A-IDENTITIES FOR THE GRASSMANN ALGEBRA: THE CONJECTURE OF HENKE AND REGEV

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ABSTRACT. Let K be an algebraically closed field of characteristic 0, and let E be the infinite dimensional Grassmann (or exterior) algebra over K . Denote by P_n the vector space of the multilinear polynomials of degree n in $x_1, \dots, x_n$ in the free associative algebra $K(X)$. The symmetric group S_n acts on the left-hand side on P_n , thus turning it into an S_n -module. This fact, although simple, plays an important role in the theory of PI algebras since one may study the identities satisfied by a given algebra by applying methods from the representation theory of the symmetric group. The S_n -modules P_n and KS_n are canonically isomorphic. Letting A_n be the alternating group in S_n , one may study KA_n and its isomorphic copy in P_n with the corresponding action of A_n . Henke and Regev described the A_n -codimensions of the Grassmann algebra E , and conjectured a finite generating set of the A_n -identities for E . Here we answer their conjecture in the affirmative.

1. INTRODUCTION

Let K be a fixed algebraically closed field of characteristic 0; all algebras, vector spaces and modules we consider will be over this field. Denote by E the Grassmann algebra of the infinite dimensional vector space V over K . If V has a basis $e_1, e_2, \dots$, then E has a basis consisting of 1 and the elements $e_{i_1}e_{i_2}\dots e_{i_k}$ where $i_1 < i_2 < \dots < i_k$, $k \geq 1$. The multiplication in E is induced by $e_ie_j = -e_je_i$ for all i and j . Thus the centre E_0 of E is the span of 1 and all elements $e_{i_1}e_{i_2}\dots e_{i_k}$ with even k . Let E_1 be the span of all such elements with odd k . Then the elements of E_1 anticommute. It is an immediate observation that for every $a, b, c \in E$ one has $[a, b] \in E_0$ and therefore $[a, b, c] = 0$. We suppose all commutators to be left normed; that is, $[a, b, c]$ stands for $[[a, b], c]$ and so on.

Let $K(X)$ be the free associative algebra with 1, freely generated over K by the set of variables $X = \{x_1, x_2, \dots\}$. A polynomial $f \in K(X)$ is a *polynomial identity*, or simply an identity for the algebra R if $f(r_1, \dots, r_m) = 0$ whenever $r_i \in R$. The set of all identities for R is an ideal $T(R)$; it is the T -ideal of R . Since $\text{char } K = 0$ one has that $T(R)$ is generated as a T -ideal by its multilinear polynomials. Denote by P_n the set of all multilinear polynomials of degree n in $x_1, \dots, x_n$. Then $T(R)$ is generated by all $T(R) \cap P_n$, $n \geq 1$. The vector space P_n has a basis consisting

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of $\{x_{\sigma(1)} \dots x_{\sigma(n)} \mid \sigma \in S_n\}$. Here and in what follows S_n is the symmetric group acting on $1, 2, \dots, n$, and A_n is the alternating subgroup of S_n .

The (left) regular S_n -module KS_n is isomorphic to P_n as follows: if $\sigma \in S_n$, then $\sigma \mapsto x_{\sigma(1)} \dots x_{\sigma(n)}$. Thus one may identify KS_n with P_n as S_n -modules. Since $KA_n \subseteq KS_n$, one considers P_n^A , the image of KA_n in P_n . The elements of P_n^A are called A -polynomials. If R is any PI algebra with T -ideal $T = T(R)$, then the A -identities of R are the polynomials in $T \cap P_n^A$, $n \geq 1$. It was pointed out in [2] that every PI algebra R satisfies some A -identity: if $f(x_1, \dots, x_n)$ is a multilinear identity for R , then $f(x_1x_2, \dots, x_{2n-3}x_{2n-2}, x_{2n-1})$ will be an A -identity for R . We recall that in [1] the authors described the module structure of the group algebra KA_n in terms of Young tableaux and related it to the one of KS_n ; see [1] and its references for further information concerning the relation between A_n and PI algebras.

Letting R be as above, one defines the n -th codimension $c_n(R)$ of R as follows: $c_n(R) = \dim P_n / (T \cap P_n)$. The n -th A -codimension is $c_n^A(R) = \dim P_n^A / (T \cap P_n^A)$. It was proved in [3] that for the Grassmann algebra one has $c_n(E) = 2^{n-1}$. It was also proved in [3] that $T(E)$ is generated as a T -ideal by the single polynomial $[x_1, x_2, x_3]$, and a detailed description of the module structure of the S_n -module $P_n(E) = P_n / (T(E) \cap P_n)$ was obtained as well. Furthermore one of the main results in [2] states that $c_n^A(E) = 2^{n-1} - 1$; see [2, Theorem 2.2].

We recall that since E satisfies the identity $[x_1, x_2, x_3]$, then it also satisfies the A -identity of degree five $[x_1x_2, x_3x_4, x_5]$. But in [2, Example 1.1] it was shown that E satisfies the A -identity

$$(1) \quad p = p(x_1, x_2, x_3, x_4) = [x_1, x_2x_3]x_4 - x_4[x_1, x_3x_2]$$

and it is of minimal degree. Furthermore in [2, Conjecture 1.2] the authors conjectured that the above polynomial generates all A -identities of E in the following *strong* sense.

Conjecture 1. *Let $\sigma \in A_n$ and $0 \leq r \leq n - 4$; put $p_{r,\sigma} = x_{\sigma(1)} \dots x_{\sigma(r)}$ and $q_{r,\sigma} = x_{\sigma(r+5)} \dots x_{\sigma(n)}$. Define $f_{r,\sigma}$ as the polynomial*

$$p_{r,\sigma}([x_{\sigma(r+1)}, x_{\sigma(r+2)}x_{\sigma(r+3)}]x_{\sigma(r+4)} - x_{\sigma(r+4)}[x_{\sigma(r+1)}, x_{\sigma(r+3)}x_{\sigma(r+2)}])q_{r,\sigma}.$$

*Then the polynomials $f_{r,\sigma}$ span **all** A -identities for E . (Observe that all $f_{r,\sigma}$ are A -identities.)*

In this paper we give an affirmative answer to the above conjecture.

Let $V(n)$ be the span of all polynomials $f_{r,\sigma}$, $0 \leq r \leq n - 4$, and $\sigma \in A_n$. Our main result is the following theorem.

Theorem 2. *If $n \geq 4$, then $\dim P_n^A / V(n) = 2^{n-1} - 1$.*

We observe that Theorem 2 implies immediately the affirmative answer of Conjecture 1. Indeed $V(n) \subseteq P_n^A \cap T(E)$, and if for some n the inclusion were proper, then one would have $\dim P_n^A / V(n) > \dim P_n^A / (P_n^A \cap T(E)) = c_n^A(E)$. But the latter A -codimension equals $2^{n-1} - 1$ according to [2, Theorem 2.2].

2. PROOF OF THEOREM 2

We prove the theorem by induction on n . First we establish some properties of $V(n)$. We consider the quotient module $P_n^A / V(n)$. We shall use the letters y for

the images of x in the quotient. That is,

$$y_{\sigma(1)}y_{\sigma(2)}\cdots y_{\sigma(n)} = x_{\sigma(1)}x_{\sigma(2)}\cdots x_{\sigma(n)} + V(n).$$

When dealing with the monomial $m = y_{\sigma(1)}y_{\sigma(2)}\cdots y_{\sigma(n)}$, if we are interested only in some submonomial m' , say $m = m_1m'm_2$, then we shall substitute m_1 and m_2 by $*$, that is $m = *m'*$. Observe that $*$ may stand for the empty monomial as well. Thus from the basic identity (1) we have:

Lemma 3. *The equality*

$$(2) \quad *y_a y_b y_c y_d * - *y_d y_a y_c y_b * - *y_b y_c y_a y_d * + *y_d y_c y_b y_a * = 0$$

holds in $P_n^A/V(n)$ for every a, b, c, d .

We note that the above equality reads

$$m_1 y_a y_b y_c y_d m_2 - m_1 y_d y_a y_c y_b m_2 - m_1 y_b y_c y_a y_d m_2 + m_1 y_d y_c y_b y_a m_2 = 0$$

for some monomials m_1 and m_2 . In what follows, in an equality all left-hand side asterisks stand for the same monomial and similarly for the right-hand side asterisks.

Lemma 4. *The vector space $P_n^A/V(n)$ is spanned by monomials of the type $y_{\sigma(1)}y_{\sigma(2)}\cdots y_{\sigma(n)}$ where y_n is at one of the last three positions. In other words either $\sigma(n-2) = n$, or $\sigma(n-1) = n$, or else $\sigma(n) = n$.*

Proof. In (2), the variable y_a appears at the first position only in the first summand. Therefore applying (2) several times to a given monomial, we represent it as a linear combination of monomials where y_n is in one of the last three positions. $\square$

Denote by $H(i, j)$ the span of all $y_{\sigma(1)}y_{\sigma(2)}\cdots y_{\sigma(n)}$ such that $\sigma(i) = n$ and $\sigma(j) = n-2$. That is, $H(i, j)$ is spanned by the monomials where y_n is at position i and y_{n-2} is at position j . Here we require $1 \leq i, j \leq n$ and $i \neq j$.

Lemma 5. *The vector space $P_n^A/V(n)$ is the sum of its subspaces $H(i, j)$ such that $i \geq n-3$ and $j \geq n-3$.*

Proof. As in the proof of the previous lemma, observe the behaviour of the variables y_a and y_c in (2). $\square$

Let $H = \sum_{j=n-3}^{n-1} H(n, j) + \sum_{i=n-3}^{n-1} H(i, n)$. This means that H is the span of the monomials whose last position is occupied either by y_n or by y_{n-2} .

Proposition 6. *One has $P_n^A/V(n) = H + H(n-1, n-2) + H(n-2, n-1)$.*

Proof. We shall use the equality (2) several times:

$$*y_a y_b y_c y_d * - *y_d y_a y_c y_b * - *y_b y_c y_a y_d * + *y_d y_c y_b y_a * = 0$$

in order to eliminate $H(n-2, n-3)$, $H(n-1, n-3)$, $H(n-3, n-2)$ and $H(n-3, n-1)$.

Case 1. Let $m = y_{\sigma(1)}\cdots y_{\sigma(n-4)}y_{n-2}y_n y_{\sigma(n-1)}y_{\sigma(n)} \in H(n-2, n-3)$. We rewrite (2) as

$$*y_d y_a y_c y_b * = *y_a y_b y_c y_d * - *y_b y_c y_a y_d * + *y_d y_c y_b y_a *$$

and set $d = n-2$, $a = n$, $c = \sigma(n-1)$, $b = \sigma(n)$ in it. Therefore we obtain

$$\begin{aligned} *y_{n-2}y_n y_{\sigma(n-1)}y_{\sigma(n)} &= *y_n y_{\sigma(n)} y_{\sigma(n-1)}y_{n-2} &- &*y_{\sigma(n)} y_{\sigma(n-1)}y_n y_{n-2} \\ & &+ &*y_{n-2}y_{\sigma(n-1)}y_{\sigma(n)}y_n. \end{aligned}$$

All monomials from the right-hand side of the above equality lie in H ; hence $m \in H$.

Case 2. Let $m = y_{\sigma(1)} \cdots y_{\sigma(n-4)} y_n y_{n-2} y_{\sigma(n-1)} y_{\sigma(n)} \in H(n-3, n-2)$. We proceed as in case 1, with the only difference that we exchange n and $n-2$; that is, we set $d = n$, $a = n-2$, and repeat the argument above.

Case 3. Let $m = y_{\sigma(1)} \cdots y_{\sigma(n-4)} y_{n-2} y_{\sigma(n-2)} y_n y_{\sigma(n)} \in H(n-1, n-3)$. Here we rewrite (2) as

$$*y_d y_c y_b y_a^* = - *y_a y_b y_c y_d^* + *y_d y_a y_c y_b^* + *y_b y_c y_a y_d^*$$

and set in it $d = n-2$, $c = \sigma(n-2)$, $b = n$, $a = \sigma(n)$. Thus we obtain

$$\begin{aligned} *y_{n-2} y_{\sigma(n-2)} y_n y_{\sigma(n)} &= - *y_{\sigma(n)} y_n y_{\sigma(n-2)} y_{n-2} + *y_{n-2} y_{\sigma(n)} y_{\sigma(n-2)} y_n \\ &\quad + *y_n y_{\sigma(n-2)} y_{\sigma(n)} y_{n-2}. \end{aligned}$$

As in case 1, all monomials from the right-hand side are in H ; thus $m \in H$.

Case 4. Now let $m = y_{\sigma(1)} \cdots y_{\sigma(n-4)} y_n y_{\sigma(n-2)} y_{n-2} y_{\sigma(n)} \in H(n-3, n-1)$. As was done before (see Case 2), it is sufficient to exchange d and b in Case 3, that is, to set $d = n$, $b = n-2$.

Hence all possibilities have been considered and the proof of the proposition is complete. $\square$

Corollary 7. $P_n^A/V(n) = H + H(n-2, n-1)$.

Proof. Let $m = y_{\sigma(1)} \cdots y_{\sigma(n-4)} y_{\sigma(n-3)} y_{n-2} y_n y_{\sigma(n)} \in H(n-1, n-2)$. Write (2) as

$$*y_a y_b y_c y_d^* = *y_d y_a y_c y_b^* + *y_b y_c y_a y_d^* - *y_d y_c y_b y_a^*,$$

and set in it $a = \sigma(n-3)$, $b = n-2$, $c = n$, $d = \sigma(n)$. Thus

$$\begin{aligned} *y_{\sigma(n-3)} y_{n-2} y_n y_{\sigma(n)} &= *y_{\sigma(n)} y_{\sigma(n-3)} y_n y_{n-2} + *y_{n-2} y_n y_{\sigma(n-3)} y_{\sigma(n)} \\ &\quad - *y_{\sigma(n)} y_n y_{n-2} y_{\sigma(n-3)}. \end{aligned}$$

Then $m \in H + H(n-2, n-3) + H(n-2, n-1)$. But $H(n-2, n-3) \subseteq H$ as was shown in Case 1 of the proof of the last proposition. Therefore the corollary is proved. $\square$

Lemma 8. *The equalities $[y_1 y_2, y_3, y_4 y_5] = 0$ and $[y_1 y_2, y_3 y_4, y_5] = 0$ are verified in $P_5^A/V(5)$.*

Proof. The proof is a straightforward computation:

$$[y_1 y_2, y_3, y_4 y_5] = y_1 y_2 y_3 y_4 y_5 - y_3 y_1 y_2 y_4 y_5 - y_4 y_5 y_1 y_2 y_3 + y_4 y_5 y_3 y_1 y_2.$$

Now we transform the second summand above using (2):

$$\begin{aligned} -y_3 y_1 y_2 y_4 y_5 &= -(y_4 y_3 y_2 y_1 + y_1 y_2 y_3 y_4 - y_4 y_2 y_1 y_3) y_5 \\ &= -y_4 (y_5 y_3 y_1 y_2 + y_2 y_1 y_3 y_5 - y_5 y_1 y_2 y_3) \\ &\quad - y_1 y_2 y_3 y_4 y_5 + y_4 y_2 y_1 y_3 y_5 \\ &= -y_4 y_5 y_3 y_1 y_2 + y_4 y_5 y_1 y_2 y_3 - y_1 y_2 y_3 y_4 y_5. \end{aligned}$$

Thus $[y_1 y_2, y_3, y_4 y_5] = 0$ in $P_5^A/V(5)$. The second equality follows from the first by the Jacobi identity $[y_1 y_2, y_3 y_4, y_5] = [y_1 y_2, y_5, y_3 y_4] - [y_3 y_4, y_5, y_1 y_2]$ since both right-hand side summands vanish on $P_5^A/V(5)$. $\square$

Corollary 9. *If $n \geq 3$, then*

$$*[y_1 y_2, y_3 y_4, \dots, y_{2n-1} y_{2n}]^* = 0.$$

Proof. The proof follows immediately from Lemma 8. $\square$

Proposition 10. *Let $n \geq 4$ and let $m = y_{\sigma(1)}y_{\sigma(2)} \cdots y_{\sigma(n)}$.*

- a) Let n be even and let for some k , $1 \leq k \leq n/2$, $\sigma(2k-1) = n$ and $\sigma(2k) = n-2$. Then $m \in H$.*
- b) If n is odd and if for some k , $1 \leq k \leq (n-1)/2$, $\sigma(2k) = n$ and $\sigma(2k+1) = n-2$, then $m \in H$.*

Proof. By Corollary 9 we reduce m to a linear combination of monomials of the form $*y_n y_{n-2} y_a y_b$ and monomials in H . But $*y_n y_{n-2} y_a y_b \in H(n-3, n-2) \subseteq H$, where the last inclusion was proved in Proposition 6, Case 2. $\square$

Here we give an example. Let $m = y_6 y_4 y_1 y_2 y_3 y_5$; thus in the notation of the last proposition, $n = 6$, $k = 1$. According to Corollary 9 we have $[y_6 y_4, y_1 y_2, y_3 y_5] = 0$; therefore

$$y_6 y_4 \cdot y_1 y_2 \cdot y_3 y_5 = y_1 y_2 \cdot y_6 y_4 \cdot y_3 y_5 + y_3 y_5 \cdot y_6 y_4 \cdot y_1 y_2 - y_3 y_5 \cdot y_1 y_2 \cdot y_6 y_4.$$

The first two summands on the right-hand side lie in $H(n-3, n-2) \subseteq H$ (recall that $n = 6$ here), and the last one belongs to H by definition.

Lemma 11. *Let $m = *y_a y_b y_n y_{n-2} y_c* \in (P_n^A/V(n)) \setminus H$ be a monomial. Then there exist h_1 and $h_2 \in H$ such that:*

$$m = *y_n y_{n-2} y_a y_b y_c* + h_1; \quad m = *y_b y_c y_n y_{n-2} y_a* + h_2.$$

Proof. It follows from Proposition 10 that the right-hand side asterisk in m stands for a monomial of even length (otherwise $m \in H$). According to Lemma 8 we have $[y_1 y_2, y_3 y_4, y_5] = 0$. This yields

$$y_1 y_2 y_3 y_4 y_5 - y_3 y_4 y_1 y_2 y_5 - y_5 y_1 y_2 y_3 y_4 + y_5 y_3 y_4 y_1 y_2 = 0.$$

Now put a and b instead of the indices 1 and 2, n and $n-2$ instead of 3 and 4, and c instead of 5, and multiply by $*$ on the left and on the right. Pay attention to the fact that $*y_c y_a y_b y_n y_{n-2}*$ and $*y_c y_n y_{n-2} y_a y_b*$ lie in H by Proposition 10, since the right-hand $*$ is of even length. This proves the existence of h_1 .

For h_2 we use the equality $[y_1 y_2, y_3, y_4 y_5] = 0$ obtained in Lemma 8. We substitute the indices 1 and 2 by n and $n-2$, and 3, 4, 5 for a , b , c , respectively. Then observe that $*y_n y_{n-2} y_a y_b y_c* \equiv *y_a y_b y_n y_{n-2} y_c* \pmod{H}$ according to the first equality since the right-hand $*$ stands for a monomial of even length. Thus the second and the fourth summand in

$$[y_n y_{n-2}, y_a, y_b y_c] = y_n y_{n-2} y_a y_b y_c - y_a y_n y_{n-2} y_b y_c - y_b y_c y_n y_{n-2} y_a + y_b y_c y_a y_n y_{n-2}$$

are in H , and the first and the third give us exactly the second equality. $\square$

Lemma 12. *Let $m = *y_a y_n y_{n-2} y_b y_c* \in (P_n^A/V(n)) \setminus H$ be a monomial. Then $m = *y_c y_n y_{n-2} y_a y_b* + h_3$ for some $h_3 \in H$. In other words, modulo H one can make cyclic permutations of the letters y_a, y_b, y_c in m .*

Proof. The right-hand $*$ of m is a monomial of odd length since otherwise $m \in H$. Then we repeat the argument of the second equality in Lemma 11 substituting in $[y_1 y_2, y_3, y_4 y_5] = 0$ the indices 1, 2, 3 by a, b, c , and 4 and 5 by n and $n-2$, respectively. Thus

$$*(y_a y_b y_c y_n y_{n-2} - y_c y_a y_b y_n y_{n-2} - y_n y_{n-2} y_a y_b y_c + y_n y_{n-2} y_c y_a y_b)* = 0.$$

The last two summands lie in H since the right-hand side $*$ is of odd length. We transform the first two as follows. The right-hand asterisk above is $*$ $= y_d m_1$ for some monomial m_1 . So we apply the first equality of Lemma 11 first to $y_b y_c y_n y_{n-2} y_d$ and then to $y_a y_b y_n y_{n-2} y_d$:

$$\begin{aligned} *y_a y_b y_c y_n y_{n-2} y_d m_1 - *y_a y_n y_{n-2} y_b y_c y_d m_1 &\in H, \\ *y_c y_a y_b y_n y_{n-2} y_d m_1 - *y_c y_n y_{n-2} y_a y_b y_d m_1 &\in H, \end{aligned}$$

and thus we are done. $\square$

Remark 13. We note that in all the above statements we used even permutations of the variables. That is, the resulting monomials will always be even as long as the initial ones are even.

Lemma 14. *One has $\dim P_4^A/V(4) = 2^{4-1} - 1 = 7$.*

Proof. This was shown to be true in [2]; see the remarks after Conjecture 1.2 in [2]. $\square$

Lemma 15. *Let $m = y_{\sigma(1)} \dots y_{\sigma(n-3)} y_n y_{n-2} y_{\sigma(n)} \in H(n-2, n-1)$ be a monomial. Then*

$$m = y_1 y_2 \dots y_{n-4} y_{n-3} y_n y_{n-2} y_{n-1} + h$$

for some polynomial $h \in H$.

Proof. We induct on n . When $n = 4$ either $m = y_1 y_4 y_2 y_3$ or $m = y_3 y_4 y_2 y_1$ and the second cannot happen as it corresponds to an odd permutation. If $n = 5$ we have $m = y_{\sigma(1)} y_{\sigma(2)} y_5 y_3 y_{\sigma(5)}$. Applying Lemma 11 we permute cyclically $y_{\sigma(1)}$, $y_{\sigma(2)}$, $y_{\sigma(3)}$ in order to get $\sigma(1) = 1$; since $\sigma \in A_5$, then $\sigma(2) = 2$ and $\sigma(5) = 4$.

Letting $n > 5$ and $\sigma(j) = 1$, we shall show that modulo H one can rewrite m so that $j = 1$. Assume $j \geq 2$. According to Lemma 11 we move the block $y_n y_{n-2}$ to the left and either $m = *y_n y_{n-2} y_1 * + h_1$ or $m = *y_1 y_n y_{n-2} * + h_1$. Now by the second part of Lemma 11 we can move y_1 one or more positions to the left, and so on. If we obtain $m \equiv y_a y_n y_{n-2} y_1 y_b * \pmod{H}$ we cannot apply Lemma 11, but we use Lemma 12 instead and move y_1 to the left.

When $\sigma(1) = 1$ we apply the induction to the monomial m_1 where $m = y_1 m_1$. Since y_1 comes first the monomial m_1 corresponds to an even permutation of $(2, \dots, n)$. $\square$

Proof. (of Theorem 2). Suppose that $\dim P_{n-1}^A/V(n-1) = 2^{n-2} - 1$ for some $n \geq 5$. By Lemma 15, the vector space $H(n-2, n-1)$ is one dimensional modulo H and is spanned by the monomial $y_1 y_2 \dots y_{n-4} y_{n-3} y_n y_{n-2} y_{n-1}$. But we have established that $P_n^A/V(n) = H(n-2, n-1) + H$; see Corollary 7. Thus $\dim P_n^A/V(n) \leq 1 + \dim H$.

But by the induction we have $\dim H \leq (2^{n-2} - 1) + (2^{n-2} - 1)$; hence

$$\dim P_n^A/V(n) \leq 1 + \dim H \leq 1 + 2^{n-1} - 2 = 2^{n-1} - 1.$$

As we already observed this proves our theorem. In other words one has $V(n) = T(E) \cap P_n^A$, and thus Henke and Regev's conjecture is true. $\square$

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