|K| = 1$ and center of mass at the origin. Let $\psi : [0, \infty) \rightarrow [0, \infty)$ be a convex, increasing function with $\psi(0) = 0$. For every bounded measurable function $f : K \rightarrow \mathbb{R}$, define

$$(1.1) \quad \|f\|_\psi = \inf \left\{ t > 0 : \int_K \psi(|f(x)|/t) dx \leq 1 \right\}.$$

We will be interested in the ψ_α -norm of linear functionals $y \mapsto \langle y, x \rangle$ on K , where $1 \leq \alpha \leq 2$ and $\psi_\alpha(t) = e^{t^\alpha} - 1$. We say that $x \neq 0$ defines a ψ_α -direction for K with constant $B > 0$ if

$$(1.2) \quad \|\langle \cdot, x \rangle\|_{\psi_\alpha} \leq B \|\langle \cdot, x \rangle\|_1.$$

It is not hard to check that this holds true if and only if

$$(1.3) \quad \|\langle \cdot, x \rangle\|_q \leq cBq^{1/\alpha} \|\langle \cdot, x \rangle\|_1$$

for every $q \geq 1$, where $c > 0$ is an absolute constant. By Borell's lemma (see [13], Appendix III), there exists an absolute constant $C > 0$ such that if K is a convex body in $\mathbb{R}^n$, then every $x \neq 0$ is a ψ_1 -direction for K with constant C .

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The study of ψ_2 -directions for linear functionals on convex bodies is motivated by the study of isotropic convex bodies and Bourgain's approach to the isotropic constant problem. A convex body K in $\mathbb{R}^n$ is called isotropic if it has volume $|K| = 1$, center of mass at the origin, and there exists a constant $L_K > 0$ such that

$$(1.4) \quad \int_K \langle y, \theta \rangle^2 dy = L_K^2$$

for every $\theta \in S^{n-1}$. Every convex body with center of mass at the origin has a linear image which is isotropic (see [12]). This image is unique up to orthogonal transformations, and hence, the isotropic constant L_K is well-defined for the linear class of K . The isotropic constant problem asks if there exists an absolute constant $C > 0$ such that $L_K \leq C$ for every isotropic convex body in any dimension. One can easily see that $L_K = O(\sqrt{n})$ for every K . Uniform boundedness of L_K is known for some classes of bodies: unit balls of spaces with 1-unconditional basis, zonoids and their polars, etc. Bourgain (see [4]) proved that $L_K = O(\sqrt[4]{n} \log n)$ and, very recently, Klartag (see [8]) improved this bound to $L_K = O(\sqrt[4]{n})$. Moreover, in [5] Bourgain proved that if every $x \neq 0$ is a ψ_2 -direction for K with constant B , then L_K is bounded by $cB \log(B + 1)$.

A question of Milman, related to this line of thought, is whether, for every isotropic convex body K in $\mathbb{R}^n$, most $\theta \in S^{n-1}$ define a ψ_2 -direction for K with a "good" constant (for example, logarithmic in n). Until recently, it was not known if there exists an absolute constant $C > 0$ such that every isotropic convex body has at least one ψ_2 -direction with constant C . Some positive results are known for special classes of convex bodies. Bobkov and Nazarov (see [2] and [3]) have proved that if K is an isotropic 1-unconditional convex body, then $\|\langle \cdot, x \rangle\|_{\psi_2} \leq c\sqrt{n}\|x\|_\infty$ for every $x \neq 0$. This shows that the diagonal direction is a ψ_2 -direction. For the class of zonoids, the existence of good ψ_2 -directions was established in [14]. Another partial result, which gives more information in the case of isotropic convex bodies with "small diameter", was obtained in [15]: If $K \subseteq (\gamma\sqrt{n}L_K)B_2^n$ for some $\gamma > 0$, then

$$(1.5) \quad \sigma(\theta \in S^{n-1} : \|\langle \cdot, \theta \rangle\|_{\psi_2} \geq c_1 \gamma t L_K) \leq \exp(-c_2 \sqrt{n} t^2 / \gamma)$$

for every $t \geq 1$, where σ is the rotationally invariant probability measure on S^{n-1} and $c_1, c_2 > 0$ are absolute constants.

Klartag (see [9]) gave a positive answer to this question, showing that every isotropic convex body admits at least one almost subgaussian linear functional. Our aim is to give a second (short) proof of this fact.

Theorem 1.1. *Let K be an isotropic convex body in $\mathbb{R}^n$. There exists $x \neq 0$ such that*

$$(1.6) \quad |\{y \in K : |\langle y, x \rangle| \geq t \|\langle \cdot, x \rangle\|_1\}| \leq \exp(-ct^2 / \log^\tau(t + 1))$$

for all $t \geq 1$, where $c, \tau > 0$ are absolute constants.

It is clear that if x defines a ψ_α -direction for K and if $T \in SL(n)$, then T^*x defines a ψ_α -direction (with the same constant) for $T(K)$. It follows that Theorem 1.1 provides almost subgaussian directions for every convex body: If K is a convex body in $\mathbb{R}^n$ with volume one and center of mass at the origin, there exists $x \neq 0$ such that (1.6) holds true for all $t \geq 1$.

The argument of Klartag is based on the study of the level sets of the logarithmic Laplace transform of log-concave functions. The argument we present here is based

on the study of the L_q -centroid bodies of an isotropic convex body. This family of bodies was studied and used by the third named author in [15], and in particular in [16], where the following sharp dimension-dependent concentration of volume estimate was proved: There exists an absolute constant $c > 0$ such that if K is an isotropic convex body in $\mathbb{R}^n$, then

$$(1.7) \quad |\{x \in K : \|x\|_2 \geq c\sqrt{n}L_K t\}| \leq \exp(-\sqrt{nt})$$

for every $t \geq 1$, where $\|\cdot\|_2$ is the Euclidean norm. The tools which are developed in [16] allow us to give a very simple proof of Theorem 1.1. We present an argument which gives $\tau = 2$, i.e. the upper bound in (1.6) is $\exp(-ct^2/\log^2(t+1))$.

Notation. We work in $\mathbb{R}^n$, which is equipped with a Euclidean structure $\langle \cdot, \cdot \rangle$. We denote by $\|\cdot\|_2$ the corresponding Euclidean norm, and write B_2^n for the Euclidean unit ball, and S^{n-1} for the unit sphere. Volume is denoted by $|\cdot|$. If K is a convex body in $\mathbb{R}^n$, we set $\overline{K} = K/|K|^{1/n}$; this is the dilation of K which has volume one. We write σ for the rotationally invariant probability measure on S^{n-1} . The Grassmann manifold $G_{n,k}$ of k -dimensional subspaces of $\mathbb{R}^n$ is equipped with the Haar probability measure $\mu_{n,k}$.

A convex body is a compact convex subset C of $\mathbb{R}^n$ with non-empty interior. We say that C has center of mass at the origin if $\int_C \langle x, \theta \rangle dx = 0$ for every $\theta \in S^{n-1}$. The support function $h_C : \mathbb{R}^n \rightarrow \mathbb{R}$ of C is defined by $h_C(x) = \max\{\langle x, y \rangle : y \in C\}$. The mean width of C is defined by

$$(1.8) \quad w(C) = \int_{S^{n-1}} h_C(\theta) \sigma(d\theta).$$

The letters c, c', c_1, c_2 , etc. denote absolute positive constants which may change from line to line. We refer to the books [18], [13] and [17] for basic facts from the Brunn-Minkowski theory and the asymptotic theory of finite dimensional normed spaces.

2. NORMALIZED L_q -CENTROID BODIES

Let K be a convex body of volume 1 in $\mathbb{R}^n$. For every $q \geq 1$ we define the L_q -centroid body $Z_q(K)$ of K by its support function:

$$(2.1) \quad h_{Z_q(K)}(x) = \|\langle \cdot, x \rangle\|_q := \left(\int_K |\langle y, x \rangle|^q dy \right)^{1/q}.$$

Since $|K| = 1$, we readily see that $Z_1(K) \subseteq Z_p(K) \subseteq Z_q(K) \subseteq Z_\infty(K)$ for every $1 \leq p \leq q \leq \infty$, where $Z_\infty(K) = \text{conv}\{K, -K\}$. On the other hand, one has the reverse inclusions

$$(2.2) \quad Z_q(K) \subseteq \frac{cq}{p} Z_p(K)$$

for every $1 \leq p < q < \infty$, as a consequence of the ψ_1 -behavior of $y \mapsto \langle y, x \rangle$. Observe that $Z_q(K)$ is always symmetric, and $Z_q(TK) = T(Z_q(K))$ for every $T \in SL(n)$ and $q \in [1, \infty]$. Also, if K has its center of mass at the origin, then $Z_q(K) \supseteq cZ_\infty(K)$ for all $q \geq n$, where $c > 0$ is an absolute constant.

It should be mentioned that L_q -centroid bodies were introduced in [10] under a different normalization. Lutwak, Yang and Zhang (see [11] and [7] for a different proof) have established the L_q affine isoperimetric inequality

$$(2.3) \quad |Z_q(K)|^{1/n} \geq |Z_q(\overline{B_2^n})|^{1/n} \geq c\sqrt{q/n}$$

for every $1 \leq q \leq n$, where $c > 0$ is an absolute constant.

We will need upper estimates for the quermassintegrals of the L_q -centroid bodies of an isotropic convex body. These follow immediately from estimates on the projections of $Z_q(K)$, which are obtained in [16]. Fix $1 \leq k \leq n$ and a k -dimensional subspace F of $\mathbb{R}^n$, and denote by E the orthogonal subspace of F . For every $\phi \in S_F$, define $E(\phi) = \{y \in \text{span}\{E, \phi\} : \langle y, \phi \rangle \geq 0\}$. By a theorem of K. Ball (see [1] and [12]), for every convex body K of volume one in $\mathbb{R}^n$, for every $q \geq 0$ and every $\phi \in F$, the function

$$(2.4) \quad \phi \mapsto \|\phi\|_2^{1+\frac{q}{q+1}} \left(\int_{K \cap E(\phi)} |\langle y, \phi \rangle|^q dy \right)^{-\frac{1}{q+1}}$$

is a gauge function on F (see also [6] for the not necessarily symmetric case). If we denote by $B_q(K, F)$ the convex body in F whose gauge function is defined by (2.4), then the volume of $B_q(K, F)$ is given by

$$(2.5) \quad |B_q(K, F)| = |B_2^k| \int_{S_F} \left(\int_{K \cap E(\phi)} |\langle x, \phi \rangle|^q dx \right)^{\frac{k}{q+1}} d\sigma_F(\phi).$$

The following identity was proved in [16].

Proposition 2.1. *Let K be a convex body of volume 1 in $\mathbb{R}^n$ and let $1 \leq k \leq n-1$. For every $F \in G_{n,k}$ and every $q \geq 1$ we have that*

$$(2.6) \quad P_F(Z_q(K)) = (k+q)^{1/q} |B_{k+q-1}(K, F)|^{1/k+1/q} Z_q(\overline{B}_{k+q-1}(K, F)).$$

Using this identity and exploiting (2.5) in order to estimate the volume of $B_q(K, F)$, one gets the following estimate (see [16]).

Proposition 2.2. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $F \in G_{n,k}$ and $E = F^\perp$ then, for every $q \in \mathbb{N}$ we have that*

$$(2.7) \quad P_F(Z_q(K)) \subseteq \frac{c(k+q)}{k} L_K Z_q(\overline{B}_{k+q-1}(K, F))$$

where $c > 0$ is an absolute constant.

Definition 2.3. Let K be an isotropic convex body in $\mathbb{R}^n$. For every integer $q \geq 1$ we define the *normalized L_q -centroid body* K_q of K by

$$(2.8) \quad K_q = \frac{1}{\sqrt{q} L_K} Z_q(K).$$

Since $|Z_q(\overline{B}_{k+q-1}(K, F))| \leq |Z_\infty(\overline{B}_{k+q-1}(K, F))| \leq 4^k$, Proposition 2.2 shows that

$$(2.9) \quad |P_F(K_q)|^{1/k} \leq \frac{c(k+q)}{k\sqrt{q}} |Z_q(\overline{B}_{k+q-1}(K, F))|^{1/k} \leq \frac{c_1(k+q)}{k} \frac{\sqrt{k}}{\sqrt{q}} |B_2^k|^{1/k}$$

for every $F \in G_{n,k}$. If $1 \leq k \leq q$, this estimate takes the simpler form

$$(2.10) \quad |P_F(K_q)|^{1/k} \leq 2c_1 \frac{\sqrt{q}}{\sqrt{k}} |B_2^k|^{1/k}.$$

In particular, for every $F \in G_{n,q}$ we have

$$(2.11) \quad |P_F(K_q)|^{1/q} \leq 2c_1 |B_2^q|^{1/q}.$$

A standard argument implies that a similar estimate is valid for every $F \in G_{n,k}$, where $q \leq k \leq n$. To see this, observe that by the log-concavity of the quermass-integrals of $P_F(K_q)$ and by Kubota's formula,

$$(2.12) \quad \left(\frac{|P_F(K_q)|}{|B_2^k|} \right)^{1/k} \leq \left(\frac{\int |P_H(P_F(K_q))| d\mu_{k,q}(H)}{|B_2^q|} \right)^{1/q},$$

where the integration is over all q -dimensional subspaces H of F , and apply (2.11) pointwise taking into account the fact that $P_H(P_F(K_q)) = P_H(K_q)$. We summarize these observations in the next theorem.

Theorem 2.4. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $1 \leq k, q \leq n$ are integers, and if $F \in G_{n,k}$, then*

$$(2.13) \quad |P_F(K_q)|^{1/k} \leq c_1 \max\{\sqrt{q/k}, 1\} |B_2^k|^{1/k},$$

where $c_1 > 0$ is an absolute constant. In particular,

$$(2.14) \quad |K_q|^{1/n} \leq c_1 |B_2^n|^{1/n}.$$

The last ingredient of the proof is a consequence of the main result in [16]: from (1.7) it follows that

$$(2.15) \quad \left(\int_K \|y\|_2^q dy \right)^{1/q} \leq c\sqrt{n}L_K$$

for all $1 \leq q \leq \sqrt{n}$. Since

$$(2.16) \quad w(Z_q(K)) \leq \left(\int_{S^{n-1}} \int_K |\langle y, \theta \rangle|^q dy \sigma(d\theta) \right)^{1/q} \leq \frac{C\sqrt{q}}{\sqrt{n}} \left(\int_K \|y\|_2^q dy \right)^{1/q}$$

for all $1 \leq q \leq n$, we have the following lemma.

Lemma 2.5. *Let K be an isotropic convex body in $\mathbb{R}^n$. If $1 \leq q \leq \sqrt{n}$, then*

$$(2.17) \quad w(K_q) \leq C,$$

where $C > 0$ is an absolute constant.

Remark 2.6. Without using Lemma 2.5, which fully exploits the results of [16], we can prove Theorem 1.1 with $\tau = 2 + \epsilon$ for any $\epsilon > 0$.

3. COVERING NUMBERS OF K_q

Let $N(K_q, sB_2^n)$ denote the minimal number of translates of sB_2^n whose union covers K_q . A standard way to estimate the covering number $N(K_q, sB_2^n)$ is through the inequality

$$(3.1) \quad |tB_2^n| \cdot N(K_q, 2tB_2^n) \leq |K_q + tB_2^n|,$$

which is valid for every $t > 0$. We will use our information on the projections of K_q in order to give an upper bound for $|K_q + tB_2^n|$.

Proposition 3.1. *Let K be an isotropic convex body in $\mathbb{R}^n$. For every $1 \leq q \leq n$ and every $t > 0$, we have that*

$$(3.2) \quad N(K_q, 2tB_2^n) \leq 2 \exp \left(C \frac{\sqrt{qn}}{\sqrt{t}} + C \frac{n}{t} \right),$$

where $C > 0$ is an absolute constant.

Proof. From the classical Steiner formula we know that

$$(3.3) \quad |K_q + tB_2^n| = \sum_{k=0}^n \binom{n}{k} W_{[n-k]}(K_q) t^{n-k}$$

for all $t > 0$, where $W_{[n-k]}(K_q)$ is the mixed volume $V_k(K_q) = V(K_q; k, B_2^n; n-k)$ (see [18]).

We will use Kubota's integral formula to express $W_{[n-k]}(K_q)$ as an average of the volumes of the k -dimensional projections of K_q : for every $1 \leq k \leq n-1$ we have

$$(3.4) \quad W_{[n-k]}(K_q) = \frac{|B_2^n|}{|B_2^k|} \int_{G_{n,k}} |P_F(K_q)| d\mu_{n,k}(F).$$

Using (3.3), (3.4) and the estimates from Theorem 2.4, we can write

$$(3.5) \quad |K_q + tB_2^n| \leq |B_2^n| \sum_{k=0}^n \binom{n}{k} \left(c_1 \max\{\sqrt{q/k}, 1\} \right)^k t^{n-k}.$$

Then, (3.1) shows that

$$(3.6) \quad N(K_q, 2tB_2^n) \leq \sum_{k=0}^q \left(\frac{c_2 n \sqrt{q}}{k^{3/2} t} \right)^k + \sum_{k=q+1}^n \left(\frac{c_2 n}{kt} \right)^k.$$

Observe that for $1 \leq k \leq q$ we have

$$(3.7) \quad \left(\frac{c_2 n \sqrt{q}}{k^{3/2} t} \right)^k \leq \left(\frac{c_2 n q}{k^2 t} \right)^k \leq \frac{\left(c_3 \sqrt{nq/t} \right)^{2k}}{(2k)!},$$

while, for $q \leq k \leq n$ we have

$$(3.8) \quad \left(\frac{c_2 n}{kt} \right)^k \leq \frac{(c_4 n/t)^k}{k!}.$$

It follows that

$$(3.9) \quad N(K_q, 2tB_2^n) \leq \exp \left(c_3 \frac{\sqrt{qn}}{\sqrt{t}} \right) + \exp \left(c_4 \frac{n}{t} \right),$$

and the result follows by the elementary inequality $x + y \leq 2xy$ for $x, y \geq 1$. $\square$

Remark 3.2. The proof actually gives $N(K_q, 2tB_2^n) \leq \exp \left(C \frac{n^{2/3} q^{1/3}}{t^{2/3}} + C \frac{n}{t} \right)$ for every $t > 0$, but this would play no role in the proof of the main result.

4. PROOF OF THE THEOREM

Let K be an isotropic convex body in $\mathbb{R}^n$. Consider the convex body

$$(4.1) \quad T = \text{conv} \left(\bigcup_{i=1}^{\lfloor \log_2 n \rfloor} \frac{1}{i} K_{2^i} \right).$$

We will use the following standard fact.

Lemma 4.1. *Let $A_1, \dots, A_s$ be subsets of RB_2^n . For every $t > 0$ we have that*

$$(4.2) \quad N(\text{conv}(A_1 \cup \dots \cup A_s), 2tB_2^n) \leq \left(\frac{cR}{t} \right)^s \prod_{i=1}^s N(A_i, tB_2^n).$$

Sketch of the proof. For $i = 1, \dots, s$, let N_i be a subset of $\mathbb{R}^n$ with cardinality $|N_i| = N(A_i, tB_2^n)$, so that $A_i \subseteq \bigcup_{x_i \in N_i} (x_i + tB_2^n)$. Let B_1^s denote the unit ball of ℓ_1^s and fix $Z \subseteq B_1^s$ of minimal cardinality, so that $B_1^s \subseteq \bigcup_{z \in Z} (z + (t/R)B_1^n)$. It is well-known that $|Z| \leq (cR/t)^s$, where $c > 0$ is an absolute constant. Consider the set $N = \{w = z_1x_1 + \dots + z_sx_s : x_i \in N_i, z = (z_1, \dots, z_s) \in Z\}$. Then, $\text{conv}(A_1 \cup \dots \cup A_s) \subseteq \bigcup_{w \in N} (w + 2tB_2^n)$. $\square$

Let $s = \lfloor \log_2 n \rfloor$ and $m = \lfloor \log_2(\sqrt{n}) \rfloor \simeq s/2$. We apply Lemma 4.1 with $A_i = \frac{1}{i}K_{2^i}$, $1 \leq i \leq s$, and $t = 1$. Observe that $A_i \subseteq c_1\sqrt{n}B_2^n$ for all $i \leq s$ (to see this, recall the known fact that if K is an isotropic convex body in $\mathbb{R}^n$, then $K \subseteq (cnL_K)B_2^n$). Using Sudakov's inequality (see [17]) and Lemma 2.5 we see that, since $2^i \leq \sqrt{n}$ for $i \leq m$, we have

$$(4.3) \quad N(A_i, B_2^n) = N(K_{2^i}, iB_2^n) \leq \exp(c'nw^2(K_{2^i})/i^2) \leq \exp(c_1n/i^2)$$

for all $i = 1, \dots, m$. Using also the entropy estimates of Section 3 to estimate $N(A_i, B_2^n)$ for $m < i \leq s = \lfloor \log_2 n \rfloor$, we may write

$$\begin{aligned} N(T, B_2^n) &\leq (c_2\sqrt{n})^{\lfloor \log_2 n \rfloor} \left[\prod_{i=1}^{\lfloor \log_2 n \rfloor} N(K_{2^i}, iB_2^n) \right] \\ &\leq e^{c_3n} \exp \left(C\sqrt{n} \sum_{i=m+1}^{\lfloor \log_2 n \rfloor} 2^{i/2} \right) \times \exp \left(Cn \cdot \left(\sum_{i=1}^m \frac{1}{i^2} + \sum_{i=m+1}^{2m} \frac{1}{i} \right) \right) \\ &\leq e^{cn}. \end{aligned}$$

It follows that $|T| \leq |CB_2^n|$, where $C > 0$ is an absolute constant. Therefore, there exists $x \neq 0$ such that

$$(4.4) \quad h_T(x) \leq C\|x\|_2,$$

and hence,

$$(4.5) \quad \|\langle \cdot, x \rangle\|_{2^i} \leq C2^{i/2}iL_K\|x\|_2$$

for every $i = 1, 2, \dots, \lfloor \log_2 n \rfloor$. This easily implies the following.

Theorem 4.2. *Let K be an isotropic convex body in $\mathbb{R}^n$. There exists $\theta \in S^{n-1}$ such that*

$$(4.6) \quad \|\langle \cdot, \theta \rangle\|_q \leq C\sqrt{q} \log q \|\langle \cdot, \theta \rangle\|_2$$

for every $q \geq 2$, where $C > 0$ is an absolute constant.

A standard argument shows that Theorem 4.2 implies Theorem 1.1 (it is actually equivalent to Theorem 1.1 with $\tau = 2$).

Remark 4.3. The proof of Theorem 4.2 carries over to the case of an arbitrary log-concave measure: the approach of [16] and all the arguments we have used in this note depend only on the Brunn–Minkowski theory. It follows that if μ is an isotropic log-concave measure in $\mathbb{R}^n$, then there exists $\theta \in S^{n-1}$ such that

$$(4.7) \quad \|\langle \cdot, \theta \rangle\|_{L^q(\mu)} \leq C\sqrt{q} \log q \|\langle \cdot, \theta \rangle\|_{L^2(\mu)}$$

for all $2 \leq q \leq n$, where $C > 0$ is an absolute constant.

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