A NOTE ON SCALING ASYMPTOTICS FOR BOHR-SOMMERFELD LAGRANGIAN SUBMANIFOLDS

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ABSTRACT. This paper deals with the asymptotic expansions describing the quantum states associated to Bohr Sommerfeld Lagrangian submanifolds of a compact Kähler manifold, in the context of geometric quantization. More precisely, it provides an improvement on a result of the work of Debernardi and the author (2006), describing a natural factorization of the expansion and providing certain remainder estimates.

INTRODUCTION

The purpose of this paper is to improve an expansion in [5] for the asymptotics associated to Bohr-Sommerfeld Lagrangian submanifolds of a compact Hodge manifold, in the context of geometric quantization (see e.g. [1], [2], [6], [9]). We adopt the general framework for quantizing Bohr-Sommerfeld Lagrangian submanifolds presented in [2], based on applying the Szegő kernel of the quantizing line bundle to certain delta functions concentrated along the submanifold.

Let M be a d -dimensional complex projective manifold, with complex structure J ; consider an ample line bundle A on it, and let h be a Hermitian metric on A such that the unique compatible connection has curvature $\Omega = -2i\omega$, where ω is a Kähler form. Then the unit circle bundle $X \subseteq A^*$, endowed with the connection one-form α , is a contact manifold. A Bohr-Sommerfeld Lagrangian submanifold of M (or, more precisely, of (M, A, h)) is then simply a Legendrian submanifold $\Lambda \subseteq X$, conceived as an immersed submanifold of M .

In a standard manner, X inherits a Riemannian structure for which the projection $\pi : X \rightarrow M$ is a Riemannian fibration. In view of this, we shall implicitly identify (generalized) functions, densities and half-densities on X at certain places.

Referring to §2 of [5] for a more complete description of the preliminaries involved, we recall that if $\Lambda \subseteq X$ is a compact Legendrian submanifold and λ is a half-density on it, there is a naturally induced generalized half-density $\delta_{\Lambda, \lambda}$ on X supported on Λ . Following [2], we can then define a sequence of CR functions

$$u_k =: P_k(\delta_{\Lambda, \lambda}) \in \mathcal{H}(X)_k,$$

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where $\mathcal{H}(X)_k$ is the k -th isotype of the Hardy space with respect to the S^1 -action, and $P_k : L^2(X) \rightarrow \mathcal{H}(X)_k$ is the orthogonal projector (extended to $\mathcal{D}'(X) \rightarrow \mathcal{H}(X)_k$). In the present setting there are natural unitary structures on $\mathcal{H}(X)_k$ and the space of global holomorphic sections $H^0(M, A^{\otimes k})$, and a natural unitary isomorphism $\mathcal{H}(X)_k \cong H^0(M, A^{\otimes k})$. One thinks of the u_k 's as representing the quantizations of (Λ, λ) at Planck's constant $1/k$. It is easily seen that u_k is rapidly decaying as $k \rightarrow +\infty$ on the complement of $S^1 \cdot \Lambda = \pi^{-1}(\pi(\Lambda))$; here $\pi : X \rightarrow M$ is the projection. On the other hand, the asymptotic concentration of the u_k 's along $S^1 \cdot \Lambda$ poses an interesting problem, already considered in Theorem 3.12 of [2].

This theme was revisited in [5] in a somewhat different technical setting; in particular, Corollary 1.1 of [5] shows that the scaling asymptotics of u_k (to be defined shortly) near any $x \in S^1 \cdot \Lambda$ admit an asymptotic expansion, and explicitly computes the leading order term. We shall presently give a more precise description of this expansion as a function on the tangent space of M at $m = \pi(x)$. Namely, we shall show that this asymptotic expansion may be factored as an exponentially decaying term in the component $w^\perp$ of $w \in T_m M$ orthogonal to Λ times an asymptotic expansion with polynomial coefficients in w (more precisely, the expansion is generally given by a finite sum of terms of this form, one from each branch of Λ projecting to m); the exponential term also contains a symplectic pairing between $w^\perp$ and the component of w along Λ , $w^\parallel$. Furthermore, we shall give some relevant remainder estimates not mentioned in [5].

Before stating the results of this paper, let us recall that for any $x \in X$ we can find a Heisenberg local chart for X centered at x ,

$$\rho : B_{2d}(\epsilon) \times (-\pi, \pi) \rightarrow X, \quad (p, q, \theta) \mapsto r_{e^{i\theta}}(\epsilon(p, q));$$

here $B_{2d}(\epsilon) \subseteq \mathbb{R}^{2d}$ is a ball of radius ϵ centered at the origin, $\varrho : B_{2d}(\epsilon) \rightarrow M$ is a preferred local chart for M centered at $m =: \pi(x)$, meaning that it trivializes the holomorphic and symplectic structures at m , and ϵ is a unitary local frame of A^* , given by the unitarization of a preferred local frame (complete definitions are in [8]). Finally, $r_{e^{i\theta}} : X \rightarrow X$ is the diffeomorphism induced by the circle action. It is in this kind of local coordinate that the scaling limits of Szegő kernels exhibit their universal nature (Theorem 3.1 of [8]). If ρ is a system of Heisenberg local coordinates centered at x , and $p, q \in \mathbb{R}^d$, $w = p + iq$, one poses

$$x + w =: \rho((p, q), 0).$$

For any θ , we have

$$u_k(\rho(p, q, \theta)) = e^{ik\theta} u_k(\rho(p, q, 0)) = e^{ik\theta} u_k(x + w).$$

Given that a system of Heisenberg local coordinates induces a unitary isomorphism of $T_m M$ and $\mathbb{C}^d$, with this understanding we can also consider the expression $x + w$ with $w \in T_m M$.

If $x \in S^1 \cdot \Lambda$, there are only finitely many elements $h_1, \dots, h_{N_x} \in S^1$ such that $x_j =: r_{h_j}(x) \in \Lambda$. Since Λ is Legendrian, hence horizontal for the connection, for any j we may naturally identify the tangent space $T_{x_j} \Lambda \subseteq T_{x_j} X$ with a subspace of $T_{\pi(x)} M$. With this in mind, if $w \in T_{\pi(x)} M$ we can write $w = w_j^\parallel + w_j^\perp$ for unique $w_j^\parallel \in T_{x_j} \Lambda$ and $w_j^\perp \in T_{x_j} \Lambda^\perp$; the latter denotes the orthocomplement of $T_{x_j} \Lambda$ in $T_{\pi(x)} M$ in the Riemannian metric of M .

Finally, let $\text{dens}_\Lambda^{(1/2)}$ be the Riemannian half-density on Λ (for the induced metric); thus if λ is a $\mathcal{C}^\infty$ half-density on Λ we can write $\lambda = F_\lambda \cdot \text{dens}_\Lambda^{(1/2)}$ for a unique $F_\lambda \in \mathcal{C}^\infty(\Lambda)$.

Theorem 1. *Let $\Lambda \subseteq X$ be a compact Legendrian submanifold, and suppose λ is a smooth half-density on it. For every $k = 1, 2, \dots$, let $u_k =: P_k(\delta_{\Lambda, \lambda})$. Suppose $x \in S^1 \cdot \Lambda$, and choose a system of Heisenberg local coordinates for X centered at x . Let $h_1, \dots, h_{N_x} \in S^1$ be the finitely many elements such that $r_{h_j}(x) \in \Lambda$. Then:*

1. *Suppose $a > 0$. Uniformly for $\min_j \{\|w_j^\perp\|\} \gtrsim k^a$, we have*

$$u_k \left(x + \frac{w}{\sqrt{k}} \right) = O(k^{-\infty}).$$

2. *There exist polynomials a_{lj} on $\mathbb{C}^d$ such that the following holds: for $w \in T_{\pi(x)}M$ and $k, \ell = 1, 2, \dots$, let us define*

$$\begin{aligned} R_{k, \ell}(x, w) = & u_k \left(x + \frac{w}{\sqrt{k}} \right) \\ & - \left(\frac{2k}{\pi} \right)^{d/2} \sum_{j=1}^{N_x} h_j^{-k} e^{-\|w_j^\perp\|^2 - i\omega_{\pi(x)}(w_j^\perp, w_j^\parallel)} F_\lambda(x_j) \\ & \cdot \left(1 + \sum_{l=1}^{\ell} k^{-l/2} a_{lj}(w) \right). \end{aligned}$$

Then uniformly for $\|w\| \lesssim k^{1/6}$ we have

$$(1) \quad |R_{k, \ell}(x, w)| \leq C_\ell k^{(d-\ell-1)/2} \sum_{j=1}^{N_x} e^{-\frac{1-\epsilon}{2}\|w_j^\perp\|^2}.$$

Corollary 1. $\forall w \in T_{\pi(x)}M$, the following asymptotic expansion holds as $k \rightarrow +\infty$:

$$\begin{aligned} u_k \left(x + \frac{w}{\sqrt{k}} \right) \sim & \left(\frac{2k}{\pi} \right)^{d/2} \sum_{j=1}^{N_x} h_j^{-k} e^{-\|w_j^\perp\|^2 - i\omega_{\pi(x)}(w_j^\perp, w_j^\parallel)} F_\lambda(x_j) \\ & \cdot \left(1 + \sum_{l \geq 1} k^{-l/2} a_{lj}(w) \right). \end{aligned}$$

This agrees with Corollary 1.1 of [5] to leading order but gives a clearer picture of the asymptotic expansion, as well as an explicit estimate on the remainder.

The proof of Theorem 1 is based on the scaling asymptotics of Szegő kernels in Theorem 3.1 of [8], whereas the proofs in [5] are based on microlocal arguments that encompass the equivariant setting, involving a direct use of the parametrix developed in [3]. In view of the scaling asymptotics of equivariant Szegő kernels proved in [7], factorizations akin to Theorem 1 also hold in the equivariant setting; we shall not discuss this here.

PROOF OF THEOREM 1

Let us first prove item 2. Thus, we want to investigate the asymptotics of $u_k \left(x + \frac{w}{\sqrt{k}} \right)$ as $k \rightarrow +\infty$, assuming that $w \in T_{\pi(x)}M$, $\|w\| \leq C k^{1/6}$ for some fixed $C > 0$.

Let $\Pi_k \in \mathcal{C}^\infty(X \times X)$ be the Schartz kernel of P_k ; explicitly, if $\{s_r^{(k)}\}$ is an orthonormal basis of $\mathcal{H}(X)_k$, then

$$\Pi_k(y, y') = \sum_r s_r^{(k)}(y) \cdot \overline{s_r^{(k)}(y')} \quad (y, y' \in X).$$

Let dens_X and dens_Λ denote, respectively, the Riemannian density on X and Λ . Then, in standard distributional shorthand, by definition of $\delta_{\Lambda, \lambda}$ for any $x' \in X$ we have

$$\begin{aligned} (2) \quad u_k(x') &= \int_X \Pi_k(x', y) \delta_{\Lambda, \lambda}(y) \text{dens}_X(y) \\ &= \langle \delta_{\Lambda, \lambda}, \Pi_k(x', \cdot) \rangle = \int_\Lambda \Pi_k(x', y) F_\lambda(y) \text{dens}_\Lambda(y). \end{aligned}$$

Let us write dist_M for the Riemannian distance function on M , pulled back to a smooth function on $X \times X$ by the projection $\pi \times \pi$. Let us set:

$$\begin{aligned} V_k &=: \left\{ x' \in X : \text{dist}_M(x, x') < 4C k^{-1/3} \right\}, \\ V'_k &=: \left\{ x' \in X : \text{dist}_M(x, x') > 3C k^{-1/3} \right\}. \end{aligned}$$

If $y \in V'_k$ and $\|w\| \leq C k^{1/6}$, then $\text{dist}_M\left(x + \frac{w}{\sqrt{k}}, y\right) \geq C k^{-1/3}$ for $k \gg 0$; by the off-diagonal estimates on Szegő kernels in [4], therefore, $\Pi_k\left(x + \frac{w}{\sqrt{k}}, y\right) = O(k^{-\infty})$ uniformly for $y \in V'_k$.

For $k \gg 0$, $\Lambda \cap V_k$ has N_x connected components:

$$\Lambda \cap V_k = \bigcup_{j=1}^{N_x} \Lambda_{kj},$$

where Λ_{kj} is the connected component containing x_j . Let $\{s_k, s'_k\}$ be an S^1 -invariant partition of unity on X subordinate to the open cover $\{V_k, V'_k\}$. In view of (2) and the previous discussion, we obtain

$$\begin{aligned} (3) \quad u_k\left(x + \frac{w}{\sqrt{k}}\right) &= \int_\Lambda \Pi_k\left(x + \frac{w}{\sqrt{k}}, y\right) F_\lambda(y) \text{dens}_\Lambda(y) \\ &\sim \sum_{j=1}^{N_x} \int_{\Lambda_{kj}} \Pi_k\left(x + \frac{w}{\sqrt{k}}, y\right) F_\lambda(y) s_k(y) \text{dens}_\Lambda(y), \end{aligned}$$

where $\sim$ means that the two terms have the same asymptotics. Let us now evaluate the asymptotics of the j -th summand in (3).

To this end, recall that the Heisenberg local chart ρ centered at x depends on the choice of the preferred local chart ϱ at $\pi(x)$ and of the local frame $\mathfrak{e}$ of A^* . We obtain a Heisenberg local chart ρ'_j centered at x_j by setting $\rho'_j(p, q, \theta) =: r_{h_j}(\rho(p, q, \theta))$. By the discussion in §2 of [5] and (48) of the same paper, we can compose ρ'_j with a suitable transformation in (p, q) (that is, a change of preferred local chart for M) so as to obtain a Heisenberg local chart $\rho_j(p, q, \theta)$ centered at x_j with the following property: Λ is locally defined near x_j by the conditions $\theta = f_j(q)$ and $p = 0$, where f_j vanishes to third order at the origin. By construction, we have $\rho_j(p, q, \theta) = r_{h_j}(\rho(p', q', \theta))$ for a certain local diffeomorphism $(p, q) \mapsto (p', q')$.

Thus Λ is locally parametrized, near x_j and in the chart ρ_j , by the imaginary vectors iq ; viewing the q 's as local coordinates on Λ near x_j , we have $\text{dens}_\Lambda = D_\lambda \cdot |dq|$ for a unique locally defined smooth function D_λ . By construction of Heisenberg local coordinates, $D_\lambda(0) = 1$.

Applying a rescaling by $k^{-1/2}$, we obtain

$$\begin{aligned}
 & \int_{\Lambda_{k,j}} \Pi_k \left(x + \frac{w}{\sqrt{k}}, y \right) F_\lambda(y) s_k(y) \text{dens}_\Lambda(y) \\
 &= k^{-d/2} \int_{\mathbb{R}^d} \Pi_k \left(x + \frac{w}{\sqrt{k}}, r_{e^{if_j(q/\sqrt{k})}} \left(x_j + \frac{iq}{\sqrt{k}} \right) \right) \\
 (4) \quad & \cdot F_\lambda \left(\frac{q}{\sqrt{k}} \right) s_k \left(\frac{iq}{\sqrt{k}} \right) D_\lambda \left(\frac{q}{\sqrt{k}} \right) dq \\
 &= k^{-d/2} \int_{\mathbb{R}^d} e^{-ikf_j(q/\sqrt{k})} \Pi_k \left(x + \frac{w}{\sqrt{k}}, x_j + \frac{iq}{\sqrt{k}} \right) \\
 & \cdot F_\lambda \left(\frac{q}{\sqrt{k}} \right) s_k \left(\frac{iq}{\sqrt{k}} \right) D_\lambda \left(\frac{q}{\sqrt{k}} \right) dq.
 \end{aligned}$$

Here, $x + \frac{w}{\sqrt{k}} = \rho \left(\frac{\Re(w)}{\sqrt{k}}, \frac{\Im(w)}{\sqrt{k}}, 0 \right)$ (we use the Heisenberg chart to unitarily identify $T_m M$ with $\mathbb{C}^d$) and $x_j + \frac{iq}{\sqrt{k}} = \rho_j \left(0, \frac{q}{\sqrt{k}}, 0 \right)$. Notice that $s_k \left(\frac{iq}{\sqrt{k}} \right) = 1$ for $\|q\| \lesssim k^{1/6}$ and $s_k \left(\frac{iq}{\sqrt{k}} \right) = 0$ for $\|q\| \gtrsim k^{1/6}$. In particular, integration takes place over a ball of radius $\sim k^{1/6}$. Also, Taylor expanding F_λ and f_j at the origin, we have asymptotic expansions

$$F_\lambda \left(\frac{q}{\sqrt{k}} \right) \sim F_\lambda(x_j) + \sum_{r \geq 1} k^{-r/2} b_r(q), \quad D_\lambda \left(\frac{q}{\sqrt{k}} \right) \sim 1 + \sum_{r \geq 1} k^{-r/2} c_r(q),$$

and, since f_j vanishes to third order at the origin,

$$f_j \left(\frac{q}{\sqrt{k}} \right) \sim \sum_{r \geq 0} k^{-(3+r)/2} d_r(q), \quad e^{-ikf_j(q/\sqrt{k})} \sim 1 + \sum_{r \geq 1} k^{-r/2} e_r(q),$$

for suitable polynomials b_r, c_r, d_r, e_r .

Let $w_j \in \mathbb{C}^d$ correspond to w in the Heisenberg local coordinates ρ_j . By the above, Taylor expanding the transformation $(p, q) \mapsto (p', q')$, we obtain $x + \frac{w}{\sqrt{k}} = r_{h_j^{-1}} \left(x_j + \frac{w_j}{\sqrt{k}} + H(w, k) \right)$, where $H(w, k) \sim \sum_{f \geq 2} k^{-f/2} h_f(w)$. Without affecting the leading order term of the resulting asymptotic expansion, we may pretend for simplicity that $x + \frac{w}{\sqrt{k}} = r_{h_j^{-1}} \left(x_j + \frac{w_j}{\sqrt{k}} \right)$.

Write $w_j = p_j + iq_j$, with $p_j, q_j \in \mathbb{R}^d$. Thus $w_j^\perp = p_j$, $w_j^\parallel = iq_j$. In view of Theorem 3.1 of [8], we have

$$\begin{aligned}
 & \Pi_k \left(x + \frac{w}{\sqrt{k}}, x_j + \frac{iq}{\sqrt{k}} \right) \\
 (5) \quad &= \Pi_k \left(r_{h_j^{-1}} \left(x_j + \frac{w_j}{\sqrt{k}} \right), x_j + \frac{iq}{\sqrt{k}} \right) = h_j^{-k} \Pi_k \left(x_j + \frac{w_j}{\sqrt{k}}, x_j + \frac{iq}{\sqrt{k}} \right) \\
 & \sim h_j^{-k} \left(\frac{k}{\pi} \right)^d e^{-ip_j \cdot q - \frac{1}{2} \|p_j\|^2 - \frac{1}{2} \|q_j - q\|^2} \cdot \left(1 + \sum_{r \geq 1} k^{-r/2} R_j(w, q) \right),
 \end{aligned}$$

for certain polynomials R_j in w and q . Furthermore, by the large ball estimate on the remainder discussed in §5 of [8], the remainder after summing over $1 \leq r \leq R$ is bounded by

$$(6) \quad C_R k^{d-(R+1)} e^{-\frac{1-\epsilon}{2}(\|p_j\|^2 + \|q - q_j\|^2)}.$$

It follows that the product of these asymptotic expansions can be integrated term by term; given this, we only lose a contribution which is $O(k^{-\infty})$ by setting $s_k = 1$ and integrating over all of $\mathbb{R}^d$.

We have

$$\int_{\mathbb{R}^d} e^{-ip_j \cdot q - \frac{1}{2}\|q_j - q\|^2} dq = e^{-ip_j \cdot q_j} \int_{\mathbb{R}^d} e^{-ip_j \cdot s - \frac{1}{2}\|s\|^2} ds = (2\pi)^{d/2} e^{-ip_j \cdot q_j - \frac{1}{2}\|p_j\|^2}.$$

Given (5), this implies that (4) is given by an asymptotic expansion with leading order term

$$h_j^{-k} \left(\frac{2k}{\pi} \right)^{d/2} e^{-\|w_j^\perp\|^2 - i\omega_{\pi(x)}(w_j^\perp, w_j^\parallel)} F_\lambda(x_j).$$

To determine the general term of the expansion, on the other hand, we are led to computing integrals of the form

$$\int_{\mathbb{R}^d} q^\beta e^{-ip_j \cdot q - \frac{1}{2}\|q_j - q\|^2} dq = e^{-ip_j \cdot q_j} \int_{\mathbb{R}^d} (s + q_j)^\beta e^{-ip_j \cdot s - \frac{1}{2}\|s\|^2} ds,$$

where q^β is some monomial. Thus we are led to a sum of terms of the form

$$e^{-ip_j \cdot q_j} C_\gamma(q_j) \int_{\mathbb{R}^d} s^\gamma e^{-ip_j \cdot s - \frac{1}{2}\|s\|^2} ds,$$

and the integral is the evaluation at p_j of the Fourier transform of $s^\gamma e^{-\frac{1}{2}\|s\|^2}$. Up to a scalar factor, the latter is an iterated derivative to $e^{-\frac{1}{2}\|s\|^2}$; therefore we are left with a summand of the form $e^{-ip_j \cdot q_j} T(q_j, q_j) e^{-\frac{1}{2}\|p_j\|^2} ds$, where T is a polynomial in p_j, q_j . Given (5), this implies that the general term of the asymptotic expansion for (4) has the form

$$h_j^{-k} \left(\frac{2k}{\pi} \right)^{d/2} k^{-l/2} e^{-\|w_j^\perp\|^2 - i\omega_{\pi(x)}(w_j^\perp, w_j^\parallel)} F_\lambda(x_j) \cdot a_{lj}(w)$$

for an appropriate polynomial $a_{lj}(w)$. Finally, (1) (at x_j) follows by integrating (6).

To complete the proof of item 2, we need only sum over j .

Turning to the proof of item 1, by definition of preferred local coordinates, if $w_j^\perp \geq Ck^a$, say, then

$$\text{dist}_M \left(x + \frac{v}{\sqrt{k}}, \Lambda_{kj} \right) \geq \frac{C}{2} k^{a-\frac{1}{2}},$$

for all $k \gg 0$. By the off-diagonal estimates of [4], $\Pi_k \left(x + \frac{w}{\sqrt{k}}, y \right) = O(k^{-\infty})$ uniformly for $y \in \Lambda_{kj}$. $\square$

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