UNIVERSAL SPACES FOR ALMOST n -DIMENSIONALITY

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ABSTRACT. We find universal functions for the class of lower semi-continuous (LSC) functions with at most n -dimensional domain. In an earlier paper we proved that a space is almost n -dimensional if and only if it is homeomorphic to the graph of an LSC function with an at most n -dimensional domain. We conclude that the class of almost n -dimensional spaces contains universal elements (that are topologically complete). These universal spaces can be thought of as higher-dimensional analogues of complete Erdős space.

Unless stated otherwise all topologies in this note are assumed to be separable and metrizable.

Let n be a nonnegative integer and let X be a topological space. We say that X is *almost n -dimensional* if there exists a topology $\mathcal{W}$ on X such that $\dim(X, \mathcal{W}) \leq n$, $\mathcal{W}$ is weaker than the given topology on X , and every point of X has a neighbourhood basis in X consisting of sets that are closed in $(X, \mathcal{W})$. If we substitute $n = 0$, then we get the notion of an almost zero-dimensional space, which concept has been introduced by Oversteegen and Tymchatyn [16]; see also [13] and [6, Proposition 6.1]. The almost zero-dimensional spaces are precisely the spaces that can be imbedded in (complete) Erdős space; see [3, Theorem 3.1] and [4, Theorem 5.13]. Recall that *Erdős space* $\mathfrak{E}$ consists of all sequences of rational numbers in ℓ^2 , the Hilbert space of square summable real sequences. *Complete Erdős space* can be represented as

$$\mathfrak{E}_c = \{(x_i)_{i \in \omega} \in \ell^2 : x_i = 0 \text{ or } 1/x_i \in \mathbb{N} \text{ for each } i \in \omega\}.$$

Both spaces were introduced and shown to be one-dimensional by Paul Erdős [9] in 1940. See [11], [2], and [5] for numerous other representations of $\mathfrak{E}_c$.

Observe that our definition of almost n -dimensionality differs from the extension of almost zero-dimensionality that is featured in Levin and Tymchatyn [14], which they show to be equivalent to regular dimension. A real-valued function φ on a topological space X is called *lower semi-continuous (LSC)* if $\{x \in X : \varphi(x) > t\}$ is open for each $t \in \mathbb{R}$. In [1] the authors present the following characterization theorem.

Theorem 1. *A space is almost n -dimensional if and only if it is homeomorphic to the graph of some LSC function with a domain of dimension at most n .*

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The following result by van Mill and Pol [15] establishes the supremum function on the Hilbert cube $[0, 1]^\omega$ as universal for LSC functions.

Theorem 2. *If $\varphi: X \rightarrow [\varepsilon, 1]$ is an LSC function for some $\varepsilon > 0$, then there exists an imbedding $h: X \rightarrow [0, 1]^\omega$ such that $\sup_{i \in \omega} h_i(x) = \varphi(x)$ for each $x \in X$.*

We define $\sigma: \mathbb{R}^\omega \rightarrow (-\infty, \infty]$ by $\sigma(x) = \sup_{i \in \omega} x_i$ for $x = (x_0, x_1, \dots) \in \mathbb{R}^\omega$. Note that σ is LSC. For $n \in \omega$ we define the n -dimensional *Nöbeling space* in $\mathbb{R}^\omega$ by

$$N_n^\omega = \{x \in \mathbb{R}^\omega : |\{i : x_i \in \mathbb{Q}\}| \leq n\}.$$

It is well known that a space X is imbeddable in N_n^ω if and only if $\dim X \leq n$; see Engelking [8, pp. 44 and 120]. The following theorem shows that $\sigma \upharpoonright N_n^\omega$ is universal for LSC functions with an n -dimensional domain.

Theorem 3. *If $\varphi: X \rightarrow (-\infty, \infty]$ is an LSC function with $\dim X \leq n$, then there exists an imbedding $h: X \rightarrow N_n^\omega$ such that $\sigma \circ h = \varphi$.*

Let for $n \in \omega$,

$$\mathfrak{E}_c(n) = \{(x, \sigma(x)) : x \in N_n^\omega, \sigma(x) < \infty\}$$

with the topology inherited from the product $N_n^\omega \times \mathbb{R}$. Theorems 1 and 3 combine to:

Theorem 4. *A space is almost n -dimensional if and only if it is imbeddable in $\mathfrak{E}_c(n)$.*

Proof of Theorem 3. For technical reasons we find it convenient to prove Theorem 3 for positive functions.

Claim 5. *If $\varphi: X \rightarrow (0, \infty]$ is an LSC function with $\dim X \leq n$, then there exists an imbedding $h: X \rightarrow N_n^\omega \cap (0, \infty)^\omega$ such that $\sigma \circ h = \varphi$.*

Since $\alpha(t) = (1 - t)^{-1}$ for $t < 0$ and $\alpha(t) = t + 1$ for $t \geq 0$ defines a homeomorphism $\alpha: (-\infty, \infty] \rightarrow (0, \infty]$ such that $\alpha(\mathbb{Q}) = \mathbb{Q} \cap (0, \infty)$ we have that Theorem 3 follows immediately from Claim 5.

Let $\varphi: X \rightarrow (0, \infty]$ be an LSC function such that $\dim X \leq n$. Select a compactification K of X with $\dim K \leq n$ (see [8, Theorem 1.7.2]) and define

$$\overline{\varphi}(x) = \sup\{\inf\{\varphi(y) : y \in X \cap U\} : U \text{ is open in } K \text{ with } x \in U\}$$

for $x \in K$. Note that $\overline{\varphi}: K \rightarrow [0, \infty]$ is an LSC function that extends φ . Note that the fibre $\overline{\varphi}^{-1}(0)$ is closed in K and we may replace K by the quotient space $K/\overline{\varphi}^{-1}(0)$ if $\overline{\varphi}^{-1}(0) \neq \emptyset$. If $\overline{\varphi}^{-1}(0) = \emptyset$, then we add an isolated point to K where $\overline{\varphi}$ assumes the value 0. In conclusion, we may assume that φ has been extended over a compact space K so that $\dim K \leq n$ and $\varphi^{-1}(0) = \{z\}$ for some $z \in K \setminus X$. We may assume that K is a subspace of some normed vector space $(E, \|\cdot\|)$ such that z is the zero vector. Put $K_k = \{x \in K : \|x\| \geq 2^{-k}\}$ for $k \in \omega$.

If Z is a space, then $C(K, Z)$ denotes the space of continuous functions from K to Z with the usual compact-open topology, which coincides with the topology of uniform convergence with respect to any metric on Z . Put $Y = \{\mathbf{0}\} \cup (0, \infty)^\omega \subset \mathbb{R}^\omega$, where $\mathbf{0}$ denotes the zero vector of $\mathbb{R}^\omega$. Since Y is a G_δ -subset of $\mathbb{R}^\omega$ we have that $C(K, Y)$ is also a G_δ -subset of $C(K, \mathbb{R}^\omega)$. We now define

$$C_z(K, Y) = \{f \in C(K, Y) : f^{-1}(\mathbf{0}) = \{z\}\}$$

and note that it is a G_δ -subset of $C(K, Y)$ because

$$C_z(K, Y) = \{f \in C(K, Y) : f(z) = \mathbf{0} \text{ and } f(K_k) \cap \{\mathbf{0}\} = \emptyset \text{ for } k \in \omega\}.$$

Let $\{H_k : k \in \omega\}$ enumerate all planes in $\mathbb{R}^\omega$ of the form $\{x \in \mathbb{R}^\omega : x_{i_0} = q_0, x_{i_1} = q_1, \dots, x_{i_n} = q_n\}$, where $i_0 < i_1 < \dots < i_n$ and $q_j \in (0, \infty) \cap \mathbb{Q}$ for $j = 0, 1, \dots, n$. We assume that $\{j : H_j = H_k\}$ is infinite for each $k \in \omega$. Note that $(0, \infty)^\omega \cap N_n^\omega = (0, \infty)^\omega \setminus \bigcup \{H_k : k \in \omega\}$. If (Z, d) is a metric space and $\varepsilon > 0$, then we call a function f with domain Z an ε -mapping if $f(x) \neq f(y)$ for all $x, y \in Z$ with $d(x, y) \geq \varepsilon$. We define for $k \in \omega$,

$$O_k = \{f \in C_z(K, Y) : f|_{K_k} \text{ is a } 2^{-k}\text{-mapping with } f(K_k) \cap H_k = \emptyset\}.$$

According to [8, Lemmas 1.11.1 and 1.11.2] the set O_k is an open subset of $C_z(K, Y)$.

We define the following metric on $\mathbb{R}^\omega$:

$$\rho(x, y) = \min\{1, \sup\{2^i |x_i - y_i| : i \in \omega\}\}$$

for $x, y \in \mathbb{R}^\omega$. Subsequently, we let $\hat{\rho}(f, g) = \sup\{\rho(f(x), g(x)) : x \in Z\}$ define the corresponding metric of uniform convergence on $C(Z, \mathbb{R}^\omega)$. It is easily verified that $\hat{\rho}$ is a complete metric on $C(K, \mathbb{R}^\omega)$, and we denote $C(K, Z)$ with the $\hat{\rho}$ -topology by $C^\rho(K, Z)$ for $Z \subset \mathbb{R}^\omega$. Note that both $(\mathbb{R}^\omega, \rho)$ and $C^\rho(K, \mathbb{R}^\omega)$ are nonseparable spaces with topologies that are stronger than the topologies on $\mathbb{R}^\omega$ and $C(K, \mathbb{R}^\omega)$, respectively. Since $C_z(K, Y)$ is a G_δ -subset of $C(K, \mathbb{R}^\omega)$ we have that it is also a G_δ -subset of $C^\rho(K, \mathbb{R}^\omega)$. Thus $C_z^\rho(K, Y)$ is a topologically complete space. Similarly, every O_k is open in $C_z^\rho(K, Y)$.

We define the partial order $\leq$ on $\mathbb{R}^\omega$ by $x \leq y$ if $x_i \leq y_i$ for each $i \in \omega$. This produces also a partial order on $C(K, \mathbb{R}^\omega)$: $f \leq g$ if $f(x) \leq g(x)$ for each $x \in K$.

Claim 6. *If $k \in \omega$, $\varepsilon > 0$, and $f \in C_z^\rho(K, Y)$, then there is a $g \in O_k$ such that $\hat{\rho}(f, g) < \varepsilon$ and $g \leq f$.*

Proof. Let $H_k = \{x \in \mathbb{R}^\omega : x_{i_0} = q_0, x_{i_1} = q_1, \dots, x_{i_n} = q_n\}$, where $i_0 < i_1 < \dots < i_n$ and $q_j \in (0, \infty) \cap \mathbb{Q}$ for $j = 0, 1, \dots, n$. Select a set of $2n + 1$ coordinates $A \subset \omega$ that contains $\{i_0, \dots, i_n\}$ and let $p : \mathbb{R}^\omega \rightarrow \mathbb{R}^A$ be the projection map given by $p_i(x) = x_i$ for $i \in A$ and $x \in \mathbb{R}^\omega$. Define the following admissible metric on $\mathbb{R}^A$:

$$\rho_A(x, y) = \max\{2^i |x_i - y_i| : i \in A\}$$

for $x, y \in \mathbb{R}^A$. Let $\hat{\rho}_A$ denote the corresponding metric of uniform convergence.

Let $m \geq k$ be such that for $x \in K$, $\|x\| \leq 2^{-m}$ implies $\rho_A(\mathbf{0}, p(f(x))) < \varepsilon$. Let

$$\delta = \inf\{f_i(x) : i \in A, x \in K_m\}$$

and note that $\delta > 0$ because $f(K \setminus \{z\}) \subset (0, \infty)^\omega$ and K_m is compact. Let $\delta' = \frac{1}{2} \min\{\varepsilon, \delta\}$ and define $f' : K_m \rightarrow \mathbb{R}^A$ by $f'_i(x) = f_i(x) - \delta' 2^{-i}$ for $i \in A$ and $x \in K_m$; thus $\hat{\rho}_A(f', p \circ f|_{K_m}) = \delta'$. By [8, Lemma 1.11.3] there exists a continuous 2^{-k} -mapping $\alpha : K_m \rightarrow \mathbb{R}^A$ such that $\alpha(K_m) \cap p(H_k) = \emptyset$ and $\hat{\rho}_A(\alpha, f') < \delta'$. Observe that $\alpha(K_m) \subset (0, \infty)^A$ and $\alpha_i(x) \leq p_i(f(x))$ for $i \in A$ and $x \in K_m$. Also

$$\hat{\rho}_A(\alpha, p \circ f|_{K_m}) \leq \hat{\rho}_A(\alpha, f') + \hat{\rho}_A(f', p \circ f|_{K_m}) < 2\delta' \leq \varepsilon.$$

Consider the compactum $S = \{x \in K : \|x\| = 2^{-m}\}$ and note that if $x \in S$, then

$$\rho_A(\mathbf{0}, \alpha(x)) \leq \rho_A(\mathbf{0}, p(f(x))) < \varepsilon.$$

Thus $\alpha(S)$ is a subset of

$$P = \prod_{i \in A} (0, \varepsilon 2^{-i}) = \{y \in (0, \infty)^A : \rho_A(0, y) < \varepsilon\}$$

and by Tietze we may extend $\alpha|_S$ to a continuous $\beta: E \rightarrow P$. We define the continuous extension $\gamma: K \rightarrow \mathbb{R}^A$ of α by

$$\gamma(x) = \begin{cases} 0, & \text{if } x = z; \\ 2^m \|x\| \beta(2^{-m} \|x\|^{-1} x), & \text{if } 0 < \|x\| < 2^{-m}; \\ \alpha(x), & \text{if } x \in K_m. \end{cases}$$

Observe that $\gamma(K \setminus (K_m \cup \{z\})) \subset P$. Define the continuous map $\bar{\alpha}: K \rightarrow \mathbb{R}^A$ by $\bar{\alpha}_i(x) = \min\{\gamma_i(x), f_i(x)\}$ for $i \in A$ and $x \in K$. Observe that $\bar{\alpha}^{-1}(0) = \{z\}$, $\bar{\alpha}(K \setminus (K_m \cup \{z\})) \subset P$, and $\bar{\alpha}|_{K_m} = \alpha$, because $\gamma_i(x) = \alpha_i(x) \leq f_i(x)$ for $x \in K_m$ and $i \in A$. If $x \in K \setminus (K_m \cup \{z\})$, then $\bar{\alpha}(x) \in P$ and by the choice of m , $p(f(x)) \in P$. Thus we have $\rho_A(\bar{\alpha}(x), p(f(x))) < \varepsilon$. Since $\bar{\alpha}|_{K_m} = \alpha|_{K_m}$ and $\bar{\alpha}(z) = 0$ this leads to $\hat{\rho}_A(\bar{\alpha}, p \circ f) < \varepsilon$.

Define $g \in C_z^p(K, Y)$ by

$$g_i(x) = \begin{cases} \bar{\alpha}_i(x), & \text{if } i \in A; \\ f_i(x), & \text{if } i \in \omega \setminus A; \end{cases}$$

for $x \in K$. Note that $g \leq f$ and $\hat{\rho}(g, f) \leq \hat{\rho}_A(p \circ g, p \circ f) = \hat{\rho}_A(\bar{\alpha}, p \circ f) < \varepsilon$. Since $K_k \subset K_m$ and $p \circ g|_{K_m} = \alpha$ we have $g \in O_k$. $\square$

Since $\varphi: K \rightarrow [0, \infty]$ is LSC there exists a sequence of continuous functions $f_i: K \rightarrow [0, \infty)$ such that $\varphi(x) = \sup_{i \in \omega} f_i(x)$ for each $x \in K$; see [7, p. 153]. We may assume that $f_i \leq f_{i+1}$ for each $i \in \omega$, and hence $\lim_{i \rightarrow \infty} f_i(x) = \varphi(x)$ for each $x \in K$. Note that the function $g = \sup_{i \in \omega} \min\{2^{-i}, f_i\}$ is a continuous function on K such that $g \leq \varphi$ and $g^{-1}(0) = \varphi^{-1}(0) = \{z\}$. We may therefore assume that $f_i^{-1}(0) = \{z\}$ for $i \in \omega$. Putting $f(x) = (f_0(x), f_1(x), \dots)$ for $x \in K$ we find that $f \in C_z^p(K, Y)$. The Baire Category Theorem now guarantees that there is an $h \in \bigcap_{i \in \omega} O_i$ such that $\hat{\rho}(f, h) < 1$. In fact, if we follow the standard proof of Baire's Theorem and we obtain h as a limit of a Cauchy sequence of functions $g_k \in \bigcap_{i=0}^k O_i$, then it is clear that with Claim 6 we can arrange that $f \geq g_0 \geq g_1 \geq \dots$. So we may assume that $h \leq f$. Since $h|_{K_k}$ is a 2^{-k} -map for every $k \in \omega$ we have that $h|_{K \setminus \{z\}}$ is one-to-one. Since $h^{-1}(0) = \{z\}$ we have that h is one-to-one and hence an imbedding. If $x \in K \setminus \{z\}$ and $k \in \omega$, then there is a $j \in \omega$ such that $x \in K_j$ and $H_j = H_k$. Since $h \in O_j$ we have that $h(x) \notin H_k$ and we may conclude that $h(K \setminus \{z\}) \subset (0, \infty)^\omega \cap N_n^\omega$. Thus we have that h imbeds X in $(0, \infty)^\omega \cap N_n^\omega$ and K in $\{0\} \cup ((0, \infty)^\omega \cap N_n^\omega)$.

Let $x \in K$. Since $\hat{\rho}(f, h) < 1$ and $h \leq f$ we have $f_i(x) - 2^{-i} \leq h_i(x) \leq f_i(x)$ for every $i \in \omega$. Consequently,

$$\sigma(h(x)) = \sup_{i \in \omega} h_i(x) \geq \lim_{i \rightarrow \infty} h_i(x) = \lim_{i \rightarrow \infty} f_i(x) = \varphi(x) = \sigma(f(x)) \geq \sigma(h(x)).$$

We may conclude that $\sigma \circ h = \varphi$ and that the proof is complete. $\square$

The Nöbeling spaces are G_δ -sets in $\mathbb{R}^\omega$, so they are topologically complete. Graphs of LSC functions are easily seen to be G_δ -subsets of the product of domain and co-domain. Thus $\mathfrak{E}_c(n)$ is a G_δ -subspace of $N_n^\omega \times \mathbb{R}$ and a topologically complete space. With Theorem 4 we have:

Corollary 7. *Every almost n -dimensional space has an almost n -dimensional completion.*

By the Lavrentieff Theorem [12] we have:

Corollary 8. *Every almost n -dimensional subspace of a space can be enlarged to a G_δ -subspace that is also almost n -dimensional.*

Since complete Erdős space $\mathfrak{E}_c$ is an almost zero-dimensional and one-dimensional space, the space $\mathfrak{E}_c \times [0, 1]^n$ is an $(n+1)$ -dimensional space (see [10]) that is clearly almost n -dimensional. Since $\mathfrak{E}_c(n)$ contains a copy of $\mathfrak{E}_c \times [0, 1]^n$ and is contained in $N_n^\omega \times \mathbb{R}$ we have $\dim \mathfrak{E}_c(n) = n + 1$.

If X is almost n -dimensional, then there exists a weaker topology $\mathcal{W}$ that is at most n -dimensional. If X is moreover compact, then the two topologies must be identical. Using the Countable Sum Theorem we find that a σ -compact space X is almost n -dimensional if and only if $\dim X \leq n$. Thus finding complete universal spaces is the optimal result for the class of almost n -dimensional spaces.

As mentioned earlier, complete Erdős space $\mathfrak{E}_c$ is a universal space for the class of almost zero-dimensional spaces. This fact is implicitly contained in [16, 11] and was formally established in [4, Theorem 5.13]. The following characterization theorem has been found by Dijkstra and van Mill [5].

Theorem 9. *A nonempty space E is homeomorphic to $\mathfrak{E}_c$ if and only if there is a zero-dimensional topology $\mathcal{W}$ on E that is coarser than the given topology on E such that for every $x \in E$ and neighbourhood U of x in E there is a neighbourhood V of x in E with V closed in $(E, \mathcal{W})$, $(V, \mathcal{W})$ topologically complete, and V a nowhere dense subset of $(U, \mathcal{W})$.*

Theorem 4 and the next result suggest that we may think of the spaces $\mathfrak{E}_c(n)$ as higher-dimensional analogues of complete Erdős space.

Proposition 10. *$\mathfrak{E}_c(0)$ is homeomorphic to $\mathfrak{E}_c$.*

Proof. Note that $N_0^\omega = (\mathbb{R} \setminus \mathbb{Q})^\omega$; thus it is a zero-dimensional topologically complete space. The weak topology $\mathcal{W}$ on $\mathfrak{E}_c(0)$ is given by $\mathcal{W} = \{\mathfrak{E}_c(0) \cap (O \times \mathbb{R}) : O \text{ is open in } N_0^\omega\}$. Thus $(\mathfrak{E}_c(0), \mathcal{W})$ is homeomorphic to the zero-dimensional space $\{x \in N_0^\omega : \sigma(x) < \infty\}$. Let $(x, \sigma(x))$ be a point in $\mathfrak{E}_c(0)$ and consider a neighbourhood U of $(x, \sigma(x))$. Since σ is LSC we may assume that U has the form $\mathfrak{E}_c(0) \cap (O \times (-\infty, r))$ for some clopen subset O of N_0^ω and $r > \sigma(x)$. Select an $s \in (\sigma(x), r)$ and consider the neighbourhood

$$V = \mathfrak{E}_c(0) \cap (O \times (-\infty, s]) = \{(y, \sigma(y)) : y \in O \text{ with } \sigma(y) \leq s\}$$

of $(x, \sigma(x))$. Note that $\{y : y \in O \text{ with } \sigma(y) \leq s\}$ is a closed subset of N_0^ω because σ is LSC. Hence V is closed in $(\mathfrak{E}_c(0), \mathcal{W})$ and topologically complete in the topology $\mathcal{W}$. Let $(y, \sigma(y))$ be a point in V and let W be a neighbourhood of y in $N_0^\omega = (\mathbb{R} \setminus \mathbb{Q})^\omega$. We may assume that $W \subset O$ and that by the product topology W contains the set $\{y_0\} \times \cdots \times \{y_k\} \times (\mathbb{R} \setminus \mathbb{Q}) \times (\mathbb{R} \setminus \mathbb{Q}) \times \cdots$ for some $k \in \omega$. Select a $t \in (s, r) \setminus \mathbb{Q}$ and note that $y' = (y_0, \dots, y_k, t, t, \dots)$ is such that $(y', \sigma(y')) = (y', t) \in (W \times \mathbb{R}) \cap U \setminus V$. Thus V is a nowhere dense subset of U in the topology $\mathcal{W}$. Theorem 9 now guarantees that $\mathfrak{E}_c(0) \approx \mathfrak{E}_c$. $\square$

Note that in Theorem 2 the LSC functions are bounded below by ε whereas there is no lower bound in Claim 5. It turns out that the bound is not essential.

Theorem 11. *If $\varphi: X \rightarrow (0, \infty]$ is an LSC function, then there exists an imbedding $h: X \rightarrow (0, \infty)^\omega$ such that $\sigma \circ h = \varphi$.*

Proof. Let $f_i: X \rightarrow (0, \infty)$ for $i \in \omega$ be a sequence of continuous functions such that $\sup_{i \in \omega} f_i(x) = \varphi(x)$ for each $x \in X$. We may assume that X is a subspace of the Hilbert cube $[\frac{1}{2}, 1]^\omega$. Define $\alpha: X \rightarrow (0, \infty)^\omega$ by

$$\alpha_i(x) = \begin{cases} f_{i/2}(x), & \text{if } i \text{ is even;} \\ x_{(i-1)/2} f_0(x), & \text{if } i \text{ is odd;} \end{cases}$$

for $i \in \omega$ and $x = (x_0, x_1, \dots) \in X$. It is clear that $\sigma \circ \alpha = \varphi$. Note that the formula $\beta(y) = (y_1/y_0, y_3/y_0, y_5/y_0, \dots)$ for $y \in \alpha(X)$ defines the inverse of α , so α is an imbedding. $\square$

Of course, Theorem 11 can also be formulated as follows. If $\varphi: X \rightarrow (-\infty, \infty]$ is an LSC function, then there exists an imbedding $h: X \rightarrow \mathbb{R}^\omega$ such that $\sigma \circ h = \varphi$.

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