REMARKS ON LIPSCHITZ p -SUMMING OPERATORS

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ABSTRACT. In this paper, a nonlinear version of the Extrapolation Theorem is proved and, as a corollary, a nonlinear version of Grothendieck's Theorem is presented. Finally, we prove that if $T : X \rightarrow H$ is Lipschitz with X being a pointed metric space and $T(0) = 0$ such that $T^\#|_{H^*}$ is q -summing ($1 \leq q < \infty$), then T is Lipschitz 1-summing.

1. INTRODUCTION

The notion of Lipschitz p -summing operators was introduced by J. D. Farmer and W. B. Johnson in [FJ]. Although the concept is new, some ingredients have appeared in the paper of J. Bourgain [B]. In [FJ], Farmer and Johnson proved a nonlinear version of the Pietsch Factorization Theorem and showed that the Lipschitz p -summing norm of a linear operator is the same as its p -summing norm. In 1974, B. Maurey [M] presented a beautiful consequence of Pietsch domination and factorization—the Extrapolation Theorem. In Section 2 we use the Farmer-Johnson Factorization Theorem to give a nonlinear version of Maurey's Extrapolation Theorem. As a consequence, we get a nonlinear form of the Grothendieck's Theorem. In 1970, S. Kwapien [K] proved that a linear operator from a Banach space X into a Hilbert space is 1-summing if its adjoint is q -summing for some $1 \leq q < \infty$. By considering the Lipschitz adjoint, we prove in Section 3 that a Lipschitz map from a pointed metric space into a Hilbert space that fixes the origin is Lipschitz 1-summing provided that its Lipschitz adjoint restricted to the dual of the Hilbert space is q -summing for some $1 \leq q < \infty$.

Throughout the paper, we use standard notation as can be found in [DJT] and [BL].

2. NONLINEAR VERSION OF THE EXTRAPOLATION THEOREM

Definition 2.1 ([FJ]). A Lipschitz map T from a metric space (X, d_X) into a metric space (Y, d_Y) is said to be Lipschitz p -summing ($1 \leq p < \infty$) if there is a constant $C > 0$ such that, regardless of the natural number n and regardless of the

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choices of $\{x_i\}_{i=1}^n$ and $\{y_i\}_{i=1}^n$ in X , we have

$$(2.1) \quad \left(\sum_{i=1}^n d_Y(Tx_i, Ty_i)^p \right)^{1/p} \leq C \sup_{f \in B_{X^\#}} \left(\sum_{i=1}^n |f(x_i) - f(y_i)|^p \right)^{1/p},$$

where $B_{X^\#}$ is the unit ball of $X^\#$, the Lipschitz dual of X ; i.e., $X^\#$ is the space of all real-valued Lipschitz functions under the (semi-)norm $\text{Lip}(\cdot)$. We follow the usual convention of considering X as a pointed metric space by designating a special point $0 \in X$ and identifying $X^\#$ with the Lipschitz functions on X that are zero at 0. With this convention $(X^\#, \text{Lip}(\cdot))$ is a Banach space and $B_{X^\#}$ is a compact Hausdorff space in the topology of pointwise convergence on X . The least C for which the above inequality holds is denoted by $\pi_p^L(T)$. We use $\pi_p^L(X, Y)$ to denote the set of all Lipschitz p -summing mappings from X into Y .

Let us recall that Farmer and Johnson proved in [FJ] the following nonlinear domination theorem.

Theorem 2.1. *Let $T : X \rightarrow Y$ be a map between two metric spaces. Let $1 \leq p < \infty$ and $C \geq 0$. Then $\pi_p^L(T) \leq C$ if and only if there is a probability measure μ on $B_{X^\#}$ such that*

$$d_Y(Tx, Ty)^p \leq C^p \int_{B_{X^\#}} |f(x) - f(y)|^p d\mu(f), \quad \forall x, y \in X.$$

The main purpose of this section is to present the Extrapolation Theorem in the nonlinear setting and give, as a corollary, a nonlinear version of Grothendieck's Theorem. Our proof of the Nonlinear Extrapolation Theorem is similar to Maurey's beautiful proof. For the sake of the readers who are not familiar with the original proof, we give the details here.

Theorem 2.2 (Nonlinear Extrapolation Theorem). *Let $1 < r < p < \infty$ and let X be a metric space such that*

$$\pi_p^L(X, \ell_p) = \pi_r^L(X, \ell_p).$$

Then, for any Banach space Y ,

$$\pi_p^L(X, Y) = \pi_1^L(X, Y).$$

Proof. We aim to prove that, regardless of Y , there is $C > 0$ such that for each $u \in \pi_p^L(X, Y)$, we have $\pi_1^L(u) \leq C \cdot \pi_p^L(u)$. Let $K = B_{X^\#}$ and regard X as a subset of $C(K)$. Let $\mathcal{P}(K)$ be the collection of all regular Borel probability measures on K . Given $\mu \in \mathcal{P}(K)$, let j_μ be the restriction of the canonical map $C(K) \rightarrow L_p(\mu)$ to X . Consequently, $\pi_p^L(j_\mu) \leq 1$. Since $\pi_p^L(X, \ell_p) = \pi_r^L(X, \ell_p)$, it follows from the Closed Graph Theorem that there is $c > 0$ such that $\pi_r^L(v) \leq c \cdot \pi_p^L(v)$, for all $v \in \pi_p^L(X, \ell_p)$. Since $L_p(\mu)$ is an $\mathcal{L}_{p,\lambda}$ -space for each $\lambda > 1$, we can assert that for each $\{x_i\}_{i=1}^m$ and $\{y_i\}_{i=1}^m$ in X , the subspace of $L_p(\mu)$ generated by $\{j_\mu x_1, \dots, j_\mu x_m\}$ and $\{j_\mu y_1, \dots, j_\mu y_m\}$ embeds λ -isomorphically into ℓ_p . That is,

$$E = \text{span}\{j_\mu x_1, \dots, j_\mu x_m, j_\mu y_1, \dots, j_\mu y_m\}$$

is contained in a subspace F of $L_p(\mu)$ for which there is an isomorphism $T : F \rightarrow \ell_p^{\dim F}$ with $\|T\| \cdot \|T^{-1}\| < \lambda$.

Thus

$$\begin{aligned} \left(\sum_{i=1}^m \|j_\mu(x_i) - j_\mu(y_i)\|^r \right)^{1/r} &\leq \|T^{-1}\| \left(\sum_{i=1}^m \|Tj_\mu(x_i) - Tj_\mu(y_i)\|^r \right)^{1/r} \\ &\leq \|T^{-1}\| \cdot c \cdot \pi_p^L(Tj_\mu) \cdot \sup_{x^\# \in B_{X^\#}} \left(\sum_{i=1}^m |x^\#(x_i) - x^\#(y_i)|^r \right)^{1/r} \\ &\leq c \|T^{-1}\| \cdot \|T\| \cdot \pi_p^L(j_\mu) \sup_{x^\# \in B_{X^\#}} \left(\sum_{i=1}^m |x^\#(x_i) - x^\#(y_i)|^r \right)^{1/r}, \end{aligned}$$

so $\pi_r^L(j_\mu) \leq c \cdot \lambda \cdot \pi_p^L(j_\mu) \leq c \cdot \lambda$, for all $\lambda > 1$. That is, $\pi_r^L(j_\mu) \leq c$.

By Theorem 2.1, we are able to produce a $\hat{\mu} \in \mathcal{P}(K)$ such that

$$\begin{aligned} \|j_\mu(x) - j_\mu(y)\|_{L_p(\mu)} &\leq c \left(\int_K |x^\#(x) - x^\#(y)|^r d\hat{\mu}(x^\#) \right)^{1/r} \\ &= c \|j_{\hat{\mu}}(x) - j_{\hat{\mu}}(y)\|_{L_r(\hat{\mu})} \end{aligned}$$

for all $x, y \in X$.

Fix $u \in \pi_p^L(X, Y)$. The Farmer-Johnson Domination Theorem again provides us with a $\mu_0 \in \mathcal{P}(K)$ such that $\forall x, y \in X$,

$$\|u(x) - u(y)\| \leq \pi_p^L(u) \|j_{\mu_0}(x) - j_{\mu_0}(y)\|_{L_p(\mu_0)}.$$

Now it is enough to show that

$$\|j_{\mu_0}(x) - j_{\mu_0}(y)\|_{L_p(\mu_0)} \leq C \|j_\lambda(x) - j_\lambda(y)\|_{L_1(\lambda)}$$

for some $\lambda \in \mathcal{P}(K)$ and some constant C depending only on X .

Starting with μ_0 , we define $(\mu_n)_{n=0}^\infty$ in $\mathcal{P}(K)$ by setting $\mu_{n+1} = \hat{\mu}_n$, $n = 0, 1, 2, \dots$. Define $\lambda = \sum_{n=0}^\infty 2^{-n-1} \mu_n$. Then $\lambda \in \mathcal{P}(K)$. Since $1 < r < p$, we can produce $0 < \theta < 1$ with

$$\frac{1}{r} = \theta + \frac{1-\theta}{p}.$$

Using Hölder's Inequality, we get

$$\begin{aligned} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_r(\mu_n)} &= \left(\int_K |j_{\mu_n}(x)(x^\#) - j_{\mu_n}(y)(x^\#)|^r d\mu_n \right)^{1/r} \\ &\leq \left(\int_K |j_{\mu_n}(x)(x^\#) - j_{\mu_n}(y)(x^\#)| d\mu_n \right)^\theta \\ &\quad \cdot \left(\int_K |j_{\mu_n}(x)(x^\#) - j_{\mu_n}(y)(x^\#)|^p d\mu_n \right)^{\frac{1-\theta}{p}} \\ &= \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_1(\mu_n)}^\theta \cdot \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_p(\mu_n)}^{1-\theta}. \end{aligned}$$

Then,

$$\begin{aligned}
& \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_p(\mu_n)} \\
& \leq c \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_r(\mu_{n+1})} \\
& \leq c \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_1(\mu_{n+1})}^{\theta} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_p(\mu_{n+1})}^{1-\theta} \\
& \leq c \left(\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_1(\mu_{n+1})} \right)^{\theta} \\
& \quad \cdot \left(\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_p(\mu_{n+1})} \right)^{1-\theta} \\
& \leq c \left(\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_1(\mu_{n+1})} \right)^{\theta} \\
& \quad \cdot \left(2 \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_p(\mu_n)} \right)^{1-\theta}.
\end{aligned}$$

Thus,

$$\begin{aligned}
& \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_p(\mu_n)} \\
& \leq c^{1/\theta} \cdot 2^{\frac{1-\theta}{\theta}} \cdot \left(\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_{n+1}}(x) - j_{\mu_{n+1}}(y)\|_{L_1(\mu_{n+1})} \right) \\
& \leq c^{1/\theta} \cdot 2^{\frac{1-\theta}{\theta}} \cdot 2 \cdot \sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_1(\mu_n)}.
\end{aligned}$$

Note that

$$\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_1(\mu_n)} = \|j_{\lambda}(x) - j_{\lambda}(y)\|_{L_1(\lambda)}.$$

So,

$$\sum_{n=0}^{\infty} 2^{-n-1} \|j_{\mu_n}(x) - j_{\mu_n}(y)\|_{L_p(\mu_n)} \leq (2c)^{1/\theta} \|j_{\lambda}(x) - j_{\lambda}(y)\|_{L_1(\lambda)}.$$

In particular,

$$\frac{1}{2} \|j_{\mu_0}(x) - j_{\mu_0}(y)\|_{L_p(\mu_0)} \leq (2c)^{1/\theta} \|j_{\lambda}(x) - j_{\lambda}(y)\|_{L_1(\lambda)}.$$

We complete the proof by taking $C = 2 \cdot (2c)^{1/\theta}$. □

In the nonlinear setting of Grothendieck's Theorem, a metric tree plays the role of ℓ_1 . The reason is that every finite subset of a metric tree is contained in a larger finite subset with the lifting property. Here we say that a Lipschitz map $f : X \rightarrow Y$ has the lifting property if for every $\epsilon > 0$ and for every A -Lipschitz quotient $Q : Z \rightarrow Y$, there is a Lipschitz map $\tilde{f} : X \rightarrow Z$ so that $Q\tilde{f} = f$ and $\text{Lip}(\tilde{f}) \leq (1 + \epsilon)A \text{Lip}(f)$. A Lipschitz surjection f between the metric spaces X and Y is called a Lipschitz quotient if there is a $C > 0$ (the smallest such C , the co-Lipschitz constant, is denoted by $\text{coLip}(f)$) such that for every $x \in X$ and $r > 0$, $f(B_X(x, r)) \supseteq B_Y(f(x), \frac{r}{C})$. If a Lipschitz quotient f from X onto Y satisfies $\text{Lip}(f) \cdot \text{coLip}(f) \leq A$, then we say that f is an A -Lipschitz quotient. We refer the reader to [JLPS] for details.

Corollary 2.3. *Let X be a metric tree and let T be a Lipschitz map from X into ℓ_2 . Then, for any $1 < r \leq 2$,*

$$\pi_r^L(T) \leq B_{r^*} A_1^{-1} \text{Lip}(T),$$

where $1/r + 1/r^* = 1$ and B_{r^*}, A_1 are the respective Khintchine Constants. As a consequence,

$$\text{Lip}(X, \ell_2) = \pi_r^L(X, \ell_2) = \pi_1^L(X, \ell_2).$$

Proof. It follows from the definition of the Lipschitz p -summing norm,

$$\pi_p^L(T) = \sup_{\substack{K \subseteq X \\ K \text{ is finite}}} \pi_p^L(T|_K) \quad (\forall 1 \leq p < \infty).$$

Let K be a finite subset of X . Since every finite subset of a metric tree is contained in a larger finite subset of the metric tree with the lifting property, without loss of generality, we assume that K has the lifting property. Fix $1 < r \leq 2$. The formal identity $i_r : L_\infty[0, 1] \rightarrow L_r[0, 1] = (i_{r^*, 1})^*$, where $i_{r^*, 1} : L_{r^*}[0, 1] \rightarrow L_1[0, 1]$ is the formal identity. For any $1 \leq q < \infty$, we define $R_q : \ell_2 \rightarrow L_q[0, 1]$ by $R[(a_n)] = \sum_n a_n r_n$, where the r_n 's are the Rademacher functions. Then $R_1 : \ell_2 \rightarrow L_1[0, 1] = i_{r^*, 1} R_{r^*}$ is an into isomorphism. Thus $R_1^* : L_\infty[0, 1] \rightarrow \ell_2 = R_{r^*}^* i_r$ is r -summing and is a linear quotient. Moreover, R_1^* is an A_1^{-1} -Lipschitz quotient. Then, by the lifting property of K , we have for every $\epsilon > 0$ that there exists a Lipschitz map $S : K \rightarrow L_\infty[0, 1]$ such that $T|_K = R_1^* S$ and $\text{Lip}(S) \leq (1 + \epsilon) A_1^{-1} \text{Lip}(T|_K)$. Thus

$$\begin{aligned} \pi_r^L(T|_K) &\leq \pi_r^L(R_1^*) \text{Lip}(S) \\ &\leq (1 + \epsilon) A_1^{-1} \pi_r^L(R_{r^*}^* i_r) \text{Lip}(T) \\ &\leq (1 + \epsilon) A_1^{-1} \|R_{r^*}\| \cdot \text{Lip}(T) \\ &\leq (1 + \epsilon) A_1^{-1} \cdot B_{r^*} \cdot \text{Lip}(T). \end{aligned}$$

This means that $\pi_r^L(T) \leq A_1^{-1} B_{r^*} \cdot \text{Lip}(T)$. So, $\text{Lip}(X, \ell_2) \subseteq \pi_r^L(X, \ell_2) \subseteq \pi_2^L(X, \ell_2) \subseteq \text{Lip}(X, \ell_2)$. That is, $\text{Lip}(X, \ell_2) = \pi_2^L(X, \ell_2) = \pi_r^L(X, \ell_2)$. It follows from Theorem 2.2 that $\pi_2^L(X, \ell_2) = \pi_1^L(X, \ell_2) = \text{Lip}(X, \ell_2) = \pi_r^L(X, \ell_2)$. $\square$

Remark 2.1. For a linear operator T from ℓ_1 into ℓ_2 , Grothendieck's Theorem says that T is 1-summing and hence 2-summing. The 2-summing norm of T is bounded by a constant multiplying the operator norm of T . The least such constant is

known as the little Grothendieck's constant. Tomczak-Jaegermann [T] proved that the little Grothendieck's constant is A_1^{-1} . Corollary 2.3 gives a nonlinear analogue. However, we do not know whether A_1^{-1} is the best or not in the nonlinear setting.

3. ADJOINTS OF LIPSCHITZ MAPS

Theorem 3.1. *Let X be a pointed metric space and H be a Hilbert space. If $T : X \rightarrow H$ is a Lipschitz map with $T(0) = 0$ such that $T^\#|_{H^*}$ is q -summing for some $1 \leq q < \infty$, then T is Lipschitz 1-summing and $\pi_1^L(T) \leq A_1^{-1} B_q \cdot \pi_q(T^\#|_{H^*})$, where A_1 and B_q are the constants from Khintchine's Inequality.*

Proof. We divide the proof into two steps.

Step 1. First consider the case of a Lipschitz map $T : X \rightarrow \ell_2^n$.

Given $\{x_i\}_{i=1}^m$ and $\{y_j\}_{j=1}^m$ in X , we use Khintchine's Inequality to get

$$\begin{aligned} \sum_{j \leq m} \|Tx_j - Ty_j\| &= \sum_{j \leq m} \left(\sum_{k \leq n} |\langle Tx_j - Ty_j, e_k \rangle|^2 \right)^{1/2} \\ &\leq \sum_j A_1^{-1} \int_0^1 \left| \sum_k \langle Tx_j - Ty_j, e_k \rangle r_k(t) \right| dt \\ &= \sum_j A_1^{-1} \int_0^1 |\langle Tx_j, \sum_k r_k(t) e_k \rangle - \langle Ty_j, \sum_k r_k(t) e_k \rangle| dt \\ &= \sum_j A_1^{-1} \int_0^1 |\langle x_j, T^\# \left(\sum_k r_k(t) e_k \right) \rangle - \langle y_j, T^\# \left(\sum_k r_k(t) e_k \right) \rangle| dt \\ &= \sum_j A_1^{-1} \int_0^1 |\langle x_j, \sum_k r_k(t) T^\# e_k \rangle - \langle y_j, \sum_k r_k(t) T^\# e_k \rangle| dt \\ &\leq A_1^{-1} \int_0^1 \left\| \sum_k r_k(t) T^\# e_k \right\| \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| dt \\ &= A_1^{-1} \int_0^1 \left\| \sum_k r_k(t) T^\# e_k \right\| dt \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)|. \end{aligned}$$

Since $T^\#|_{(\ell_2^n)^*} : (\ell_2^n)^* \rightarrow X^\#$ is linear and q -summing, it follows from Pietsch's Domination Theorem that there is a Borel regular probability measure μ on $B(\ell_2^n)$ such that for all $x^* \in (\ell_2^n)^*$, we have

$$\|T^\# x^*\| \leq \pi_q(T^\#|_{(\ell_2^n)^*}) \left(\int_{B(\ell_2^n)} |\langle x, x^* \rangle|^q d\mu(x) \right)^{1/q}.$$

Thus

$$\begin{aligned}
 & \left(\int_0^1 \left\| \sum_k r_k(t) T^\# e_k \right\|^q dt \right)^{1/q} \\
 & \leq \pi_q(T^\#|_{(\ell_2^n)^*}) \left[\int_0^1 \left(\int_{B(\ell_2^n)} |\langle x, \sum_k r_k(t) e_k \rangle|^q d\mu(x) \right) dt \right]^{1/q} \\
 & = \pi_q(T^\#|_{(\ell_2^n)^*}) \left[\int_{B(\ell_2^n)} \left(\int_0^1 |\langle x, \sum_k r_k(t) e_k \rangle|^q dt \right) d\mu(x) \right]^{1/q} \\
 & \leq \pi_q(T^\#|_{(\ell_2^n)^*}) \left[\int_{B(\ell_2^n)} B_q^q \left(\sum_k |\langle x, e_k \rangle|^2 \right)^{q/2} d\mu(x) \right]^{1/q} \\
 & \leq \pi_q(T^\#|_{(\ell_2^n)^*}) B_q .
 \end{aligned}$$

Step 2. Fix $\{x_j\}_{j=1}^m$ and $\{y_j\}_{j=1}^m$ in X . Identifying the span of the Tx_j 's and Ty_j 's with ℓ_2^n for the appropriate n , let $P \in \mathcal{L}(H)$ be the orthogonal projection onto this span. Then

$$\begin{aligned}
 \sum_{j \leq m} \|Tx_j - Ty_j\| &= \sum_{j \leq m} \|PTx_j - PTy_j\| \\
 &\leq A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \int_0^1 \left\| \sum_k r_k(t) (PT)^\#(e_k) \right\| dt \\
 &\leq A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \\
 &\quad \cdot \left(\int_0^1 \left\| \sum_k r_k(t) (PT)^\#(e_k) \right\|^q dt \right)^{1/q} \\
 &\leq A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \pi_q((PT)^\#|_{(\ell_2^n)^*}) B_q \\
 &= A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \pi_q((T^\#|_{H^*})(P^\#|_{(\ell_2^n)^*})) B_q \\
 &\leq A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \pi_q(T^\#|_{H^*}) \|P^*\| B_q \\
 &\leq A_1^{-1} \sup_{f \in B_{X^\#}} \sum_j |f(x_j) - f(y_j)| \pi_q(T^\#|_{H^*}) B_q .
 \end{aligned}$$

Thus, $\pi_1^L(T) \leq A_1^{-1} B_q \pi_q(T^\#|_{H^*})$. $\square$

Remark 3.1. As mentioned in the introduction, a linear operator T from a Banach space X to a Hilbert space H is 1-summing if T^* is q -summing for some $1 \leq q < \infty$. We generalize this result to the nonlinear case. To see that Theorem 3.1 is really a generalization, one only needs to notice that the restriction of $T^\#$ to H^* is exactly

T^* when T is linear. If we assume $T^\#$ itself is q -summing, then by considering the Lipschitz free spaces $\mathcal{F}(X)$ and $\mathcal{F}(H)$ and using the linear results, it is easy to conclude that T is Lipschitz 1-summing. So the importance of our theorem is the restriction of $T^\#$ to H^* .

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